01/11/2016

F.M

Fluid Mechanics

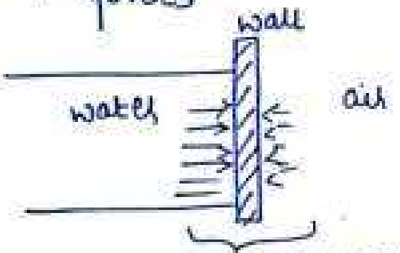
(Liq. & Gas) — Motion & Static

Syllabus

① Fluid Properties → ρ, ω, S, μ, $K \rightarrow \beta$, σ (Liq) → capillary

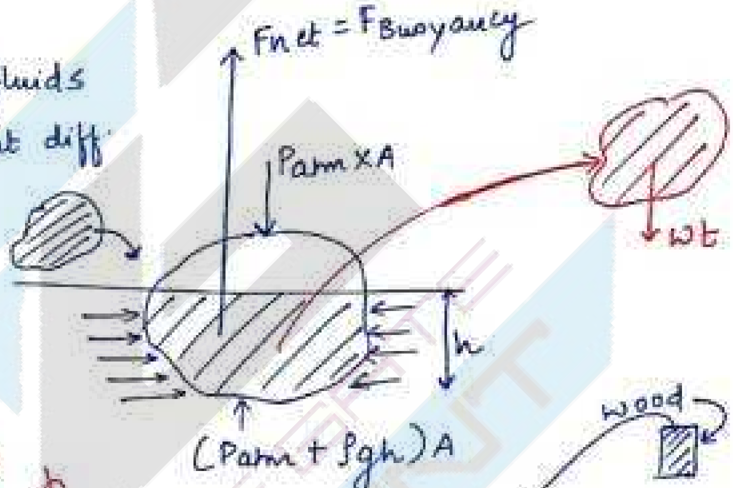
② Pressure measurement

③ Hydrostatic forces

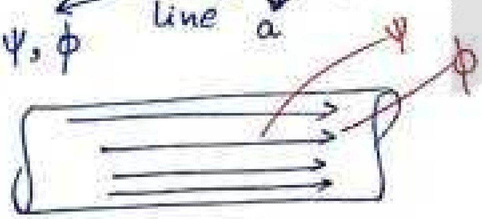


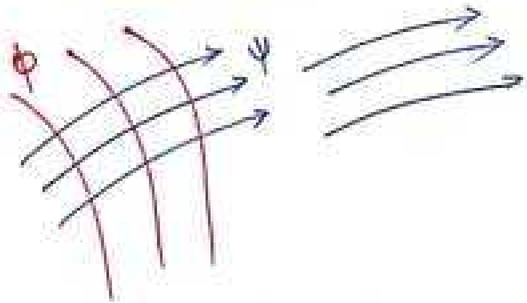
only different fluids or same fluid at diff. levels.

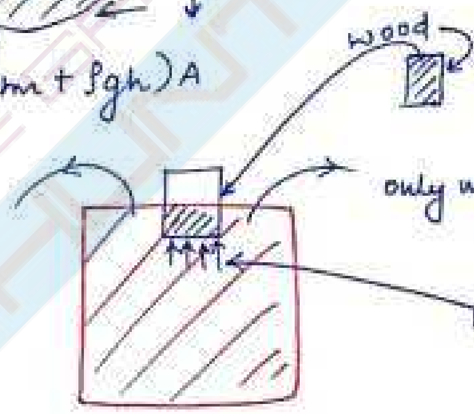
④ Buoyancy



⑤ Fluid Kinematics → ψ, ϕ, line, a, V

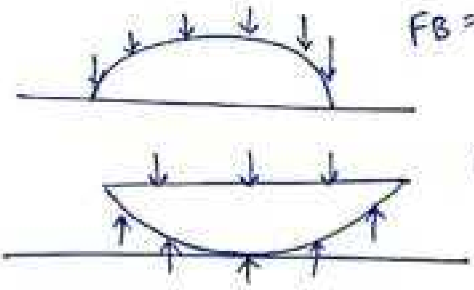






only magnitude defines Buoyancy

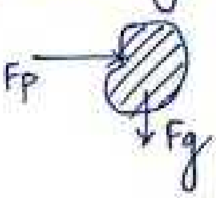
Buoyancy



$F_B = 0$

$F_B \neq 0$

⑥ Fluid dynamics



$\boxed{F_p + F_g = F_v}$

Euler's analysis

$\boxed{F_p + F_g + F_v = F_i}$ Navier - stoke's eqn.

⑦ Flow through pipes.

⑧ Laminar pipe flow

⑨ turbulent —"—————.

⑩ Boundary layer Theory × (ESE - 2017)

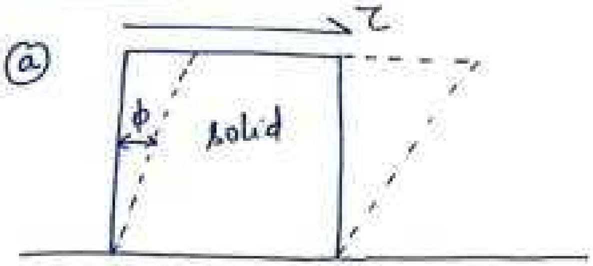
⑪ Dimensional Analysis. $T \propto (l, g)$ × (ESE - 2017)

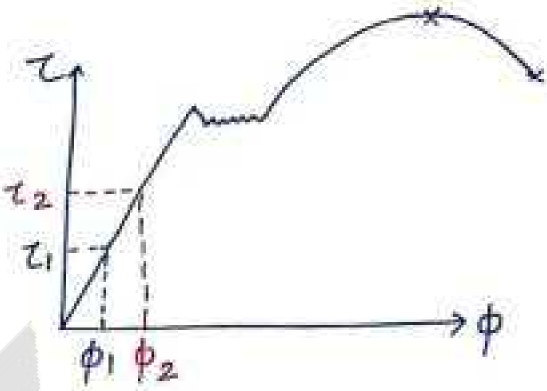
CHAPTER - 1
FLUID PROPERTIES

* **fluid Mechanics** :- It is the science which deals the fluid (Liquids and gases) in rest or in motions.

* **FLUID** :-

(a)





shear stress ← $\tau \propto \phi$ → shear Strain

$\boxed{\tau = G\phi}$

modulus of Rigidity ← $\boxed{G = \frac{\tau}{\phi}}$

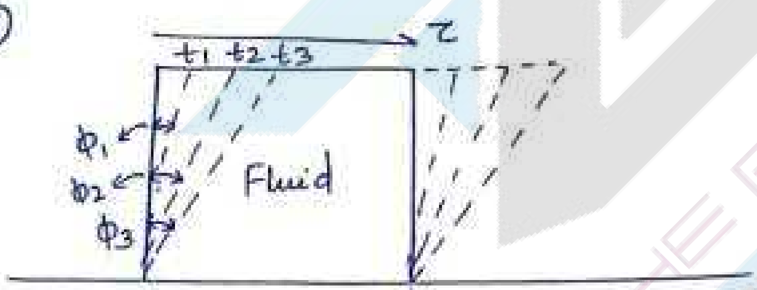
$\tau \neq 0$

$\phi \to 0$ Rigid

$G \to \infty$

$\boxed{\phi} \propto \frac{1}{\boxed{G}}$

(b)



$\boxed{\tau = \mu \frac{d\phi}{dt}}$

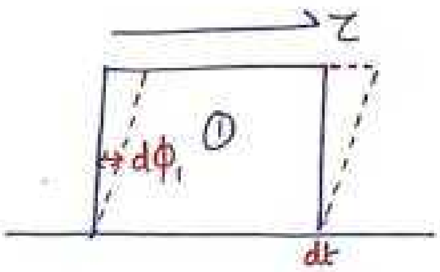
$\mu = \frac{\tau}{\left(\frac{d\phi}{dt}\right)}$

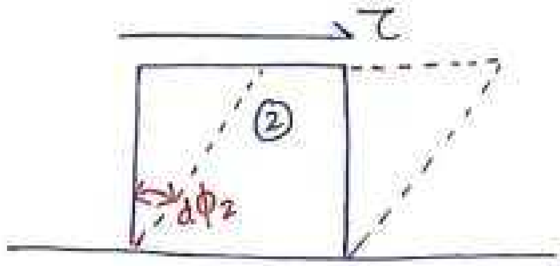
$\tau \propto \frac{\phi}{t}$

$\tau \propto \frac{d\phi}{dt}$ → Rate of shear strain

$\mu \propto \frac{1}{\left(\frac{d\phi}{dt}\right)}$

Ex :-





$\frac{d\phi_1}{dt} < \frac{d\phi_2}{dt}$

$\boxed{\mu_1 > \mu_2}$

$\boxed{\text{Fluidity} = \frac{1}{\mu}}$

$\therefore \boxed{F_1 < F_2}$

A fluid is a substance which is capable of flowing or deforming under the action of shear force (However the small force may be)

In solids, the deformation is almost constant for the given stress (shear)

In fluids, the deformation changes with time, so In fluids rate of deformation is more important than deformation.

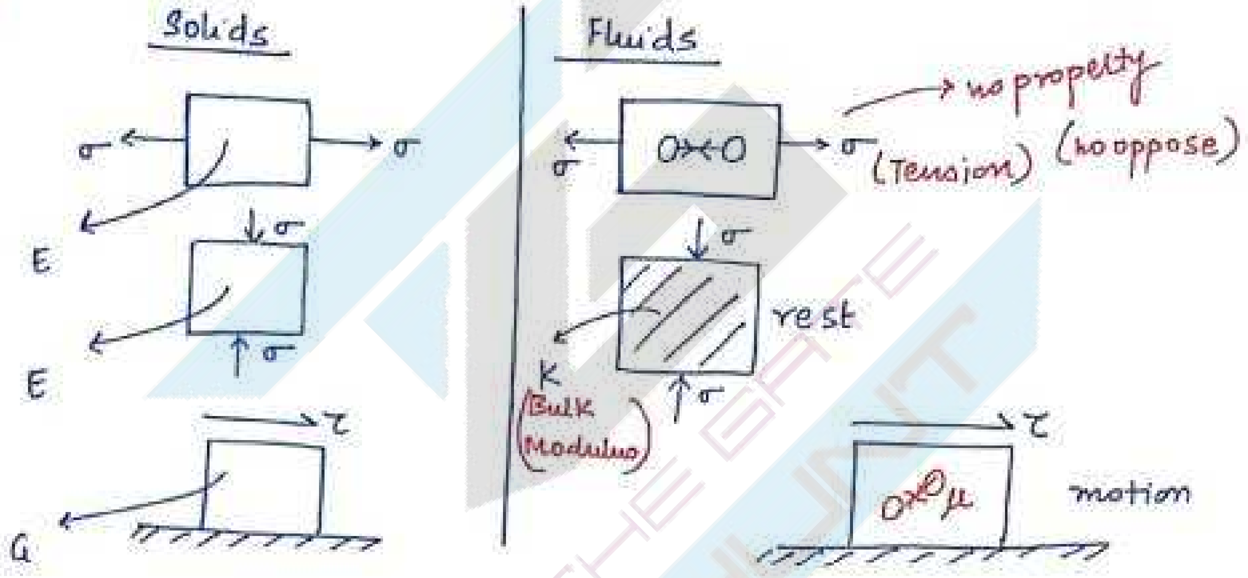
Note :-

Workbook

Q29 → UPSC → (a)

Gate → (c) → shear

Note.....

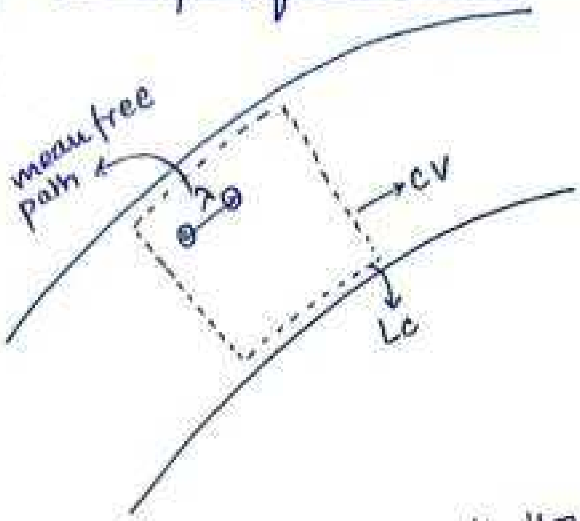


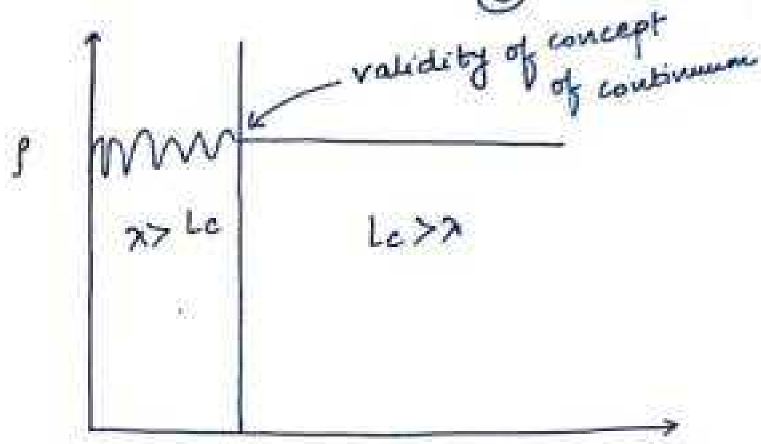
(1) Solids can resist tensile, compressive and shear forces under static condition.

Fluids have negligible resistance for Tensile loads and they can resist compressive forces under static condition but they can resist shear forces under dynamic condition due to viscosity of the fluid.

So the property known as Viscosity observed under dynamic condition.

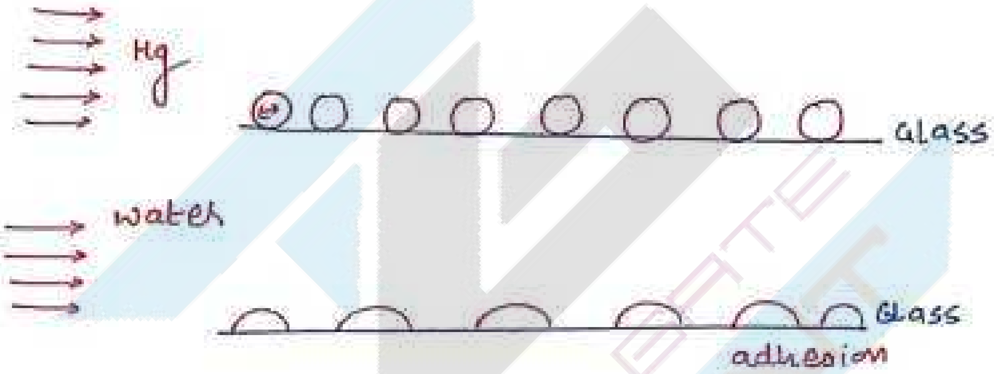
* Concept of Continuum :-





$\rho = \frac{m}{V}$

* No slip phenomenon :-



When viscous fluid flows over the solid surface, Bulk of fluid molecules acquires the Velocity of the surface. This sticking phenomenon is known as No slip phenomenon. It is defined by concept of continuum.

It is neither due to adhesion of liquid and solid surface nor due to surface roughness.

* Fluid Properties :- ① Density (ρ) → It is defined as mass per unit volume at a specified pressure and temperature in the fluid.
→ mass density

$$\rho = \frac{m}{V} \left(\frac{Kg}{m^3}\right)$$

$4°C \rightarrow 1\,atm \rightarrow \rho_w \rightarrow 1000\ Kg/m^3$

$20°C \rightarrow 1\,atm \rightarrow \rho_w \rightarrow 998\ Kg/m^3$

$20°C \rightarrow 100\,atm \rightarrow \rho_w \rightarrow 1003\ Kg/m^3$

] 0.5% change → hence incompressible

Treated as

but compressible hota hai

ओ ρ change

$20°C \rightarrow 1\,atm \rightarrow \rho_{air} \rightarrow 1.23\,kg/m^3$

$0°C \rightarrow 100\,KPa$ $\quad \rho_{air} = 1.27\,Kg/m^3$

⇓

S.T.P.

② Specific Weight / weight density (ω) :-

$$\omega = \frac{Wt}{Vol^m} \left(\frac{N}{m^3}\right)$$

$$\omega = \frac{mg}{V}$$

$$\boxed{\omega = \rho g}$$

$\omega_{water} = \rho_w \times g$

$= 1000 \times 9.81$

$$\boxed{\omega_w = 9810 \left(N/m^3\right)}$$ S.I.

$1\,kgf = 9.81\,N$

$\omega_w = 1000 \times (9.81N)/m^3$

$$\boxed{\omega_w = 1000\ kgf/m^3}\ \underline{M.K.S.}$$

③ Relative Density (R.D) :-

$$\boxed{R.D. = \frac{\rho_1}{\rho_2}}$$

④ Specific gravity (S.G.) :-

$$(S.G.)_f = \frac{\rho_f}{\rho_{standard\ fluid}}$$

Liq. → $\rho_w = 1000\,kg/m^3$

gas → $\rho_{air} = 1.27\,kg/m^3$ (S.T.P.)

Liq :- $S_f = \frac{\rho_f}{\rho_w}$

$$S_f = \frac{\rho_f . g}{\rho_w . g}$$

$$\boxed{S_f = \frac{\omega_f}{\omega_w}}$$

Ex :- Hg :- $S_{Hg} = 13.6$

$$\frac{\omega_{Hg}}{\omega_w} = S_{Hg}$$

$$\boxed{\omega_{Hg} = 13.6 \times \omega_w}$$ → 13.6 times heavier than water

$$\boxed{\omega_{Hg} = 13.6 \times 9810}\ N/m^3$$

Ex :- $\boxed{S_{wood} = 0.6}$

S_{iron} → wrt (S_w)

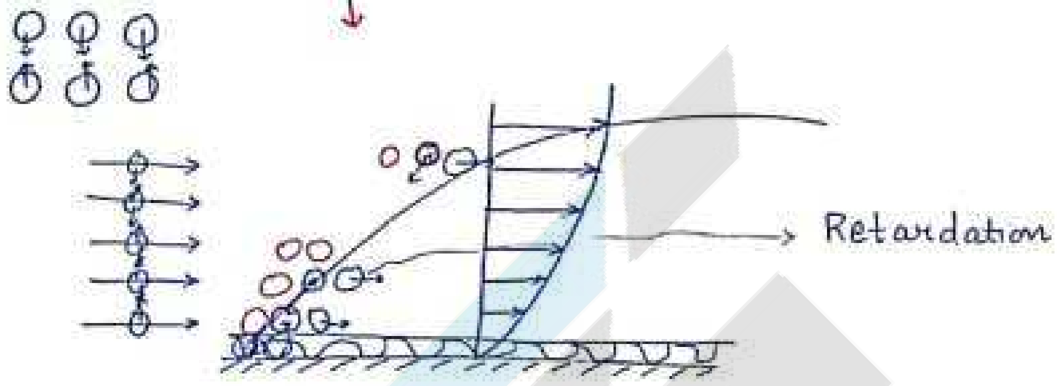
$$\boxed{S_{gasoline} = 0.75}$$

⑤ Viscosity of the Fluid :- The property which offers the resistance b/w 2 adjacent layers is known as viscosity.

viscosity is due to :-

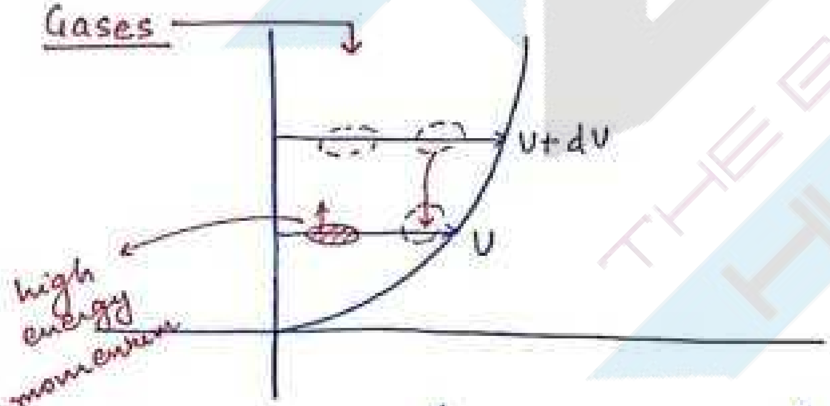
ⓐ cohesive forces between the layers (in liquids).

ⓑ due to molecular momentum transfer in ⊥r direction of the flow (in gases).



$T\uparrow \longrightarrow$ cohesive forces$\downarrow \longrightarrow \mu_{liq}\downarrow$

Gases



$V_{rms} = \sqrt{\frac{3KT}{m}}$

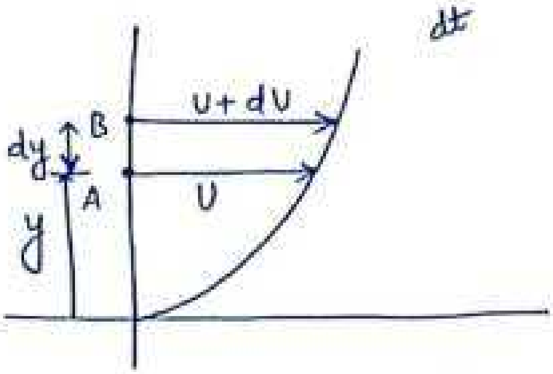
$V_{rms} \propto \sqrt{T}$

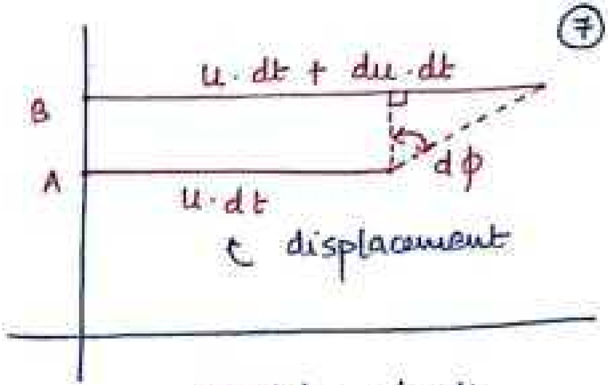
$\mu_{gas}\uparrow \propto$ randomness$\uparrow \propto V_{rms} \propto \sqrt{T}$

$\uparrow \boxed{\mu_{gas} \propto \sqrt{T}} \uparrow$

Note :-

dt





$$\tan d\phi = \frac{du\,dt}{dy}$$

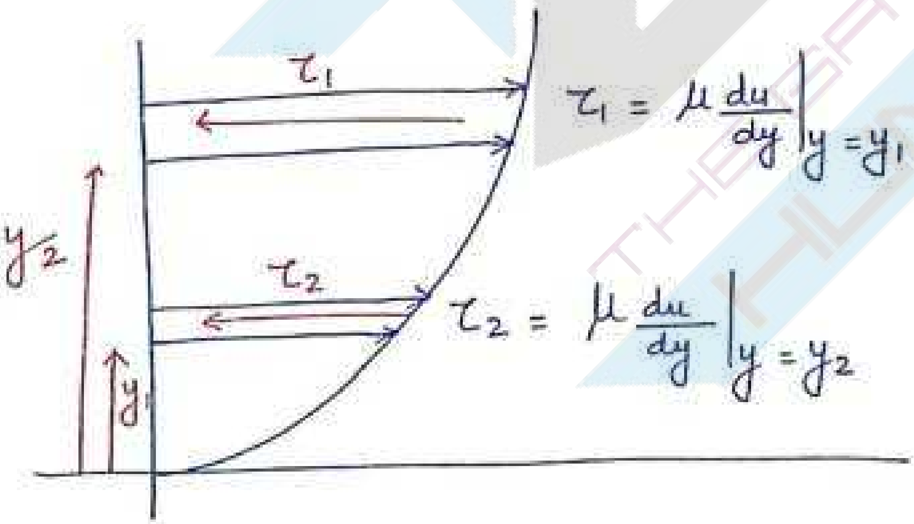
small angle

$$d\phi = \frac{du \cdot dt}{dy}$$

Rate of shear strain $\leftarrow$ $\boxed{\dfrac{d\phi}{dt} = \dfrac{du}{dy}}$ $\rightarrow$ Velocity gradients

$$\tau = \mu \frac{d\phi}{dt}$$

$\boxed{\tau = \mu \dfrac{du}{dy}}$ $\rightarrow$ only for a layer not Region



$$\tau_1 = \mu \frac{du}{dy}\bigg|_{y = y_1}$$

$$\tau_2 = \mu \frac{du}{dy}\bigg|_{y = y_2}$$

Units of Viscosity :-

① Viscosity / Dynamic Viscosity / Absolute Viscosity :-

$$\tau = \mu \frac{du}{dy}$$

forces से define property since (N) comes.

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} \quad \frac{N/m^2}{\left(\frac{m/s}{m}\right)}$$

$$\boxed{\mu \longrightarrow \frac{Ns}{m^2}}$$

$$\frac{Ns}{m^2} = \frac{Kg\,(m/s^2)\cdot s}{m^2}$$

(m·a)

mass not force

$$* \quad \boxed{\frac{Ns}{m^2} = \frac{Kg}{m \cdot s}} \quad (S.I.)$$

$$\frac{1\,Ns}{m^2} = \frac{9.81\,Ns}{9.81\,m^2}$$

kgf

$$* \quad \boxed{\frac{1\,Ns}{m^2} = \frac{1}{9.81}\,\frac{kgf - sec}{m^2}}$$

(kg force) ⟶ means force

C.G.S. :-

$$\boxed{\frac{g}{cm - sec} = \text{Poise}}$$

$$\frac{N}{m^2} = \text{Pascal} = \text{Pa}$$

$$\boxed{\text{Pa-sec} = \frac{kg}{m\text{-}s}}$$

$$\frac{10^{-3}\,Kg}{10^{-2}\,m\text{-}s} = \text{Poise}$$

$$\boxed{0.1\,\frac{Kg}{m\text{-}s} = \text{Poise}}$$

$$0.1\,\frac{Ns}{m^2} = \text{Poise}$$

v.v.v. imp

$$\boxed{\text{Poise} = 0.1\ \text{Pa-sec}}$$

Poise = 0.1 Pa sec

μ

(S.I.)

$\boxed{\frac{Ns}{m^2} = \frac{Kg}{m\text{-}s}}$

(C.G.S.)

$\boxed{\frac{dyne\text{-}sec}{cm^2} = \frac{g}{cm\text{-}s} = \text{Poise}}$

$\boxed{1\,dyne = 10^{-5} N}$

* 20°C → μ_w = 1 Centipoise
$= 10^{-2}$ Poise
$= 10^{-3}$ Pa-sec

* $\mu_{air} = 2\times10^{-5}$ Pa-sec

$$\frac{\mu_w}{\mu_{air}} = \frac{10^{-3}}{2\times10^{-5}} = \frac{100}{2} = 50$$

$$\boxed{\mu_w = 50\,\mu_{air}}$$

② Kinematic Viscosity / Momentum diffusivity (ν) :-

$$\boxed{\nu = \frac{\mu}{\rho}} \quad \frac{\cancel{kg}/ms}{\cancel{kg}/m^3}$$

$$\boxed{\nu = \frac{\mu}{\rho}} \quad \frac{m^2}{s}$$

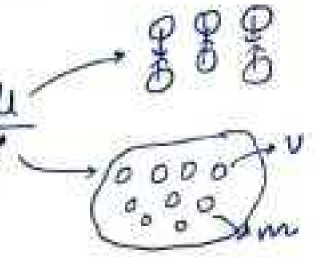
C.G.S.

$$\frac{cm^2}{s} = \text{stoke}$$

$$\frac{10^{-4}\, m^2}{s} = \text{stoke}$$

$$\boxed{\frac{m^2}{s} = 10^4 \text{ stoke}}$$

$$\nu = \frac{\mu}{\rho}$$



* $\mu_w = 10^{-3}$ Pa-s $\boxed{\mu_w > \mu_{air}}$

$\rho_w = 1000$

$\mu_{air} = 2\times10^{-5}$ Pa-s

$\rho_{air} = 1.23$

$$\nu_w = \frac{\mu_w}{\rho_w} = \frac{10^{-3}}{1000} = 10^{-6}\,\frac{m^2}{s}$$

$$\nu_{air} = \frac{2\times10^{-5}}{1.23} = 1.67\times10^{-5}\ m^2/s$$

$$\boxed{\nu_w < \nu_{air}}$$

WB

28 a ✓

statement b ✓

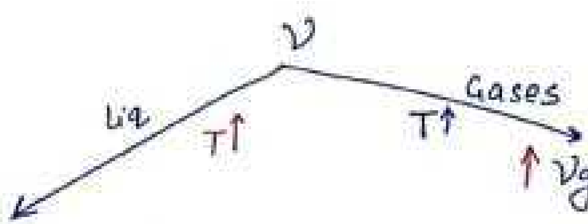
Ans → b ✓

Effect of Temperature and pressure on Viscosity :-

*

$T\uparrow \rightarrow \mu_{Liq}\downarrow$

$\uparrow \mu_{gas} \propto \sqrt{T}\uparrow$



$$\uparrow \nu_{gas} = \frac{\mu_{gas}\uparrow}{\rho_{gas}\downarrow}$$

$$\downarrow \nu_{Liq} = \frac{\mu_{Liq}\downarrow}{\rho_{Liq}\ \times}$$

$P = \rho R T$

$\rho = \frac{P}{RT}$

$\rho \propto \frac{1}{T} \rightarrow$ abs.

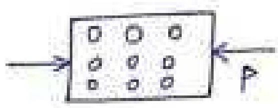
so $V_{gas} \propto \frac{\sqrt{T}}{(1/T)}$

$\boxed{\uparrow V_{gas} \propto T^{3/2} \uparrow} \rightarrow$ Abs

T ↑	Liq	Gas
μ	↓	↑
ν	↓	↑

* μ

Liq, P↑ ← μ → gases, P↑



incompressible

$\mu_{liq} \neq f(P)$

$\rho \neq f(P)$

$\mu_{gas} = \sqrt{\frac{3KT}{m}}$

$\mu_{gas} \propto \sqrt{T}$

$\mu_{gas} \neq f(P) \rightarrow$ moderate

But high pres में gas में randomness आती है।

$\rho = \frac{P}{RT}$

$\rho \propto (P)$ ✓

ν

$P\uparrow \quad\quad P\uparrow$

$\nu_{liq} = \dfrac{\mu_{liq}^{\times}}{\rho_{liq}^{\times}} \qquad\qquad \downarrow \nu_{air} = \dfrac{\mu_{air}^{\times}}{\rho_{air}\uparrow}$

incompressible

$\nu_{liq} \neq f(P)$

$P\uparrow$	Liq	Gases
μ	×	×
ν	×	$\downarrow$

WB

② → ⓐ

⑤ → ⓓ

⑥ → ⓐ if option's अलग then

$\nu_{gas} \propto T^{3/2}$

$$\frac{\nu_{70^\circ}}{\nu_{20^\circ}} = \left(\frac{70+273}{20+273}\right)^{3/2}$$

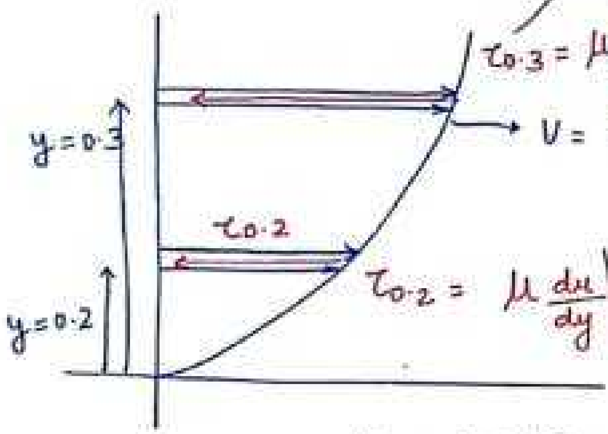
$$\nu_{70^\circ} = 1.6 \times 10^{-5}\left[\frac{70+273}{20+273}\right]^{3/2}$$

$$\approx 2.02 \times 10^{-5}$$

⑦ → b ✓

02/11/2016 WB (31) $u = \frac{3}{4}y - y^2$ $\mu = 0.84\ \frac{Ns}{m^2}$ (13)

$\tau_{0.3m} \rightarrow \tau_{0.2m}$



$\tau_{0.3} = \mu \frac{du}{dy}\Big|_{y=0.3}$

$\tau_{0.2} = \mu \frac{du}{dy}\Big|_{y=0.2}$

$\mu = 0.84\ \frac{Ns}{m^2}$

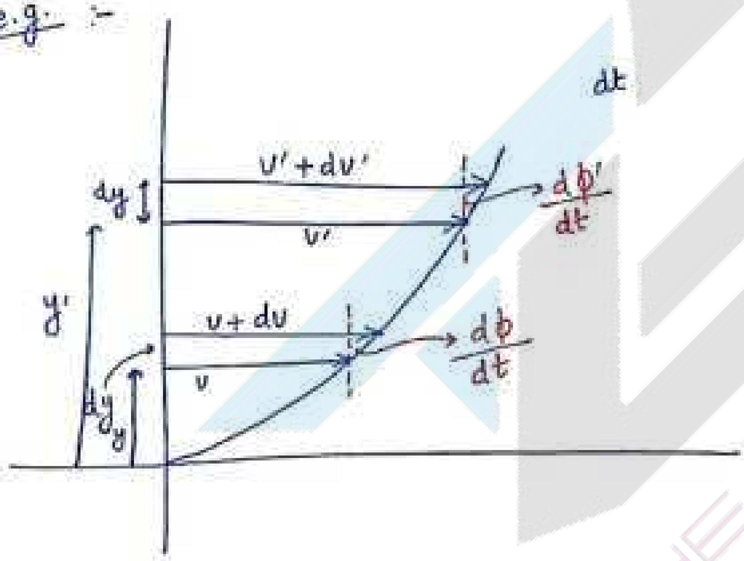
$\tau_{0.3} = N\, \tau_{0.2}$

$\cancel{\mu} \frac{du}{dy}\Big|_{y=0.3} = N \cancel{\mu} \frac{du}{dy}\Big|_{y=0.2}$

$\left[\frac{3}{4} - 2y\right]_{0.3} = N\left[\frac{3}{4} - 2\times y\right]_{0.2}$

$\boxed{N = 0.428}$

e.g. :-

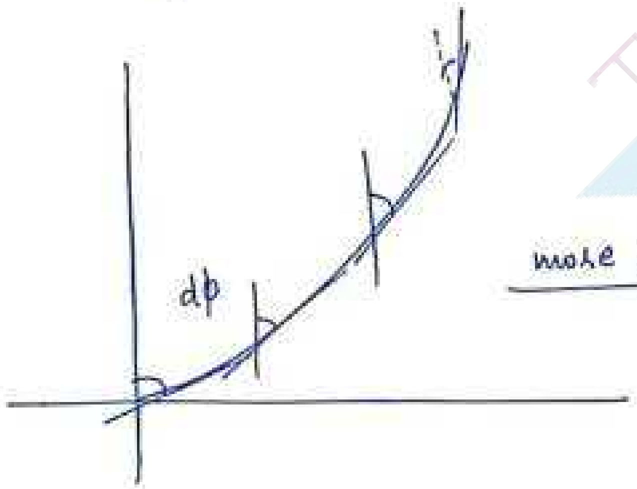


$\frac{d\phi'}{dt} < \frac{d\phi}{dt}$

$\frac{du'}{dy} < \frac{du}{dy}$

$\mu \frac{du'}{dy} < \mu \frac{dv}{dy}$

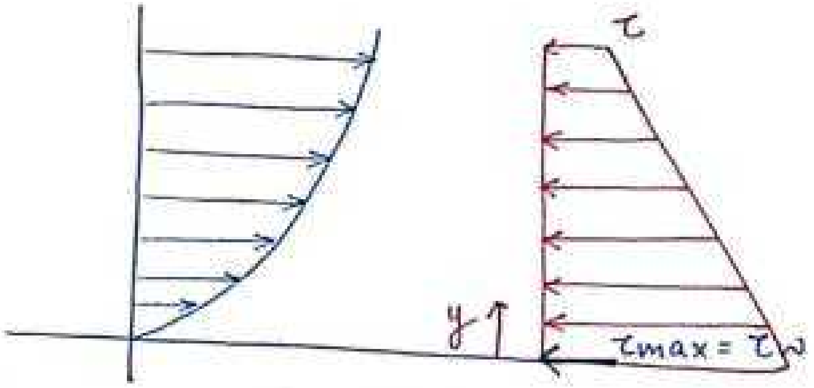
$\boxed{\tau' < \tau}$



more slope on bottom → wall shear stress

↓

max^m. stress

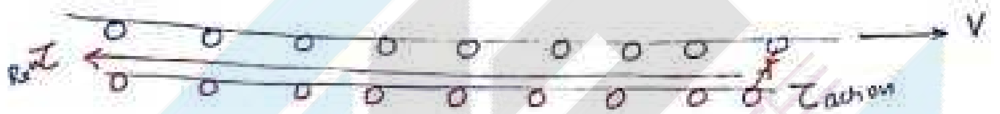


$$V = \frac{3}{4}y - y^2 \qquad \tau = \mu\frac{dv}{dy}$$

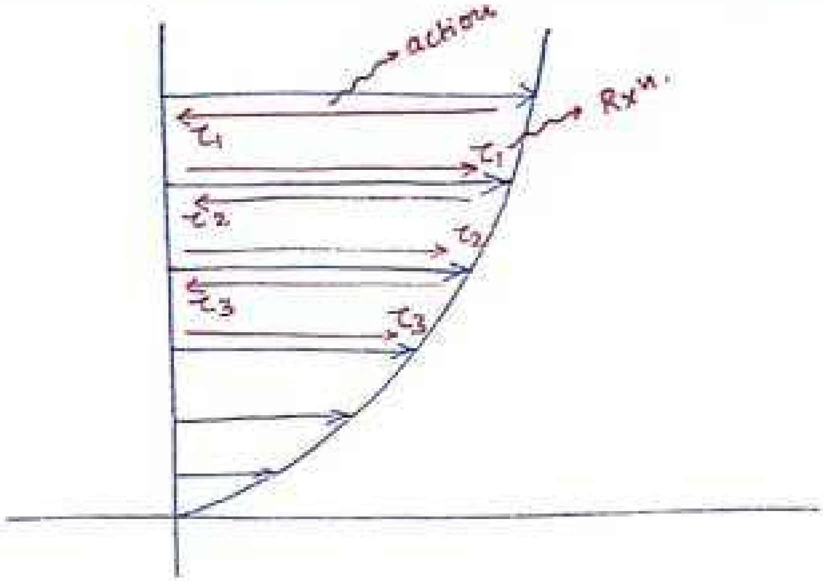
$$\tau = \mu\left[\frac{3}{4} - 2y\right]$$

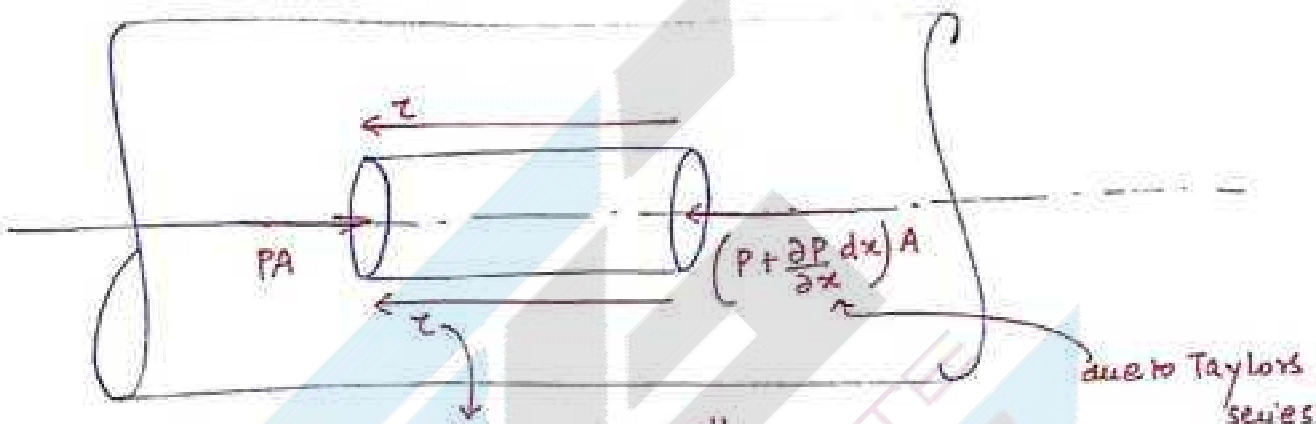
$$y = c - mx$$

* FREE BODY DIAGRAM :-



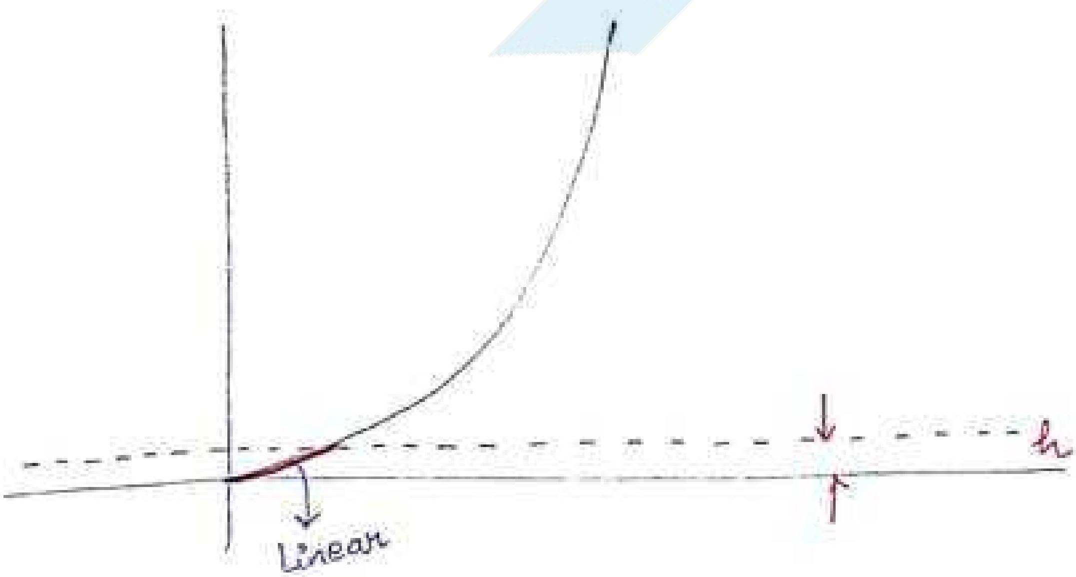
[fix वाली layer ऊपर वाली को forward push करेगी]

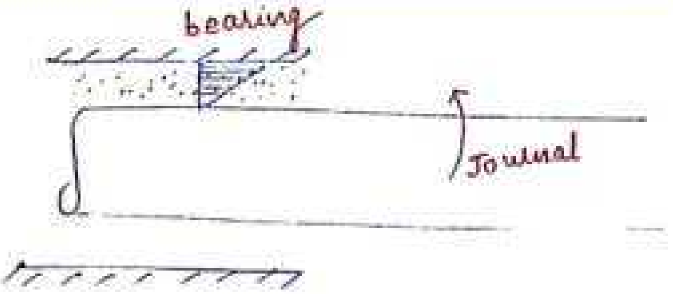


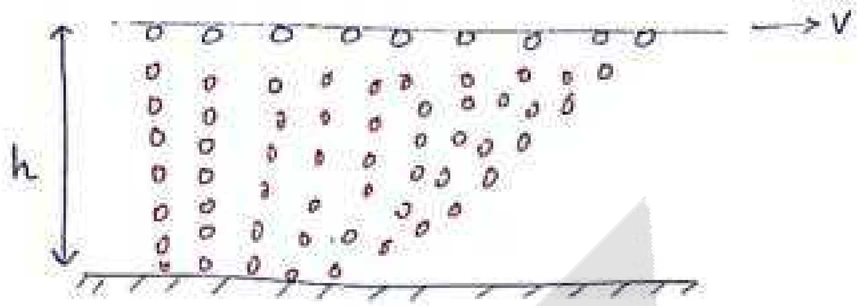


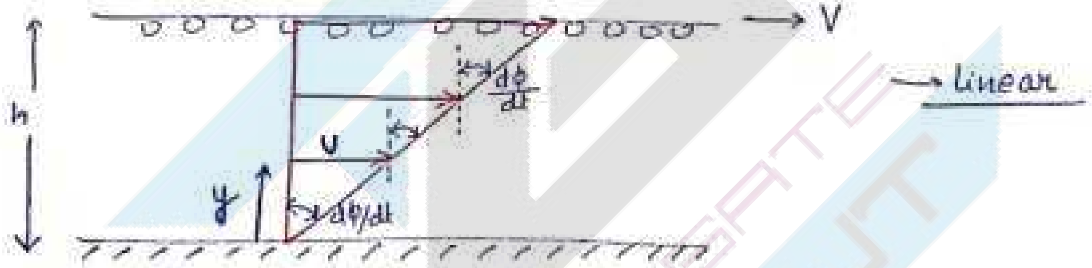
(Pipe $\frac{n}{H}$)

* LINEAR VELOCITY PROFILE ASSUMPTION :-









Similar Δs

$$\frac{u}{y} = \frac{V}{h}$$

$$\boxed{u = \frac{Vy}{h}}$$

✓ $u = a + by$ — (A)

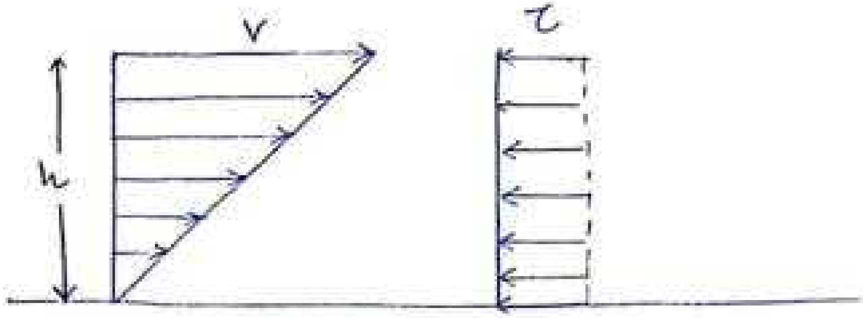
✓ $y = 0$, $u = 0$, $a = 0$

✓ $y = h$, $u = V$

✓ $V = 0 + bh$

✓ $b = V/h$

$$\boxed{u = \frac{V}{h}y}$$

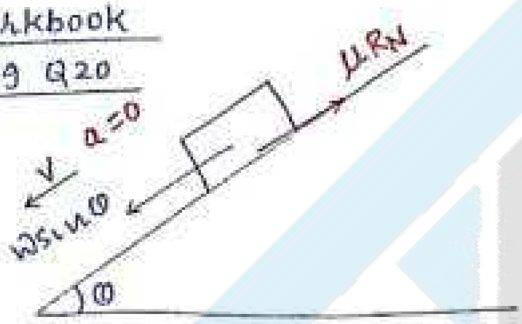


$$U = \frac{V}{h} y \qquad \tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{V}{h}$$

$\tau \neq f(y)$ ∵ same overall

Workbook
Pg 9 Q20

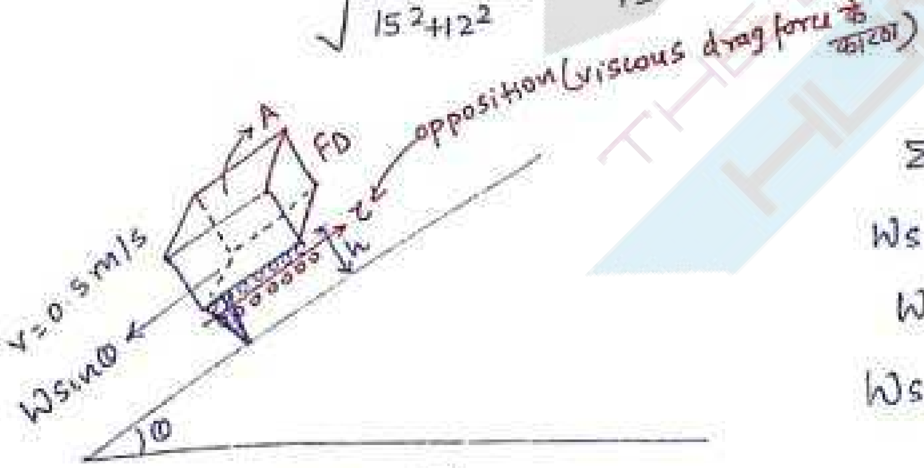


$$\Sigma F_{net} = ma \to 0$$

$$W\sin\theta - \mu R_N = 0$$

$$\boxed{W\sin\theta = \mu R_N}$$

$$\sin\theta = \frac{5}{\sqrt{15^2+12^2}} = \frac{5}{13}$$



$\sin\theta = 5/13$

$$\Sigma F_{net} = 0$$

$$W\sin\theta - F_D = 0$$

$$W\sin\theta = \tau A$$

$$W\sin\theta = \mu \frac{du}{dy} A$$

$$\left\{\tau = \mu \frac{du}{dy}\right\}$$

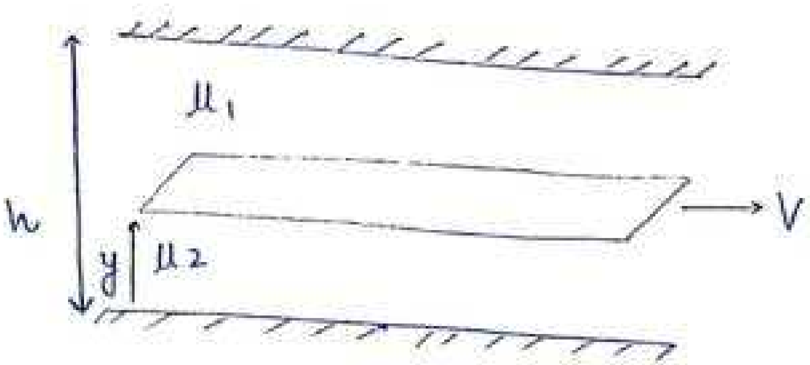
$$130 \times \frac{5}{3} = \mu \times \frac{0.5}{0.5\times10^{-2}} \times 1$$

$$\left\{\because \tau = \mu V/h\right\}$$

$$\boxed{\mu = 0.5\,Ns/m^2}$$

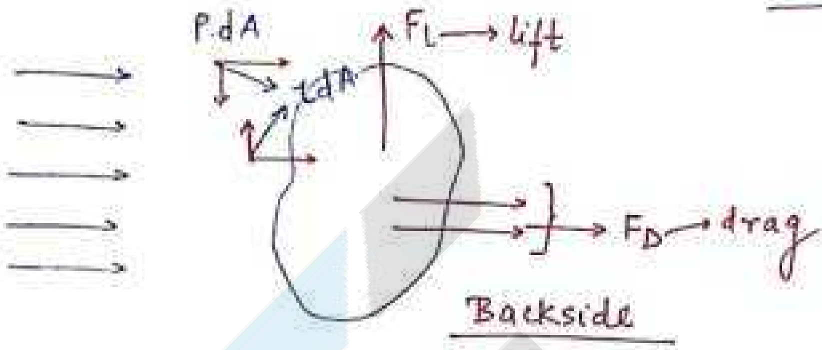
WB
38

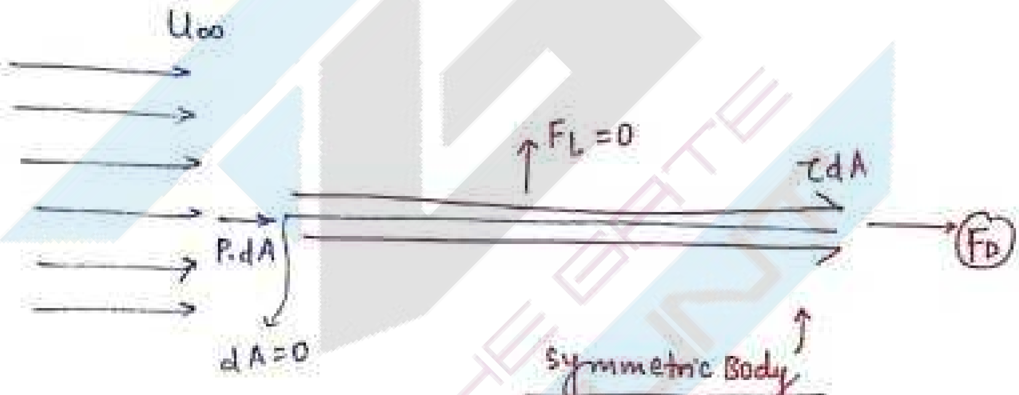
Thin plate → no pressure drag

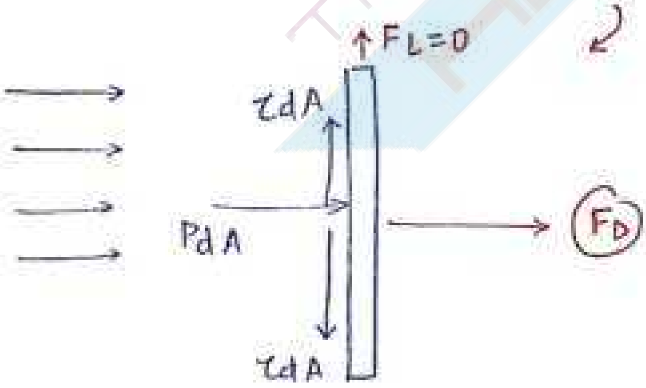


Thin → means
$PdA = 0$
$\because dA = 0.$

ans is only shear drag

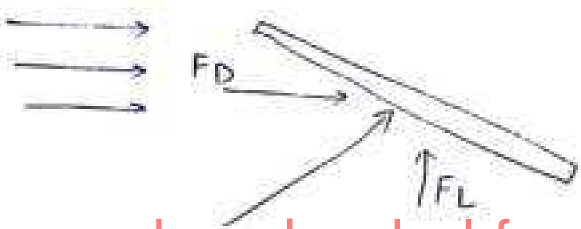




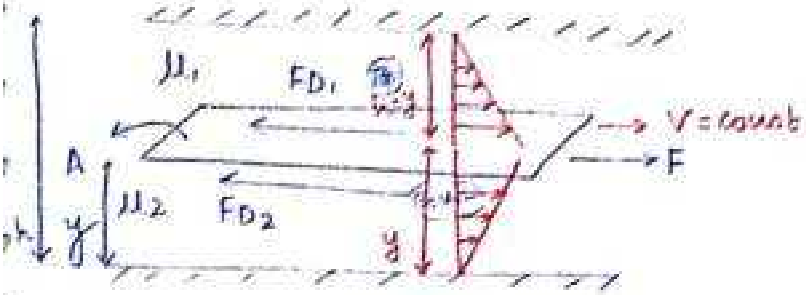


τdA = 0

(Drag रहेगा
मगर lift
न रहे)



const v से move → F-0



$\Sigma F_{net} = ma^0$ ($\because v = const$)

(19)

$F - [F_{D_1} + F_{D_2}] = 0$

$F = F_{D_1} + F_{D_2}$

$F = \tau_1 A + \tau_2 A$

$$\left[\tau = \mu \frac{V}{h}\right]$$

$$F = \mu_1 \frac{V}{(h-y)} A + \mu_2 \frac{V}{y} A$$

$$\frac{dF}{dy} = 0 \text{ (for min)}$$

$$\frac{dF}{dy} = \mu_1 \frac{V}{(h-y)^2} A + \left(-\mu_2 \frac{V}{y^2}\right) A$$

$$\frac{dF}{dy} = 0 = \mu_1 \frac{\cancel{V}}{(h-y)^2} \cancel{A} - \mu_2 \frac{\cancel{V}}{y^2} \cancel{A}$$

$$\frac{y^2}{(h-y)^2} = \frac{\mu_2}{\mu_1}$$

$$\frac{y}{h-y} = \frac{\sqrt{\mu_2}}{\sqrt{\mu_1}}$$

$$\frac{h}{y} - 1 = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\boxed{y = \frac{\sqrt{\mu_2}}{\sqrt{\mu_1} + \sqrt{\mu_2}} h}$$

if $\mu_1 = \mu_2 = \mu$

$$\boxed{y = h/2}$$

Q Find y where $F_{D_1} = F_{D_2}$

$$\tau_1 \cancel{A} = \tau_2 \cancel{A}$$

$$\mu_1 \frac{\cancel{V}}{(h-y)} = \mu_2 \frac{\cancel{V}}{y}$$

$$\frac{h-y}{y} = \frac{\mu_1}{\mu_2}$$

$$\boxed{y = \frac{\mu_2}{(\mu_1 + \mu_2)} h}$$

Q if $\mu_1 > \mu_2$ → minm के लिए μ_2 की तरफ shifting होगा।

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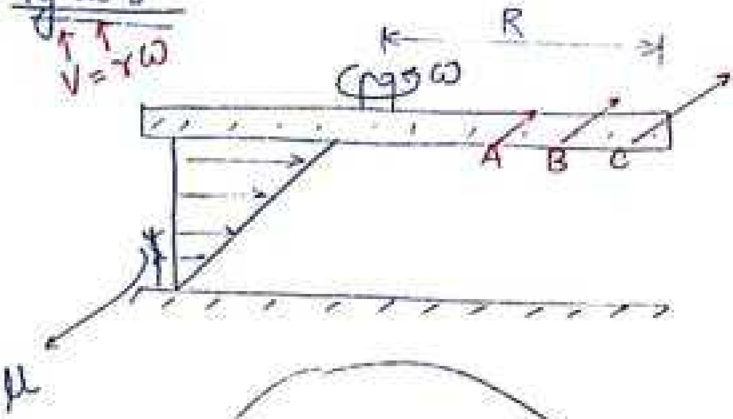
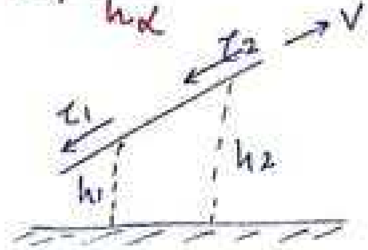
WB (17) Pg no 8

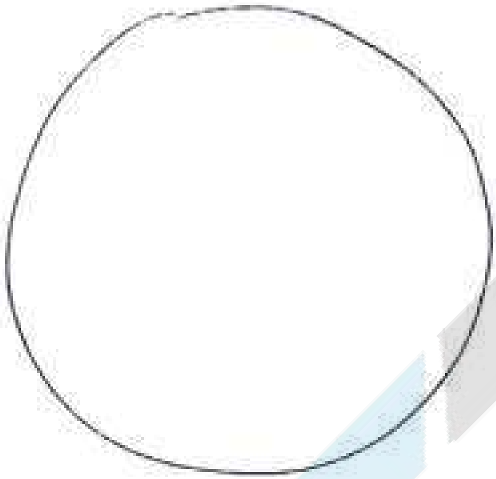


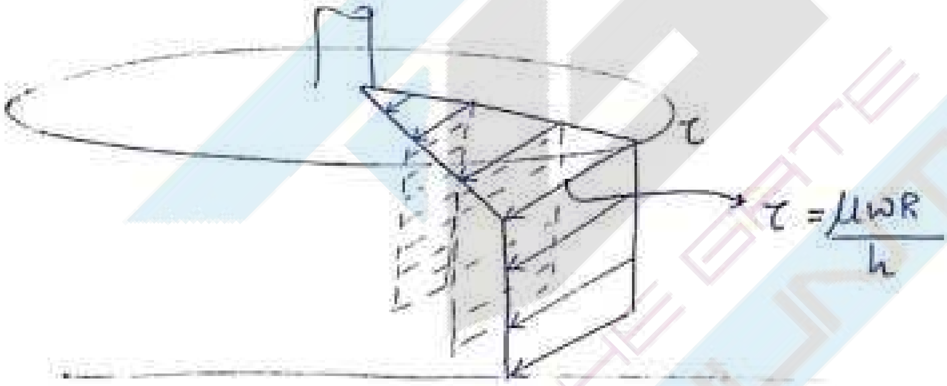
$\tau = \mu \frac{V}{h}$ ∝ varying

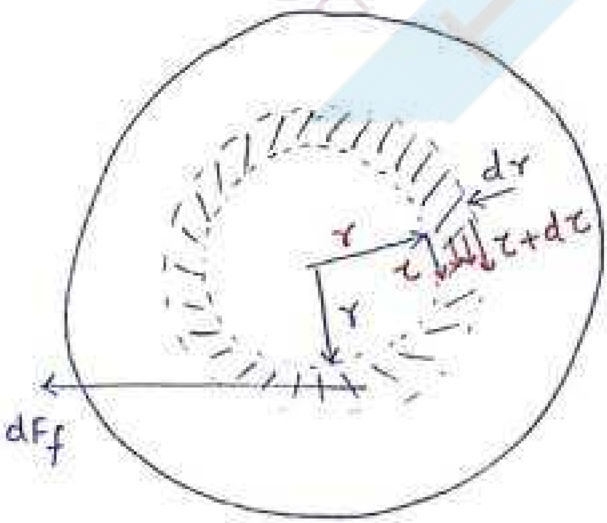


$\tau = \mu \frac{\omega r}{h}$

$\tau \propto r$







$$dF_f = \tau \times dA$$

$$T_f = \frac{\pi \mu \omega}{2h} R^4$$

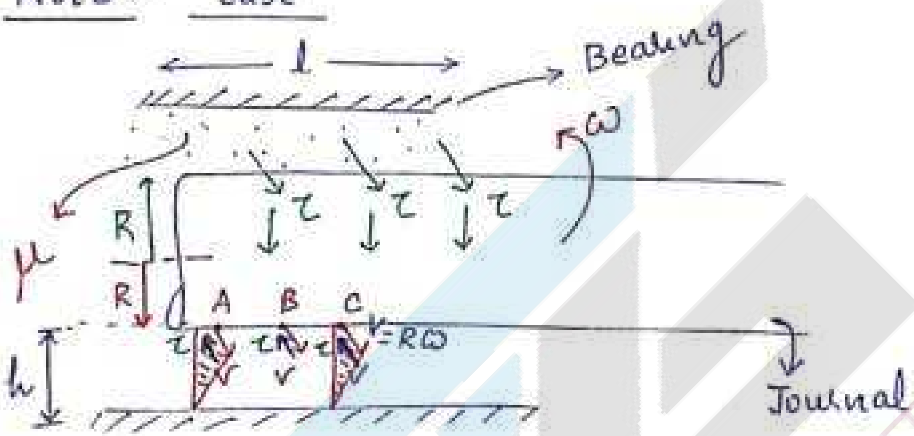
$$d\tau_f = dF_f \times r$$

$$d\tau_f = \tau \times dA \times r$$

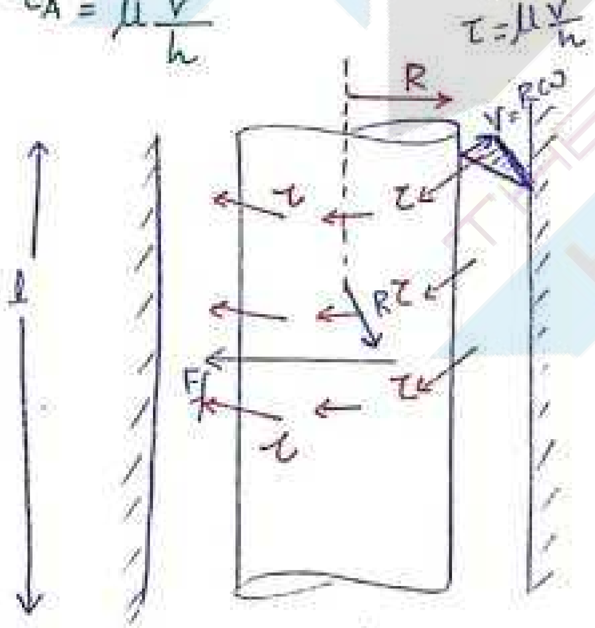
$$d\tau_f = \frac{\mu \omega r}{h} \times (2\pi r\, dr) \cdot r$$

$$\int dT_f = \frac{\mu \omega (2\pi)}{h} \int_0^R r^3\, dr$$

Note :- Case



$$\tau_A = \mu \frac{V}{h}$$

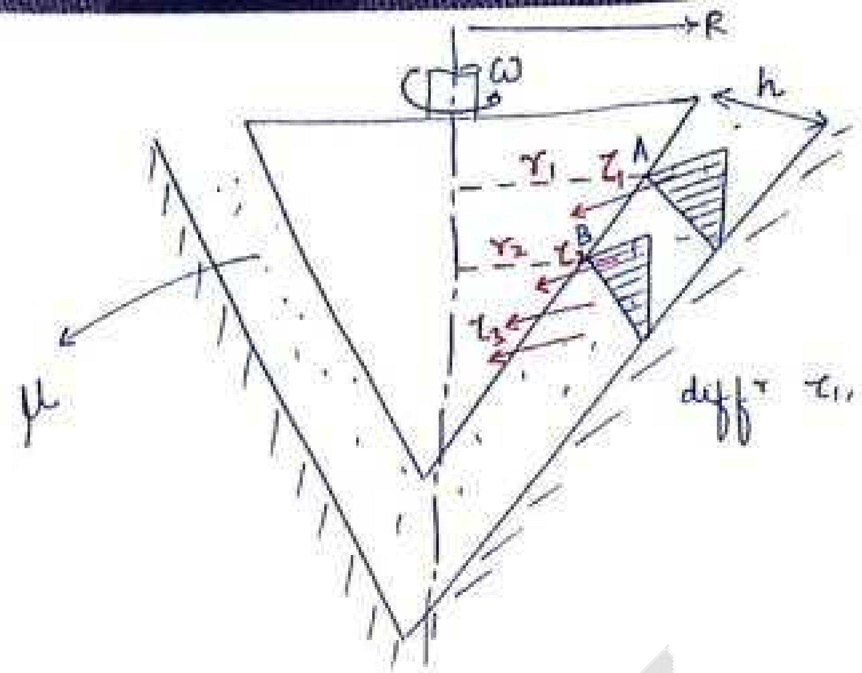


$$F_f = \tau \times A$$
$$= \tau \times (2\pi R l)$$

$$T = F_f \times R$$

$$T = \tau (2\pi R l) \times R$$
$$= \mu \frac{V}{h} (2\pi R l) R$$

$$\boxed{T = \frac{\mu R \omega}{h} (2\pi R l) R}$$



$$\tau = \frac{\mu (r)\omega}{h}$$

diff^r τ_1, τ_2, τ_3

* NEWTON'S LAW OF VISCOSITY :-

As per N.L.O.V, the shear stress at a distance y from the surface b/w two layers is proportional to Rate of shear deformation that is the proportionality coefficient known as viscosity does not depends on rate of shear deformation. This law is analogous to hooke's law.

Fluids which follows N.L.O.V. are known as newtonian fluids.

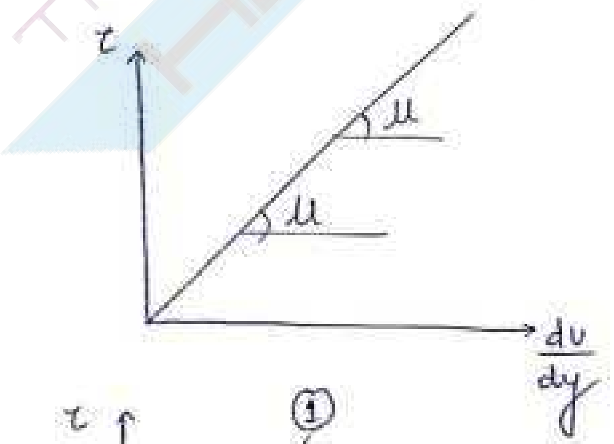
Ex :- ① water, air, diesel, kerosene, most gases, Hg etc.

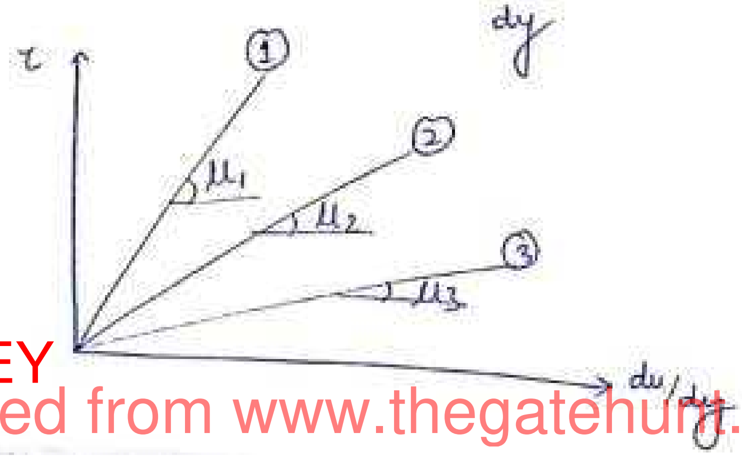
$$\tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

$$\mu \neq f\left(\frac{du}{dy}\right)$$

$$\boxed{y = mx}$$





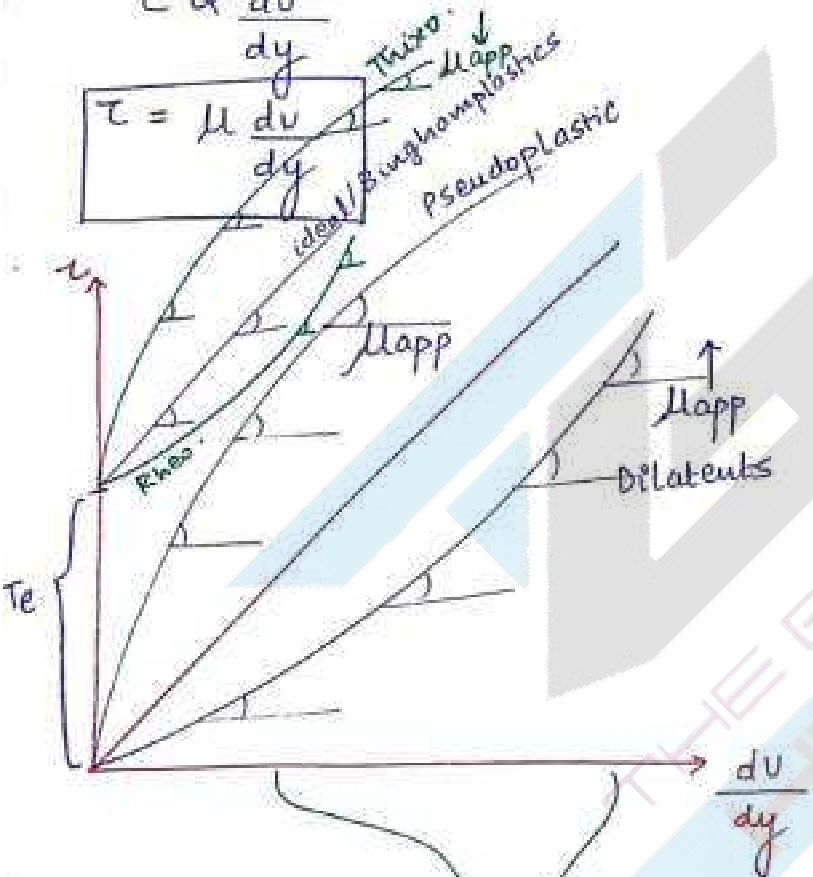
Classification

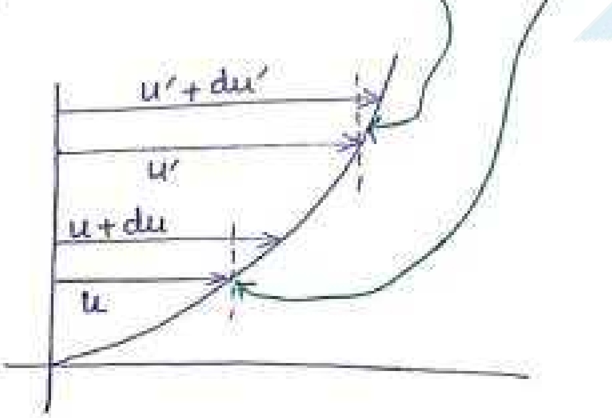
NEWTONIAN fluid

follows N.L.O.V.
(Fluid- Mechanics)

$\tau \propto \frac{du}{dy}$

$$\boxed{\tau = \mu \frac{du}{dy}}$$





NON NEWTONIAN (Rheology)

$$\boxed{\tau = B + K\left(\frac{du}{dy}\right)^n}$$

power law

Analogy

B = 0

$$\tau = K\left(\frac{du}{dy}\right)^n$$

n = flow behaviour index

$$\tau = \underbrace{\left[K\left(\frac{du}{dy}\right)^{n-1}\right]}_{\text{मान लो } \mu}\left(\frac{du}{dy}\right)$$

$$\boxed{\tau = \mu_{app}\frac{du}{dy}}$$

apparent viscosity

where $\mu_{app} = K\left(\frac{du}{dy}\right)^{n-1}$

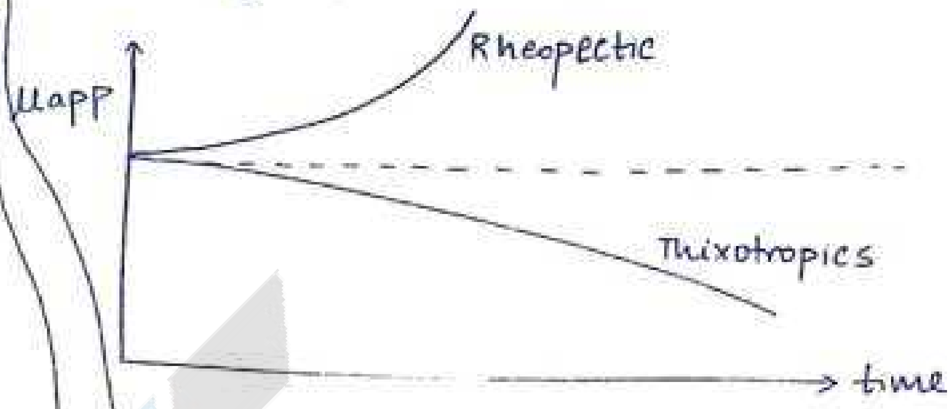
$$\boxed{\tau = B + \mu_{app}\frac{du}{dy}}$$

power law

(Rheology)

$\mu_{app} = f\left(\frac{du}{dy}\right)$

$\mu_{app} = f\left(\frac{du}{dy}, \text{time}\right)$



- Dilatent
- Pseudoplastic
- Binghamplastic / Ideal plastics

$\mu_{app} = f\left(\frac{du}{dy}, \text{time}\right)$

- Thixotropic
- Rheotropics

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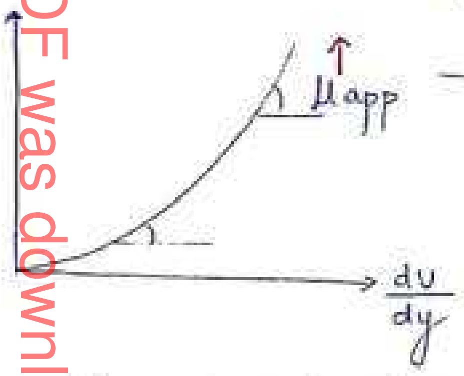
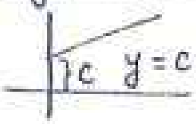
Dilatant (shear thickening fluids) -





$y = c + mx$

$$\tau = B + K\left(\frac{du}{dy}\right)^n$$

$$\overset{\uparrow}{\mu_{app}} = K\left(\frac{du}{dy}\right)^{\overset{\uparrow}{(n-1)}}$$

$$\tau = B + \mu_{app}\left(\frac{du}{dy}\right)$$

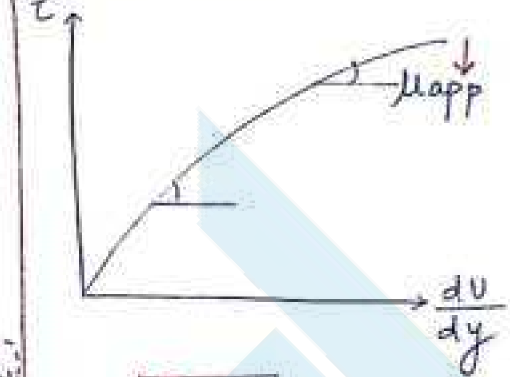
$n - 1 > 0$

$\boxed{n > 1}$

$\boxed{B = 0}$

Ex :- Honey, sugar solⁿ, Non colloidal solⁿ, Butter solⁿ, quick sand, etc.

Pseudoplastics -

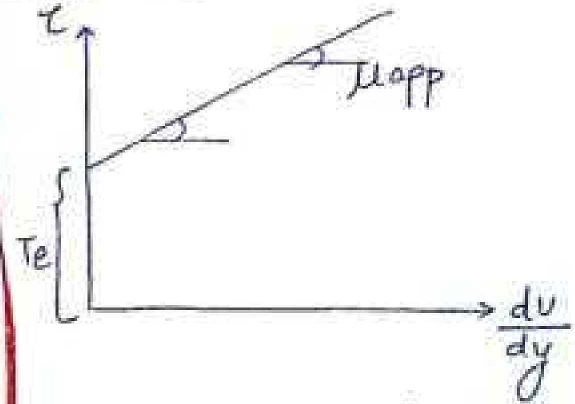


$\boxed{B = 0}$

$\boxed{n < 1}$

Ex :- Blood, milk, $\underbrace{\text{polymer solution, etc.}}_{\text{colloidal sol}^n}$

Binghamplastics



$$\tau = B + \mu_{app}\left(\frac{du}{dy}\right)$$

$B \neq 0 \quad B = \tau_e$

$$\mu_{app} = K\left(\frac{du}{dy}\right)^{(n-1)}$$

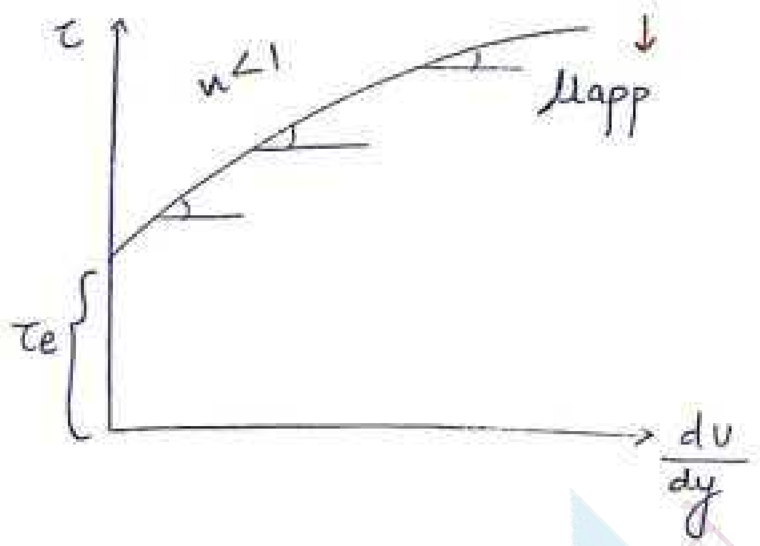
$n - 1 = 0$

$\boxed{n = 1}$

$\boxed{\mu_{app} = K}$

Ex :- Toothpaste, gel, cream, ~~quick sand~~ drilling mud.

Thixotropics



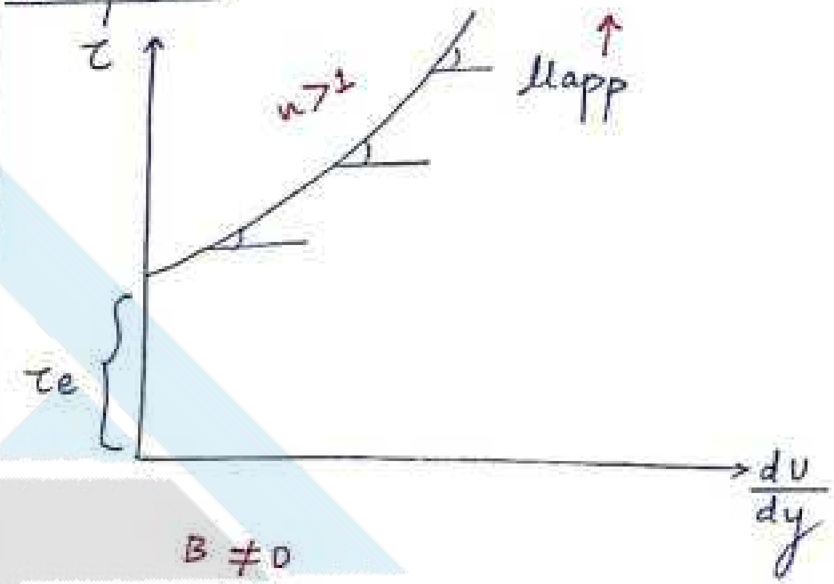
$B \neq 0$

$B = \tau_e$

$n < 1$

Ex:- paints; enamels; sewage sludge; catchup; etc.

Rheopectic



$B \neq 0$

$B = \tau_e$

$n > 1$

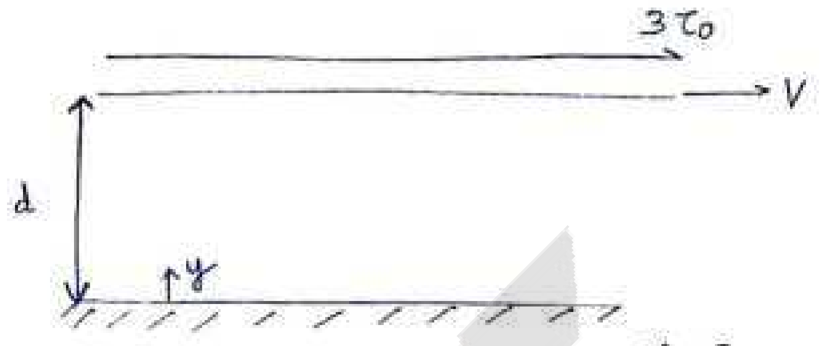
Ex:- Printers ink, gypsum soln.

Q18 WB (1A) (18) → linear profile

(b) ✓

(14) $\tau = \tau_0 + k\left(\frac{du}{dy}\right)^{1/2}$ $\boxed{n<1}$ → Thixo.

✓→ 3 variables



$V = ?$

$$3\tau_0 = \tau_0 + k\left(\frac{du}{dy}\right)^{1/2}$$

$$\frac{du}{dy} = \frac{4\tau_0^2}{k^2}$$

$$\int du = \frac{4\tau_0^2}{k^2}\int dy$$

$$u = \frac{4\tau_0^2}{k^2}y + C$$

Bcndn → $y = 0,\ u = 0,\ C = 0$

$$u = \frac{4\tau_0^2}{k^2}y$$

∴ → $y = d$

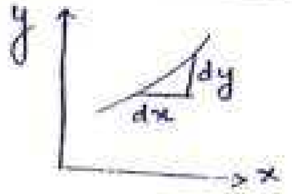
$$\boxed{V = \frac{4\tau_0^2}{k^2}d}$$

Q> What is the type of the fluid as per the given chart.

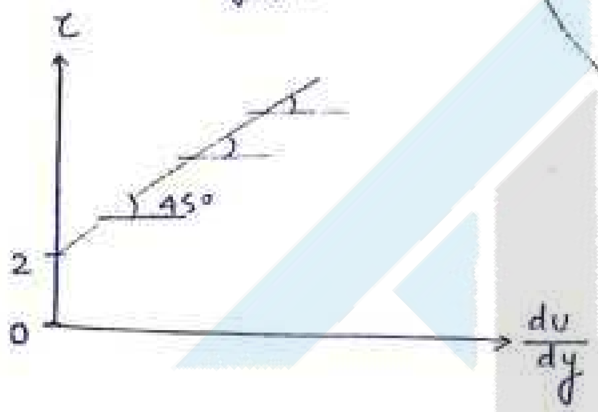
$\frac{du}{dy}$	0	1	2	3
τ	2	3	4	5

Sol Analytical method

$\frac{3-2}{1-0}$ = 1 $\frac{4-3}{2-1}$ = 1 $\frac{5-4}{3-2}$ = 1

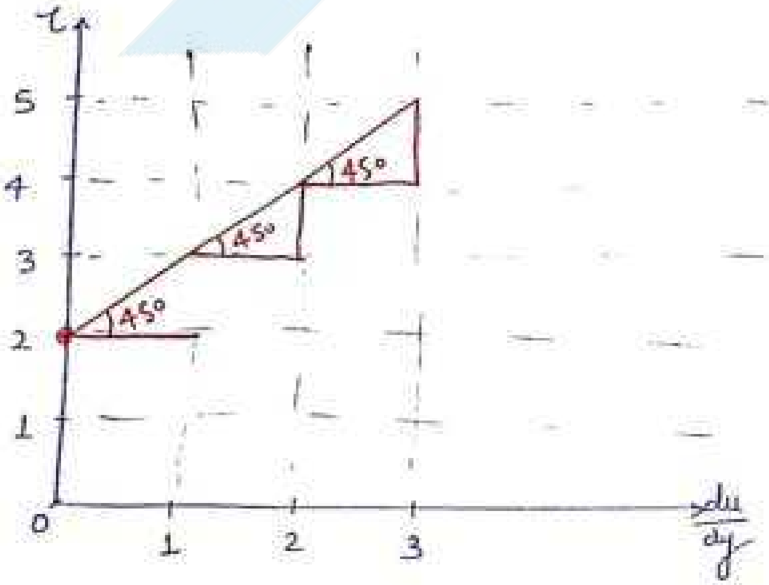


$\theta = dy/dx$



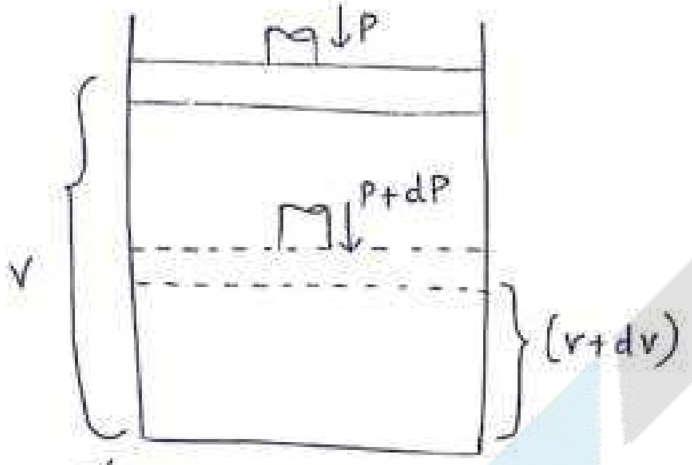
check slope

graphical Method



* **BULK MODULUS** :- It is the measure of modulus of elasticity of the fluid. It defines the compressibility of the fluid. Bulk modulus is defined as change in pressure for unit Volumetric strain. (29)

Liquids have large value of Bulk modulus (k_w of water $= k_w = 2\times10^6$ kN/m2, $k_{air} = 101$ kN/m2) than gases (—).



$$dP \propto -\left(\frac{dv}{v}\right)$$

$\Downarrow$ +ve $\qquad$ $\Downarrow$ $-(-ve)$

$$dP \propto \left(-\frac{dv}{v}\right)$$

$P + dP > P$

$dP > 0$

$v + dv < v$

$dv < 0$

$-dv > 0$

$$dP = k\left(-\frac{dv}{v}\right)$$

$$\boxed{k = \frac{dP}{\left(-\frac{dv}{v}\right)}} \quad N/m^2$$

mass conserve

$\rho v = \text{mass}$

$\ln\rho + \ln V = \ln m$

$$\frac{d\rho}{\rho} + \frac{dv}{v} = 0$$

$$\boxed{\frac{d\rho}{\rho} = \frac{-dv}{v}}$$

$$\boxed{k = \frac{dP}{\left(d\rho/\rho\right)}}$$

incompressible fluid ;

$\boxed{k \longrightarrow \infty}$ $\qquad$ $\boxed{-\frac{dv}{v} = 0}$ $\qquad$ $\beta = 0$

compressibility

$$\boxed{\beta = 1/k}$$

$$\beta = \frac{\left(-dv/v\right)}{dP} = \frac{\left(d\rho/\rho\right)}{dP}$$

Fluid is incompressible

incomp.

$\beta = 0$

$\frac{1}{\rho} \frac{d\rho}{dP} = 0$

$\Downarrow \quad \Downarrow$

$0 \quad 0$

$\cancel{\rho \to \infty}$

$\boxed{\frac{d\rho}{dP} = 0}$

$K_w = 2\times10^6 \ kN/m^2$

$K_{air} = 101 \ kN/m^2$

$\frac{K_w}{K_{air}} \simeq 20,000$

$K = \frac{1}{\beta}$

$\boxed{\frac{\beta_{air}}{\beta_w} = 20,000}$

air is 20,000 times more compressible than water.

In Expansivity, $P \leftrightarrow$ Tempr T

WB

$P = (3500 p^{1/2} + 2500) \ N/m^2$

100 kPa

(c) ✓

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3/11/2016 WB (4) $P = (3500\rho^{1/2} + 2500)\ N/m^2$

$$K = \frac{dP}{\left(\frac{-dV}{V}\right)} = \frac{dP}{\left(\frac{d\rho}{\rho}\right)}$$

$$K = \rho\left(\frac{dP}{d\rho}\right)$$

$$K = \rho\left[\frac{1750}{\rho^{1/2}}\right]$$

$$K = 1750\rho^{1/2}$$

$K \Rightarrow P = 100\,kPa$

$$K = 1750\left(\frac{P-2500}{3500}\right)$$

$$K = 48750\ N/m^2$$

---

water :

20°C 1 atm $\rho_w = 998\ kg/m^3$

20°C 100 atm $\rho_w = 1003\ kg/m^3$

$$d\rho = 1003 - 998 = 5\ kg/m^3$$

$$\frac{d\rho}{\rho} \times 100 = \frac{5}{998} \times 100 \simeq 0.5\%$$

Incompr. fluid $\boxed{\beta = 0}$

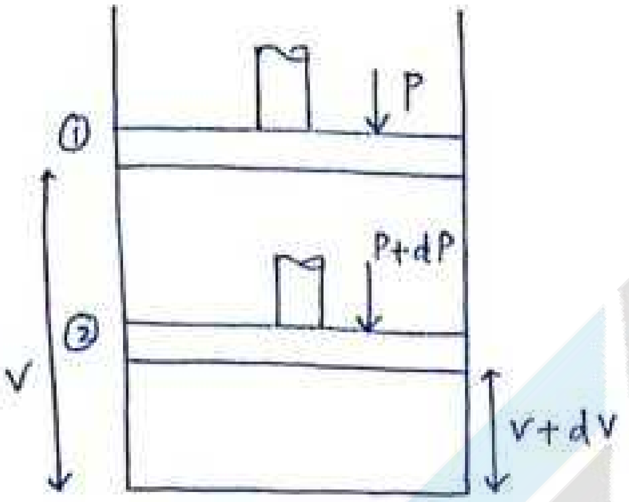
$$\frac{(d\rho/\rho)}{dP} = 0 \quad \boxed{\frac{d\rho}{dP} = 0}$$

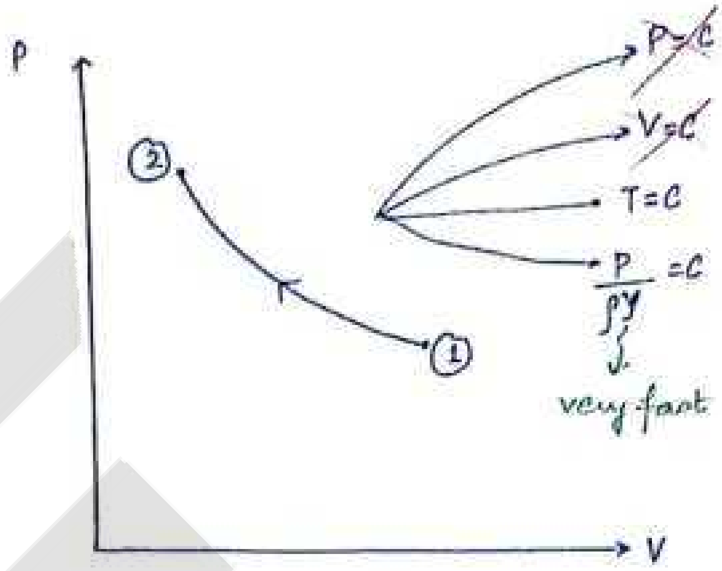
I— flow $\boxed{\frac{d\rho}{\rho} < 5\%}$

* The K of liquids are very high than gases so liquids are considered as incompressible fluid.

Bulk Modulus for Gases :-

Gases :-





Isothermal Bulk modulus

$\boxed{T=C}$

$P = \rho R T$ (ideal gas)

P → Abs, T → Abs

$$\frac{dP}{d\rho} = RT$$

$$\rho\frac{dP}{d\rho} = \rho R T$$

$$\boxed{K = P} \rightarrow \text{Abs.}$$

$\beta = 1/K$

$$\boxed{\beta = 1/P}$$

Adiabatic Bulk modulus

$$P = c\rho^{\gamma}$$

$$\frac{dP}{d\rho} = \gamma \cdot c \cdot \rho^{\gamma-1}$$

$$\rho\frac{dP}{d\rho} = \gamma \cdot c \cdot \rho^{\gamma}$$

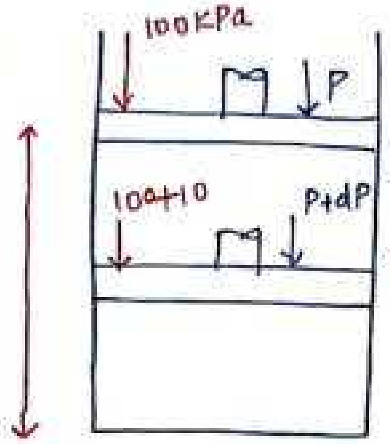
$$\boxed{K = \gamma P} \rightarrow \text{Abs}$$

↑ in A.B. process

$B = 1/K$

$$\boxed{B = 1/\gamma P}$$

Ex :-



during adiabatic process,
Bulk modulus ↑
Compressibility ↓

WB
(3) $K = P$ $P\uparrow$ $K\uparrow$ (b) ✓

(35) $P \rightarrow$ 3MPa $\rightarrow$ 3.5 MPa

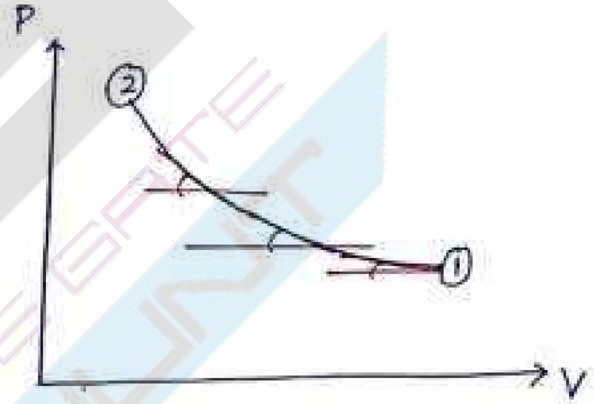
$\rho \rightarrow$ 500 kg/m3 $\rightarrow$ 501 kg/m3

SIR

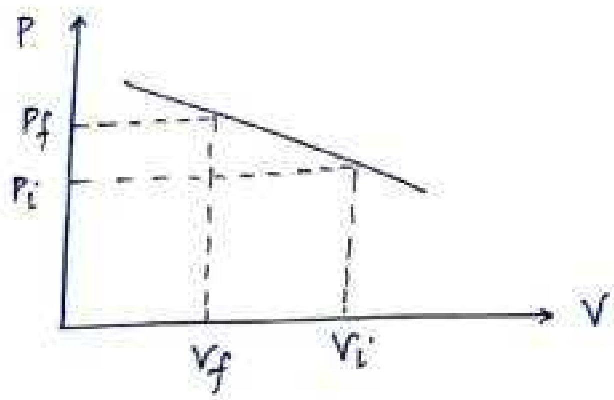
$$K = \frac{dP}{\left(\frac{-dV}{V}\right)} = \frac{dP}{\left(\frac{d\rho}{\rho}\right)}$$

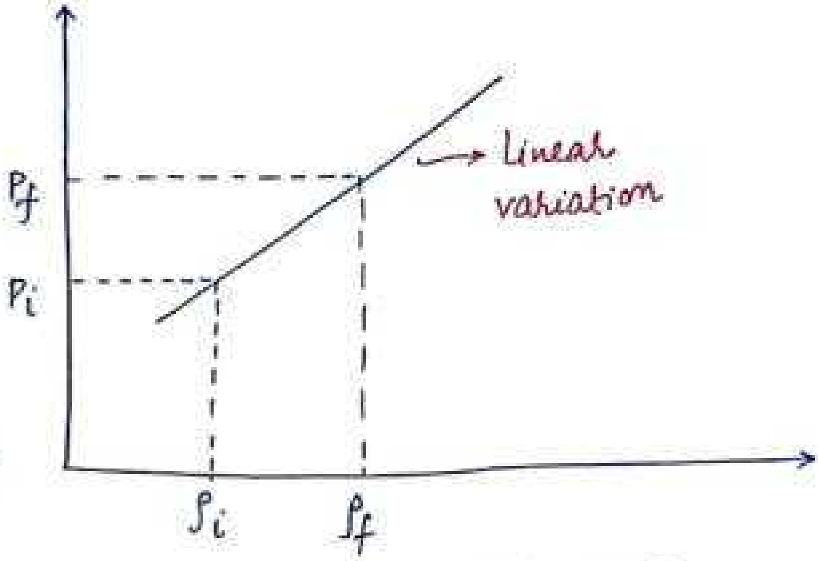
↳ initial

$$K = V_i\left(\frac{dP}{(-dV)}\right)$$



avg. K → that means not varying K → slope is not varying
hence it has
linear slope.





use this

$$K = \frac{P_f - P_i}{\frac{\rho_f - \rho_i}{\rho_i}}$$ → Average Analysis.

$$\boxed{K = 250\ \text{MPa}}$$

No deviation in case less diffr. in P and ρ's.

but

$$K = \frac{dP}{\frac{d\rho}{\rho}}$$

$$\int_{P_i}^{P_f} dP = K \int_{\rho_i}^{\rho_f} \frac{d\rho}{\rho}$$

→ This is for instantaneous.

Real analysis → True state

Note :- fluid

$$\boxed{C = \sqrt{\frac{K}{\rho}}}$$

⇓

speed of sound
or
speed of pressure wave.

MOHIT CHOUKSEY

solids :-

$$\boxed{c = \sqrt{\frac{E}{\rho}}}$$

Fluids

Liquids :-

$$\boxed{c = \sqrt{\frac{K}{\rho}}}$$

eg : water

$$c = \sqrt{\frac{2 \times 10^6 \times 10^3}{1000}}$$

$$c = 1414 \text{ m/s}$$

Gas :

$$c = \sqrt{K/\rho}$$

$$P = \rho R T$$

isothermal

$$K = P$$

$$c = \sqrt{\frac{P}{\rho}}$$

$$\boxed{c = \sqrt{RT}}$$

Newtons Result

whole environment pressure waves (isothermal)

adiabatic

$$K = \gamma P$$

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

$$\boxed{c = \sqrt{\gamma R T}}$$

air

$$c = 332 \text{ m/s}$$

Laplace Result

when we speaks/dissipates energy,

compress. exp. comp. exp. ----

sound ← adiabatically

$$\boxed{Ma = \frac{V}{c}}$$

Ma → Mach no.

c → sonic speed

$Ma < 1 \longrightarrow V < c \longrightarrow$ flow is subsonic flow

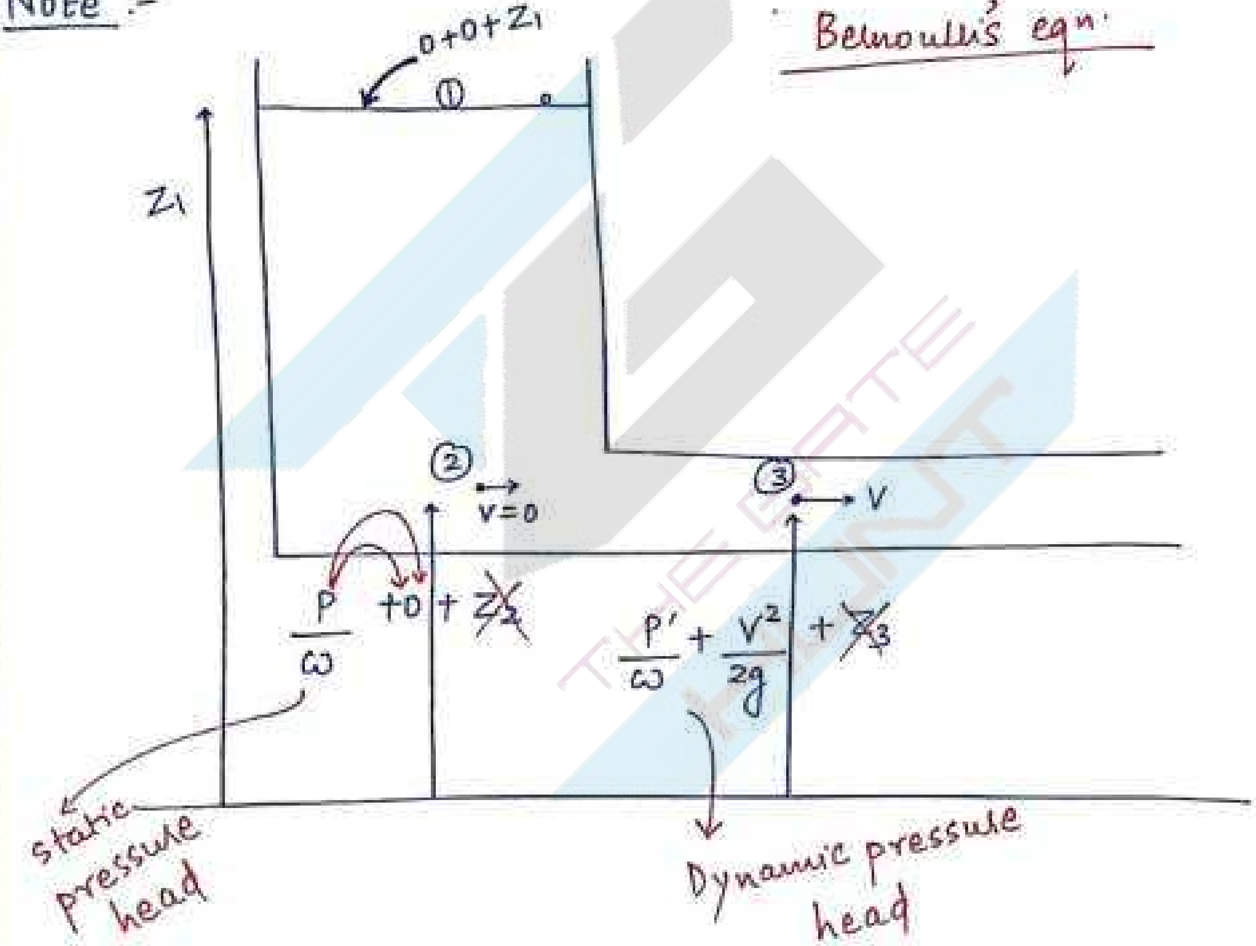
$Ma = 1 \longrightarrow V = c \longrightarrow$ flow is sonic

$Ma > 1 \longrightarrow V > c \longrightarrow$ supersonic (Brahmos)

$Ma > 5 \longrightarrow V > 5c \longrightarrow$ Hypersonic

Note :-

Bernoulli's eqn.



Bernoulli's eqn ② & ③

$$\frac{P}{\omega} + 0 + \cancel{Z_2} = \frac{P'}{\omega} + \frac{V^2}{2g} + \cancel{Z_3}$$

$$\frac{dP}{\rho \cancel{g}} = \frac{V^2}{2\cancel{g}}$$

$$\frac{1}{\rho}\left[\frac{dP}{\left(\frac{d\rho}{\rho}\right)}\right] = \frac{V^2}{2\left(\frac{d\rho}{\rho}\right)}$$

$$\frac{k}{\rho} = \frac{V^2}{2\left(\frac{d\rho}{\rho}\right)}$$

$$c^2 = \frac{V^2}{2\left(\frac{d\rho}{\rho}\right)}$$

$$\frac{V^2}{c^2} = 2 \times \frac{d\rho}{\rho}$$

$$\frac{V}{c} = \sqrt{2 \times \frac{d\rho}{\rho}}$$

incompressible flow

$$\frac{d\rho}{\rho} < 0.05$$

$$Ma < \sqrt{2 \times 0.05}$$

$$\boxed{Ma < 0.32}$$

air :

$$\frac{V}{c} < 0.32$$

$$\frac{V}{332} < 0.32$$

$$\boxed{V < 106 \text{ m/s}}$$

* If compressible fluid flows less than 0.32 of Ma, then flow is consider to be incompressible flow.

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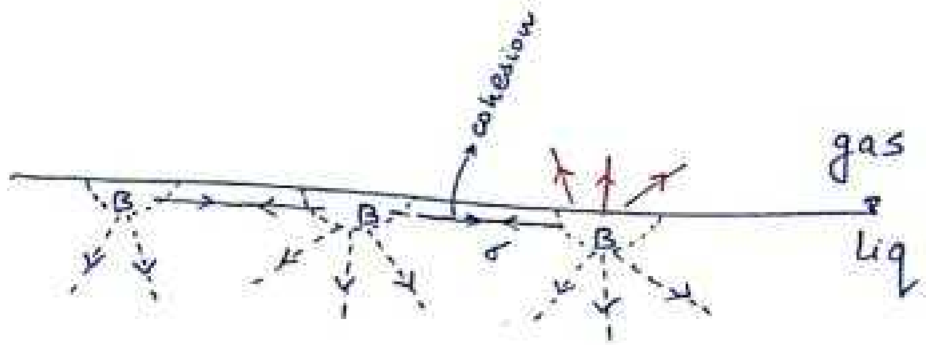
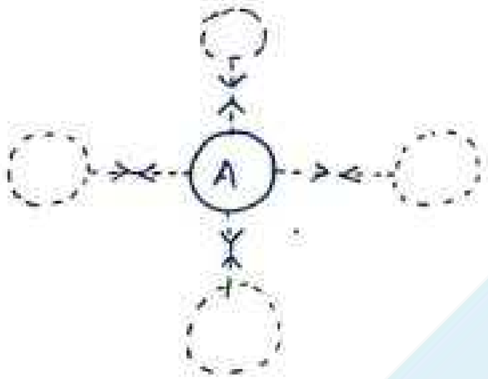
Surface Tension :-



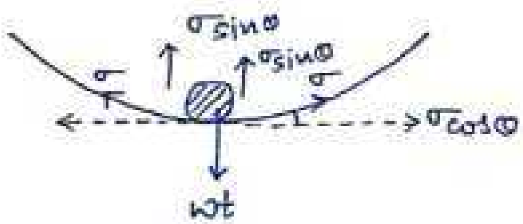


$\sigma_{W-air} = 0.073\ N/m$

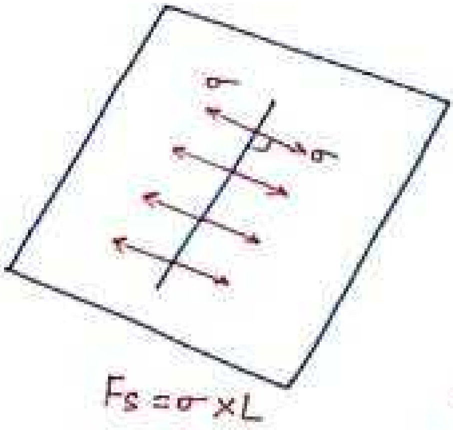
$\sigma_{Hg-air} = 0.497\ N/m$

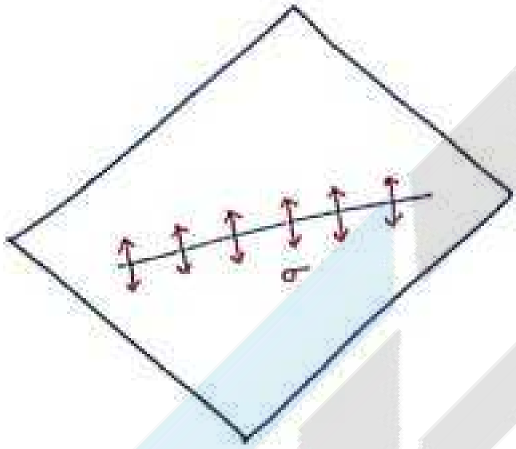
$\sigma_{alcohol-air} = 0.02\ N/m$

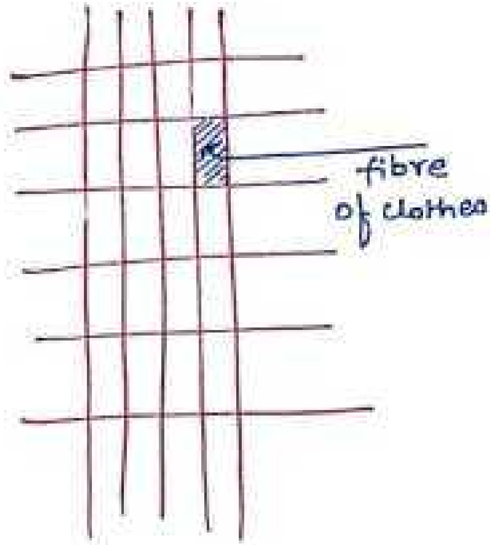
$$\boxed{\sigma = \frac{\text{surface energy}}{\text{area}}}\ J/m^2$$



$$\boxed{\sigma = \frac{F}{L}}\ N/m$$





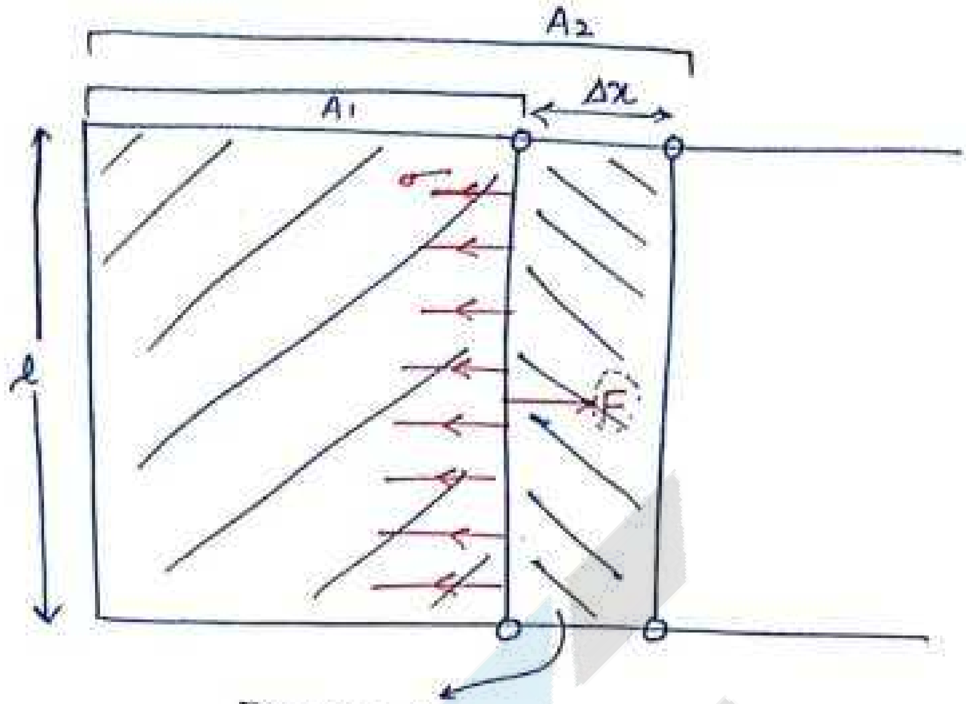


Surf. Tension is due to unbalanced cohesive forces. It is the liquid-gas, liquid-liquid interface phenomenon. It is due to cohesive forces. It is defined as surface energy per unit area. It is also defined as line force that is it is the force per unit length normal to the line drawn at the liquid surface.

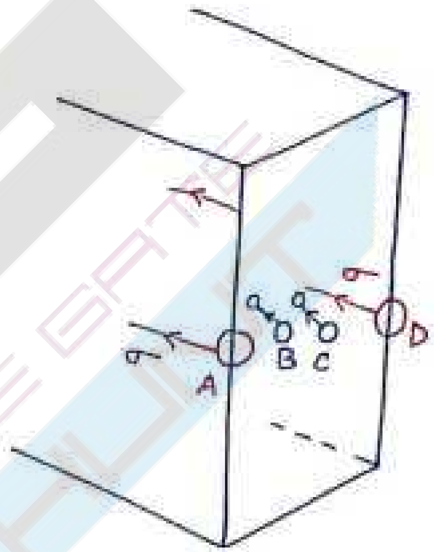
as the tempr. ↑, cohesion ↓, surface tension ↓, detergent decreases the surface tension but salt ↑.

Application :- ① floating of insects & leaves over the water surface, sap of water in the trees, separation of soil, capillarity effect, formation of Bubbles, droplets, jet, etc.

SURFACE ENERGY :-
(Concept)



Increased area = $2 \times (l \times \Delta x)$



Equilibrium

$$\boxed{F = F_s}$$

$$F_s = \sigma \times l + \sigma l$$

$$F_s = \sigma \times 2l$$

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Workdone

$$W = F \cdot \Delta x$$

$$= F_s \cdot \Delta x$$

$$W = \sigma (2l) \cdot \Delta x$$

$$W = \sigma [(2l) \Delta x]$$

$$W = \sigma \Delta A$$

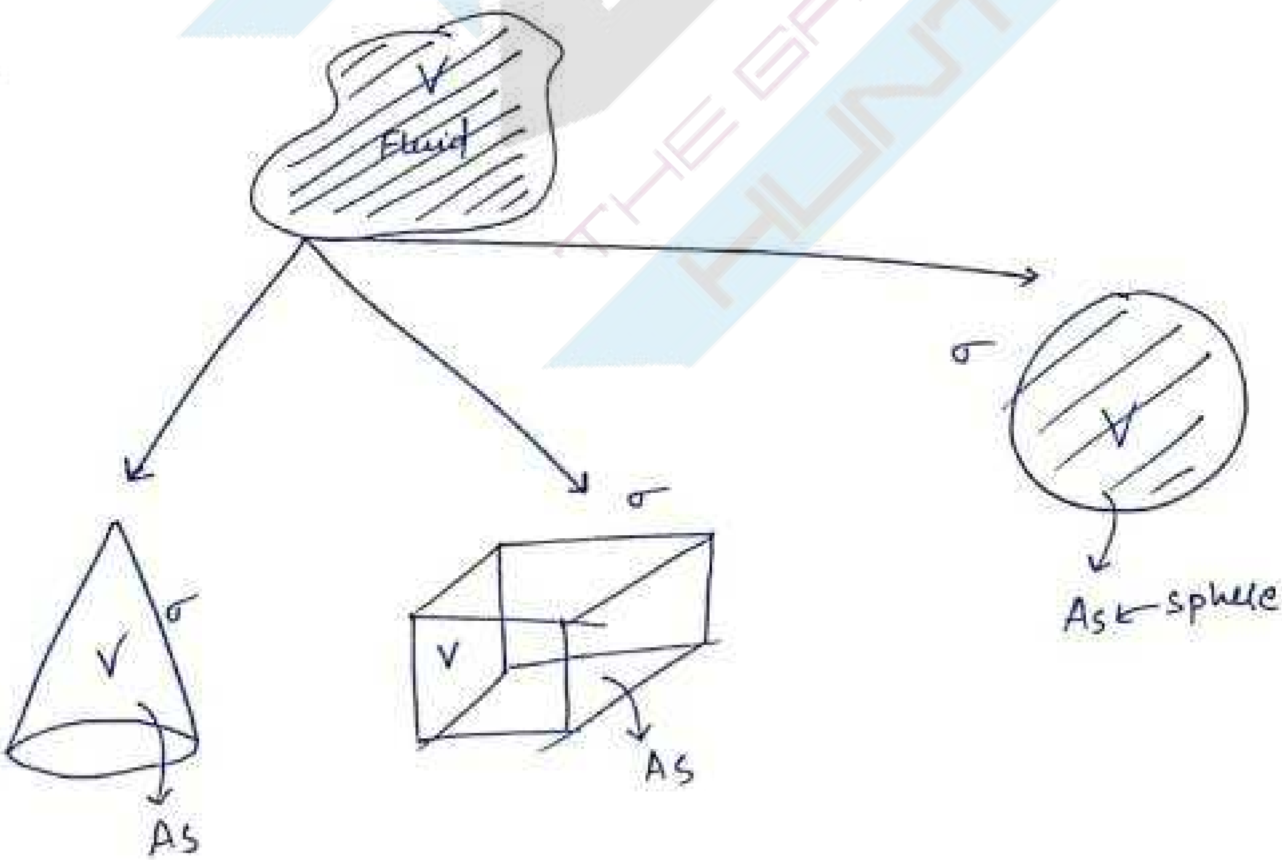
$$\boxed{\sigma = \frac{W}{\Delta A}} \quad J/m^2$$

$$W = \sigma A_2 - \sigma A_1$$

$$\boxed{W = E_2 - E_1}$$

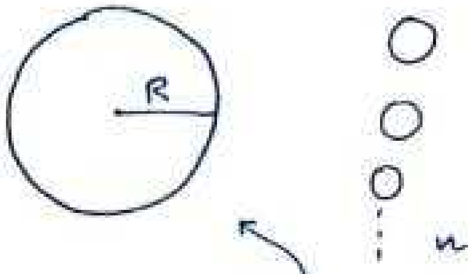
final — initial

~~structure~~ surface energy



Pg 8

Q19



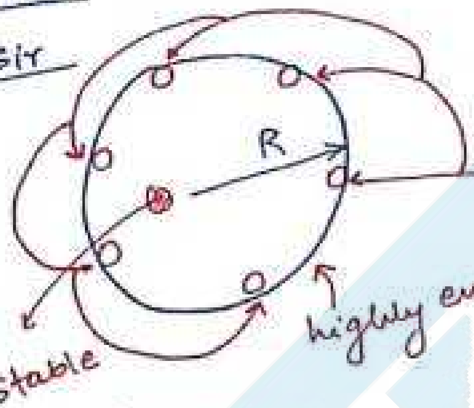
σ

$\sigma = \frac{W}{A}$

$\Rightarrow \sigma (4\pi R^2) = W$

But

sir



$\therefore$ large Energy

r
O 1, O 2, O 3 ---------- O n

$A_S = \underbrace{n \, 4\pi r^2}_{\text{more}}$

$E_2 = \sigma n 4\pi r^2$

$A_L = 4\pi R^2$

large droplet

$E_1 = \sigma_1 A_L$

Workdone $= E_2 - E_1$

$= \sigma n 4\pi r^2 - \sigma 4\pi R^2$

$= \sigma 4\pi R^2 \left[\frac{n r^2}{R^2} - 1\right]$

$= \sigma 4\pi R^2 \left[n \times \frac{1}{n^{2/3}} - 1\right]$

mass conserved

$m_L = m_S$

$\cancel{\rho} \cdot V_L = \cancel{\rho} V_S$

$\cancel{4/3 \pi} R^3 = n \cancel{4/3 \pi} r^3$

$\left(\frac{R}{r}\right)^3 = n$

$\left(\frac{r}{R}\right)^3 = 1/n$

$r/R = (1/n)^{1/3}$

$1 - 2/3 \rightarrow \frac{3-2}{3} \rightarrow 1/3$

$$\boxed{W = \sigma 4\pi R^2 [n^{1/3} - 1]}$$

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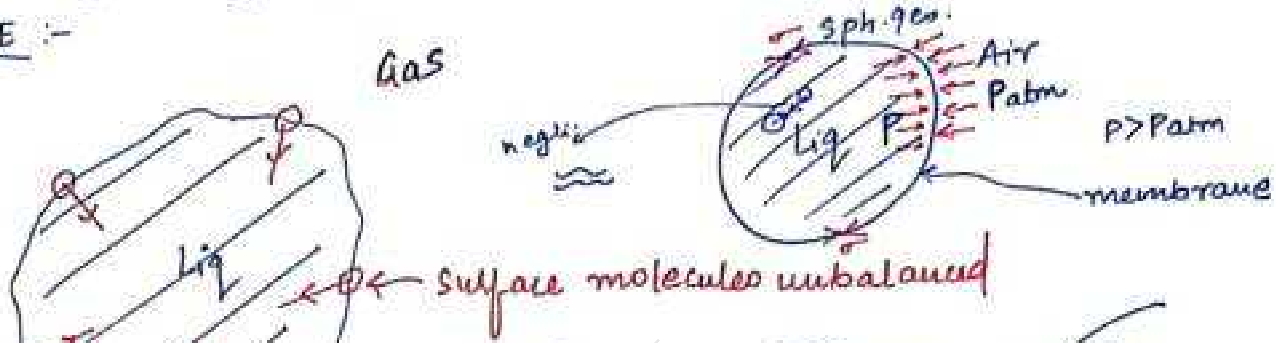
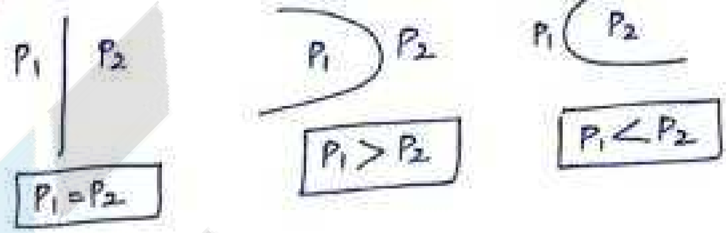
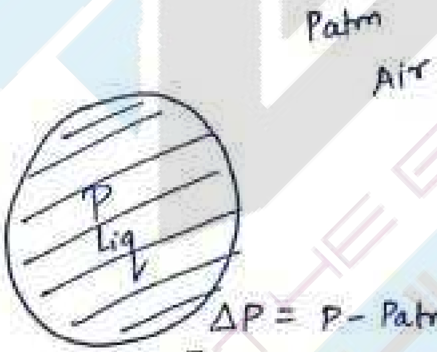
Note :- For the given volume, the surface area of spherical geometry ~~area of~~ geometry is minimum, so for formation of droplets and Bubbles, Less work is required that is geometry will have more stability.

NOTE :-



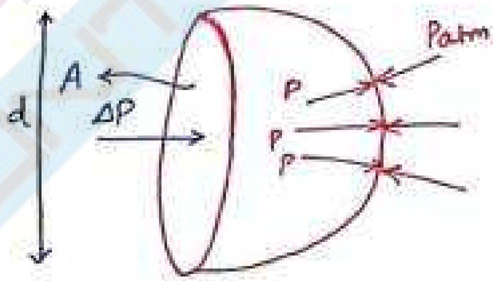


Case I → Pressure inside liquid droplet/Air Bubble :-



$\Delta P = P - P_{atm}$

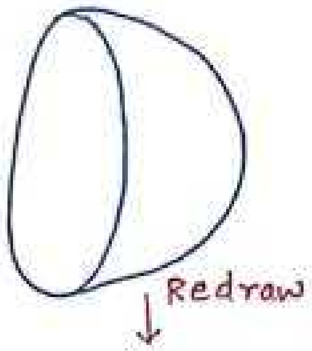
Excess pressure

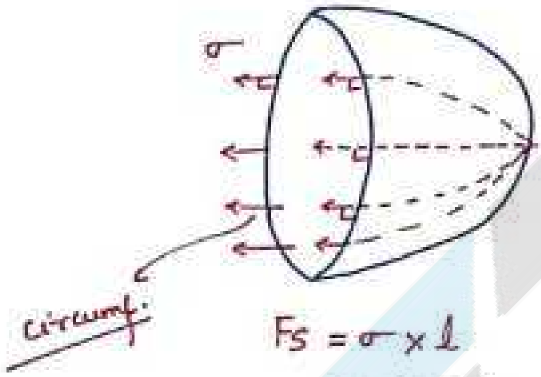


$\Delta P =$ Excess pressure

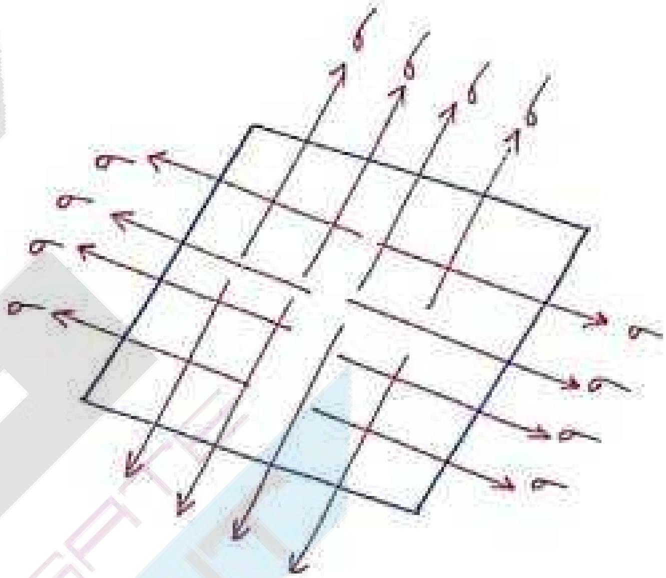
$F_P = \Delta P \times A$

$F_P = \Delta P \frac{\pi}{4} d^2$





$$F_s = \sigma \times l$$
$$= \sigma \times \pi d$$

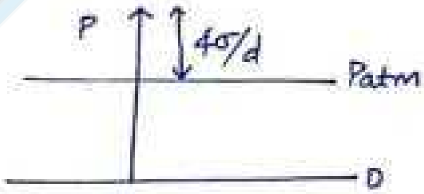


<u>Equilibrium</u>

$$F_p = F_s$$

$$\Delta P \frac{\pi}{4} d^{\cancel{2}} = \sigma \cancel{\pi} \cancel{d}$$

** $\boxed{\Delta P = \frac{4\sigma}{d}}$ **



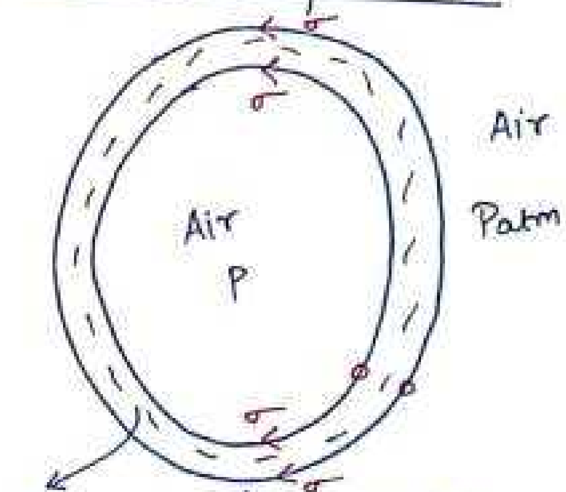
$$P - P_{atm} = \frac{4\sigma}{d}$$

** $\boxed{P = P_{atm} + \frac{4\sigma}{d}}$ ** Absolute

$P_{atm} = 0$

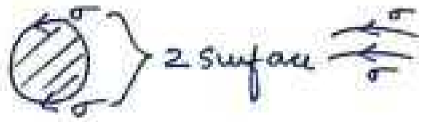
** $\boxed{P = \frac{4\sigma}{d}}$ ** Gauge

Pressure inside Soap Bubble



$\Delta P = P - P_{atm}$

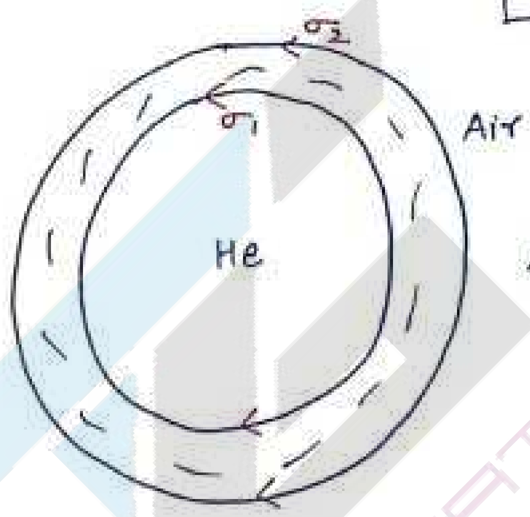
$\Delta P = \dfrac{4\sigma}{d}$



$\Delta P = \dfrac{4\sigma}{d_o} + \dfrac{4\sigma}{d_i}$

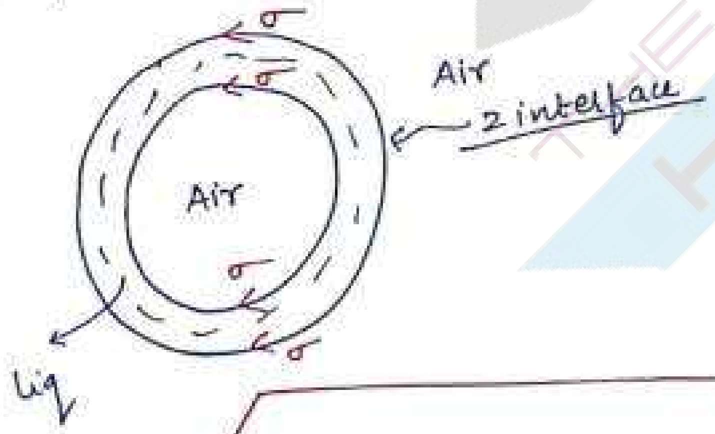
$d_o \simeq d_i \simeq d$

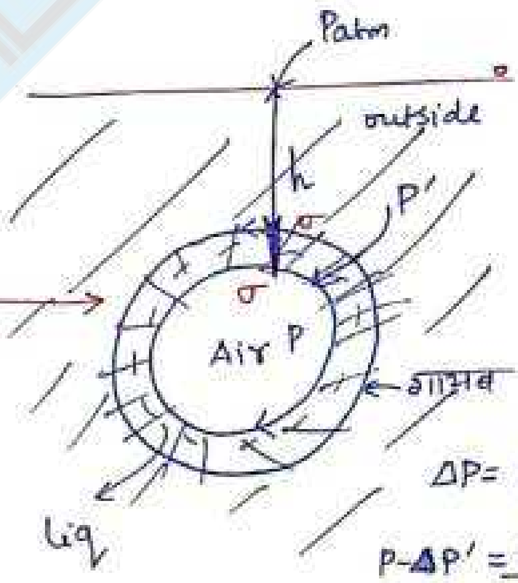
$\boxed{\Delta P = \dfrac{8\sigma}{d}}$



$\Delta P = \dfrac{4\sigma_1}{d} + \dfrac{4\sigma_2}{d}$

Now,





$\Delta P = \dfrac{4\sigma}{d}$

$P - P' = \dfrac{4\sigma}{d}$

$P = P' + \dfrac{4\sigma}{d}$

$\boxed{P = P_{atm} + \rho g h + \dfrac{4\sigma}{d}}$

$\Delta P = \dfrac{4\sigma}{d}$

air

Q36

air bubble $\longrightarrow$ 1 interface

$d = 0.001 m$

$\sigma = 0.072 N/m$

$$\Delta P = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.072}{0.001} = \frac{4 \times 72}{1} = 248 N/m^2$$

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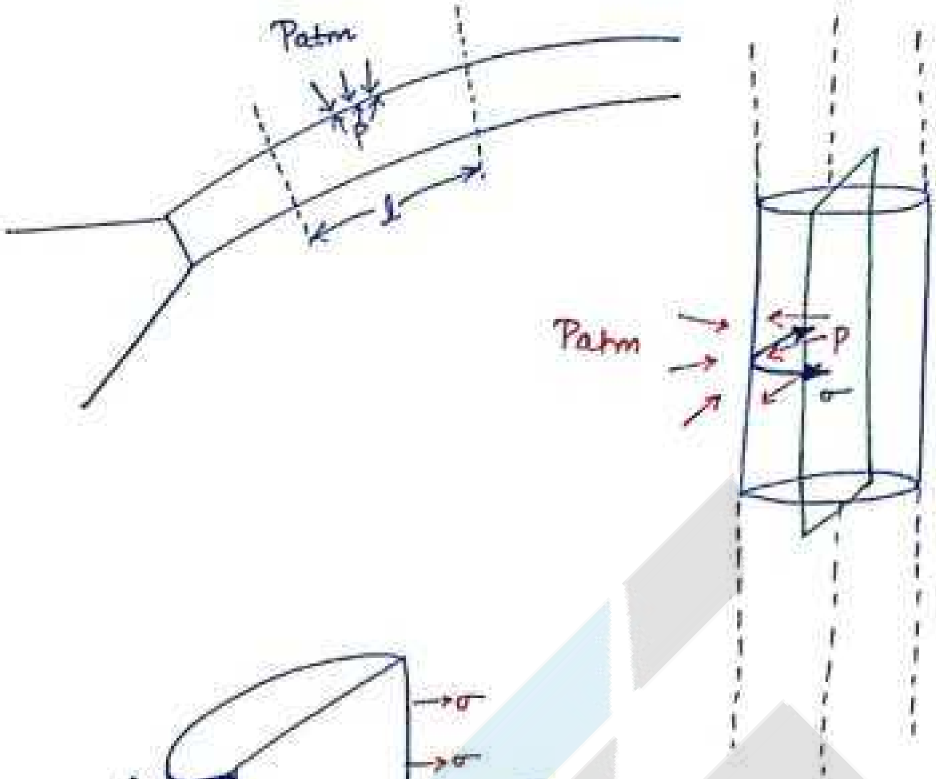
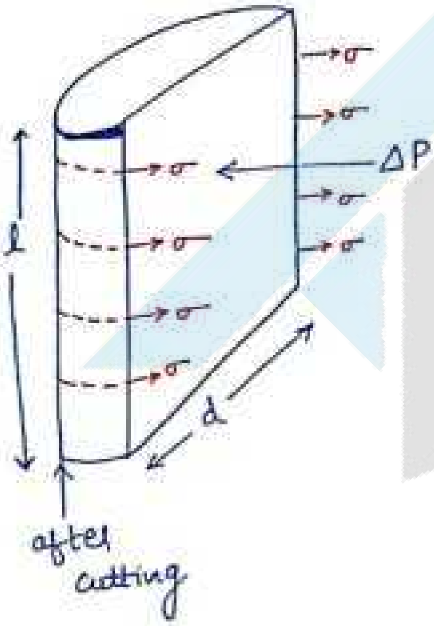
Pressure Inside a Liquid Jet -

jet का Formn → viscous action से होगा





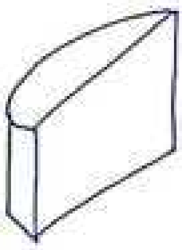
$$F_p = \Delta P \times (l \times d)$$

$$F_s = (\sigma \times l) + (\sigma \times l)$$

$$F_p = F_s$$

$$\Delta P(\cancel{l} \times d) = 2\sigma \cancel{l}$$

$$\boxed{\Delta P = \frac{2\sigma}{d}}$$



$\Delta P = \frac{8\sigma}{d}$ → soap Bubble

$\Delta P = \frac{4\sigma}{d}$ → soap की Bubble, liquid droplet

$\Delta P = \frac{2\sigma}{d}$ → liquid Jet

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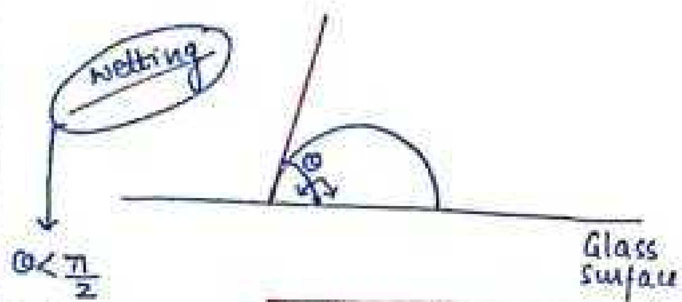
Wetting and Non-Wetting Phenomenon

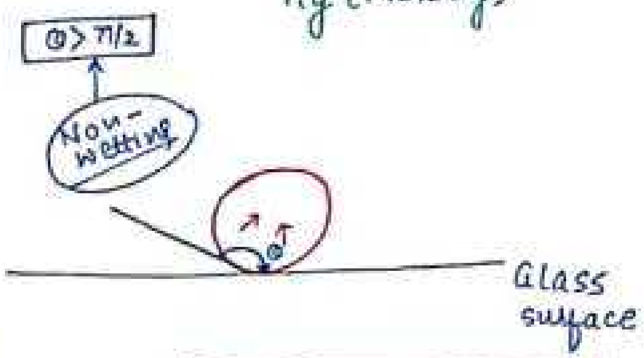
water

Hg (Mercury)



$\theta < \frac{\pi}{2}$
acute angle

$\boxed{\text{Adhesive force} > \text{cohesive force}}$

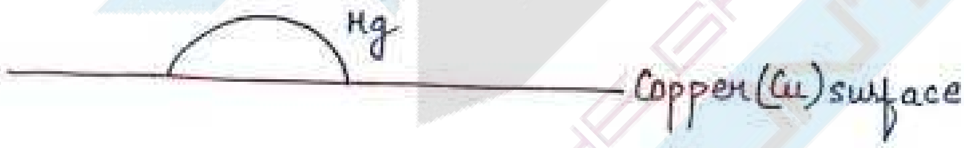


$\boxed{\text{cohesive forces} > \text{Adhesive forces}}$

---

*

*

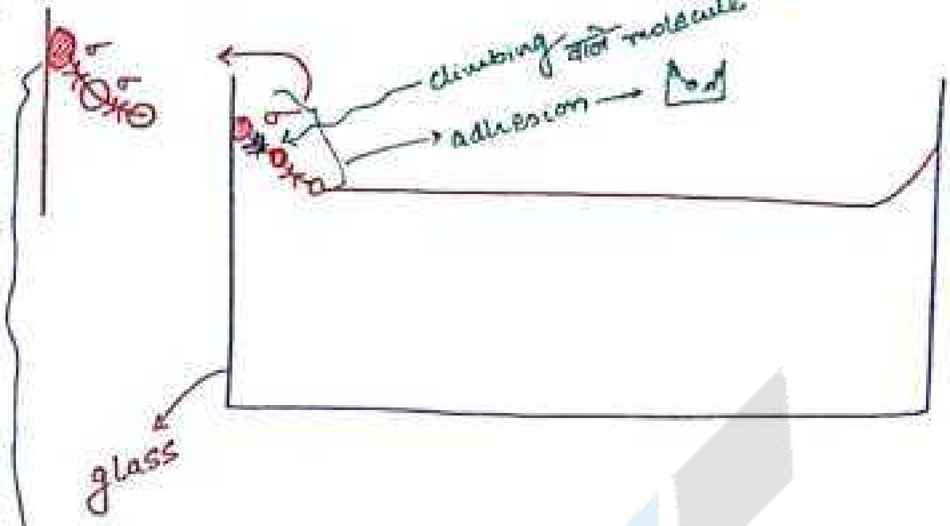


✓ WkNWP → is Relative phenomenon.

→ wetting and non-wetting ph. is relative phenomenon. It depends upon adhesion b/w the liquid and solid surface and cohesion b/n the liquid surface.

Capillarity :-

When the small diameter tube (less than 10 mm) is inserted into a liquid, there may be rise or fall of the liquid in the tube wrt outer surface is known as capillarity

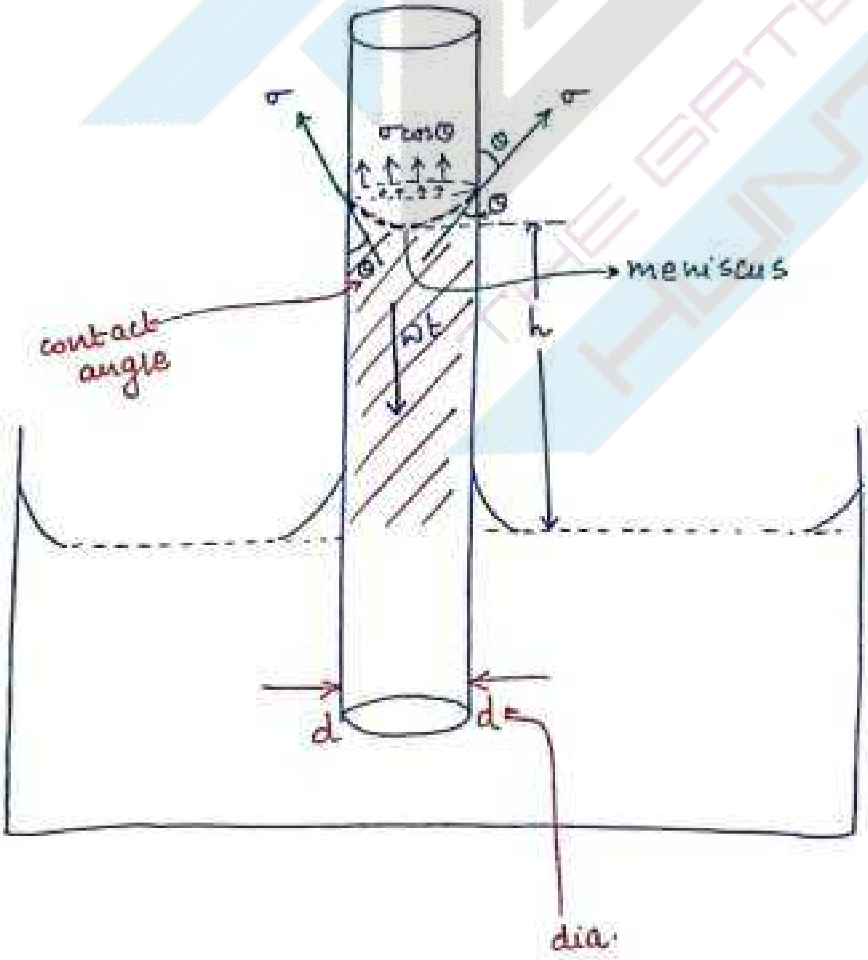


E/m
adhesive forces
↓
adhesion

adhesion के लिए cohesion dominant है |

$\therefore$ weight (wt) = ρg = $\frac{mg \times V}{V}$ (× volm)

wt = mg



$F_s = (\sigma \cos\theta \, \pi d)$

$Wt = w \times \text{Volume}$ ($w \to \rho g$)

Equilibrium :-

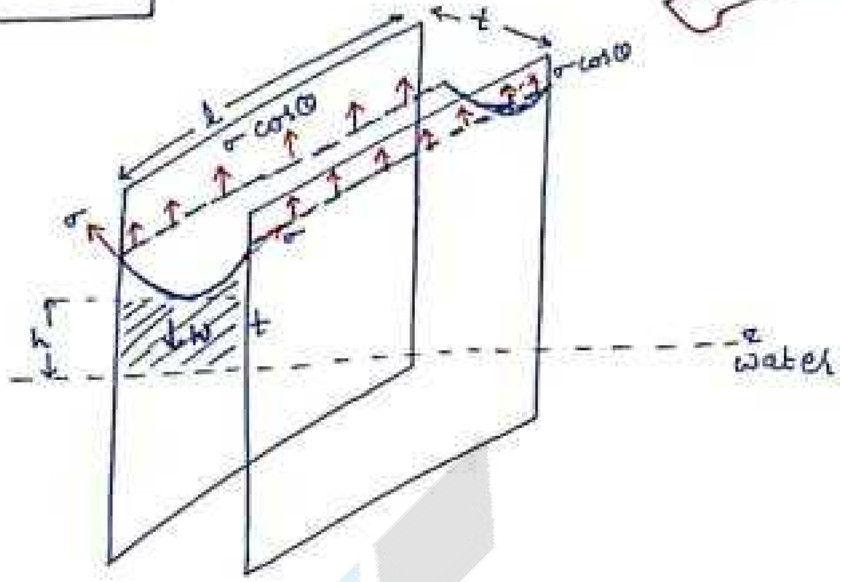
$Wt = F_s$

$$w \times \left(\frac{\pi}{4} d^2 h\right) = (\sigma \cos\theta \, \pi d)$$

$$\boxed{h = \frac{4\sigma \cos\theta}{wd}}$$

where $\boxed{w = \rho g}$

$\uparrow \boxed{h \propto \frac{1}{d \downarrow}}$

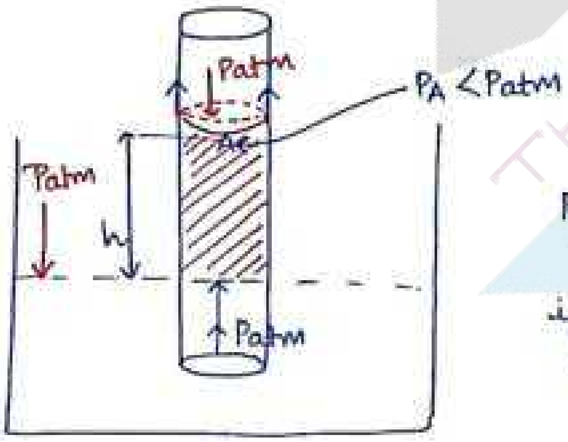


$$W_t = F_s$$

$$\omega \times (h \times l \times t) = (\sigma \cos\theta) l + (\sigma \cos\theta \times l)$$

$$\boxed{h = \frac{2\sigma \cos\theta}{\omega t}}$$

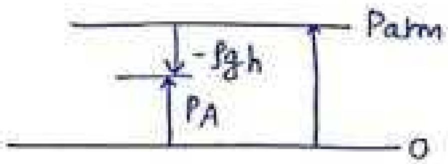
*



$$P_A = P_{atm} - \rho g h$$

if hole करे तो No ejaculation → no Jet

since vacc. under.



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∵ Angle of contact

51

$\theta = 0°$ Pure water - Glass

$\theta = 29°$ Normal water - Glass

$\theta = 130° - 138°$ Hg - Glass

$\theta = 26°$ Kerosene - Glass

lighter droplet →

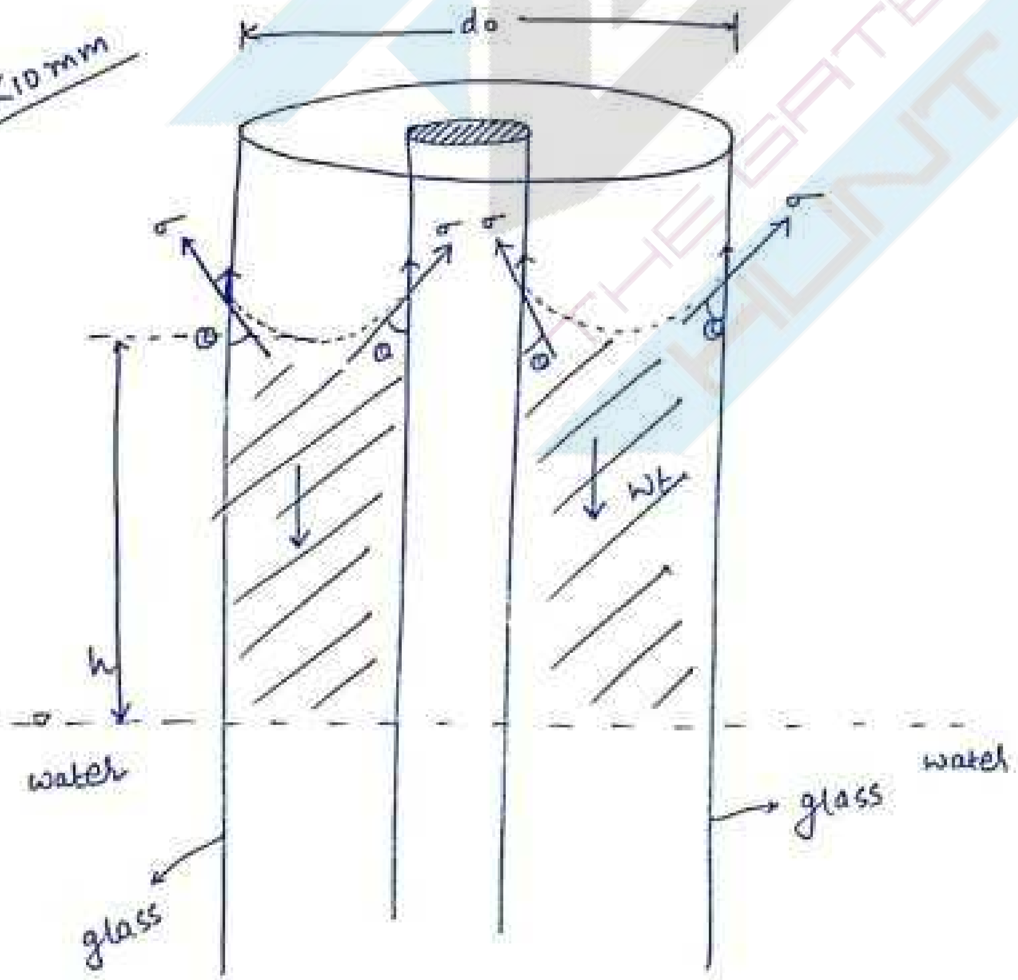
larger θ

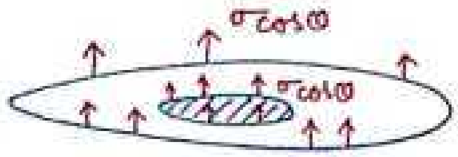
larger droplet →

Note → ① As the diameter of the tube reduces, the capillarity effect increases. In manometers, the diameter of the tube are more than 10 mm to avoid the capillarity effect in pressure measurement.

② Capillary Rise b/n two concentric tubes

$d_o < 10\,mm$





$$F_s = \sigma \cos\theta \, \pi d_o + \sigma \cos\theta \, \pi d_i$$

$$Wt = w \cdot Vol^m$$

$$= w \times \frac{\pi}{4}\left(d_o^2 - d_i^2\right)h$$

Equilibrium

$$Wt = F_s$$

$$w \frac{\pi}{4}\left(d_o^2 - d_i^2\right)h = \sigma \cos\theta \, \pi \left[d_o + d_i\right]$$

$$\boxed{h = \frac{4\sigma \cos\theta}{w(d_o - d_i)}}$$

$d_i = 0$

$$\boxed{h = \frac{4\sigma \cos\theta}{w d_o}}$$

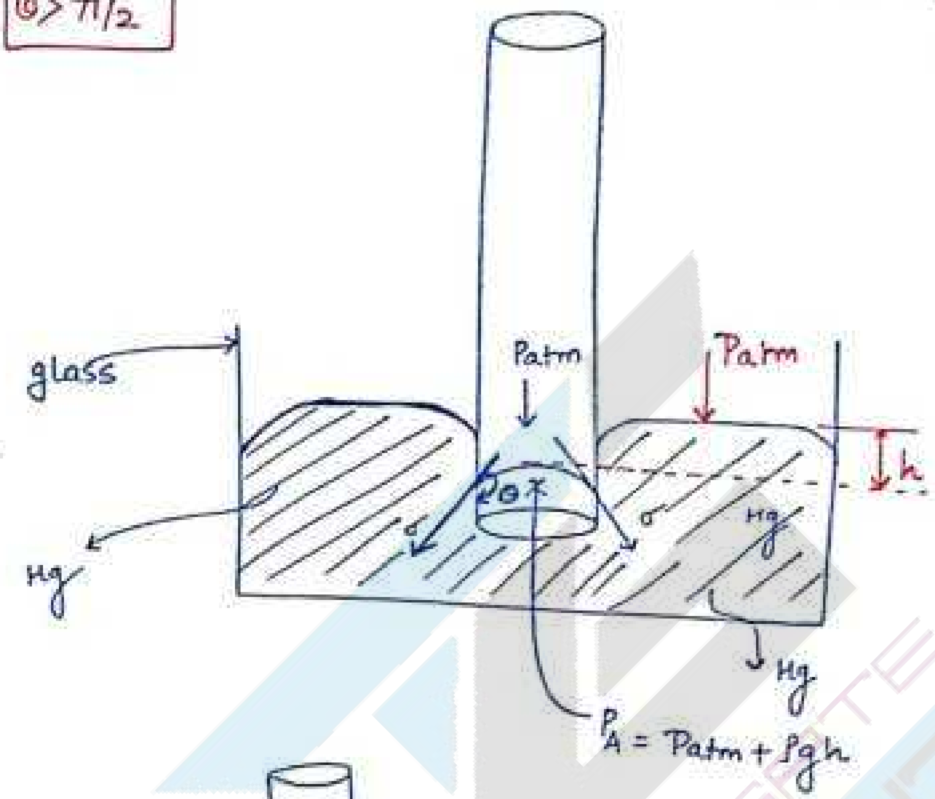
* Capillarity Fall

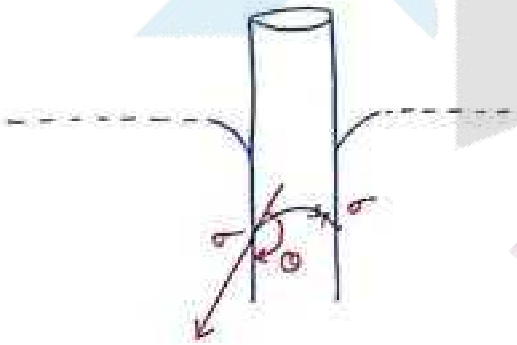
$$\boxed{h = -\frac{\sigma\, 4\cos\theta}{\omega d}}$$

fall

here, hydrostatic force balance करता

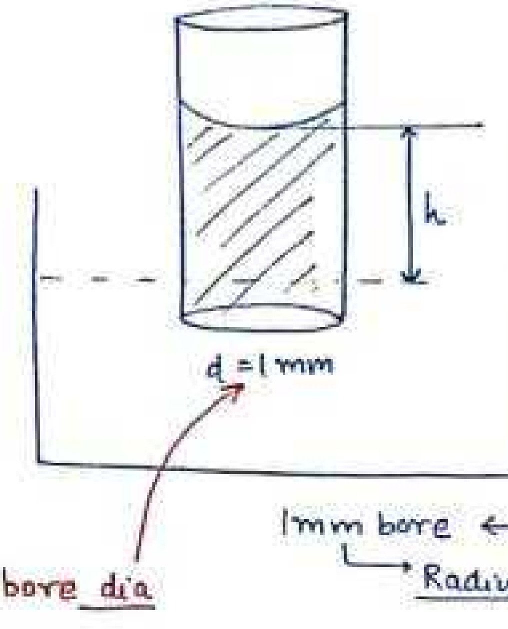
$\theta > \pi/2$





1 d	4 c	7 b	10 b	13 c	16 discuss —	19 b	22 b	25 a
2 a	5 d	8 b	11 a	14 c	17 a	20 c	23 a	26 d
3 b	6 a	9 a	12 d	15 d	18 b	21 b	24 c	27

(26)



$\sigma_w = 0.075$

$$h = \frac{4\sigma\cos\theta}{wd}$$

$\theta = 0° \leftarrow$ here

1mm bore ⟵ civil eng. ⟵ ies exam ⟶ in Real Qn.
↳ Radius

(27) b
(28) b
(29) b
(30) 822.78
(31) 0.428
(32) 20 KPa
(33) 20μ Pas ⟶ 20 milli Pas ⟶ 0.04 Ans
↓ 4 ↑ collection

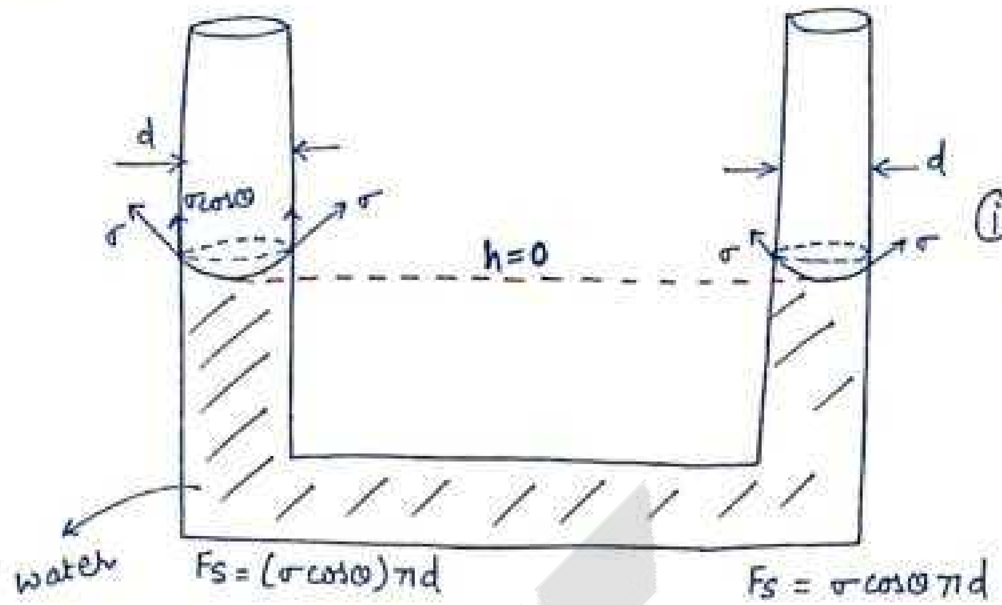
(34) HW 38.68
(35) 250 MPa
(36) 288
(37) 8.59 = 8.6 mm

(38) —

(T1) Terminal Velocity
↓
constant V
↓
accr. → 0
↓
net forces → 0

NOTE :-

$\boxed{d < 10\,mm}$



(i) $d_1 = d_2$

$h = 0$

$\boxed{d_2 < 10\,mm}$

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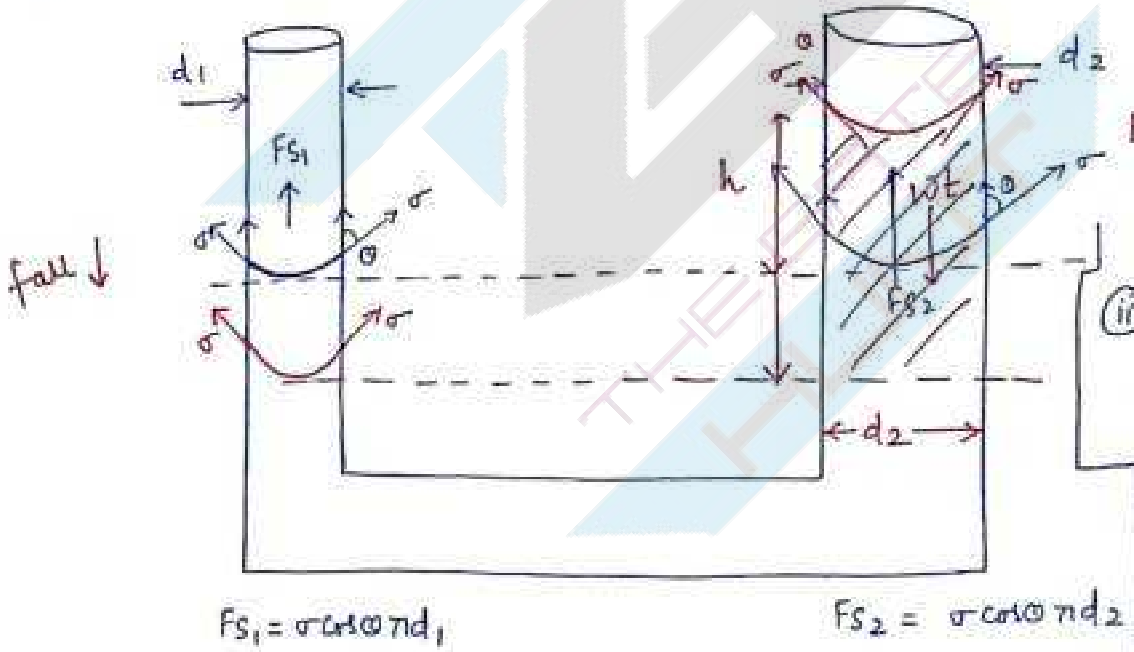


(ii) $d_1 = 0$

$h = \dfrac{4\sigma\cos\theta}{w d_2}$

Single Tube का Rise

(iii) $d_1 > d_2$

$h < 0$

दूसरी तरफ Rise होगा

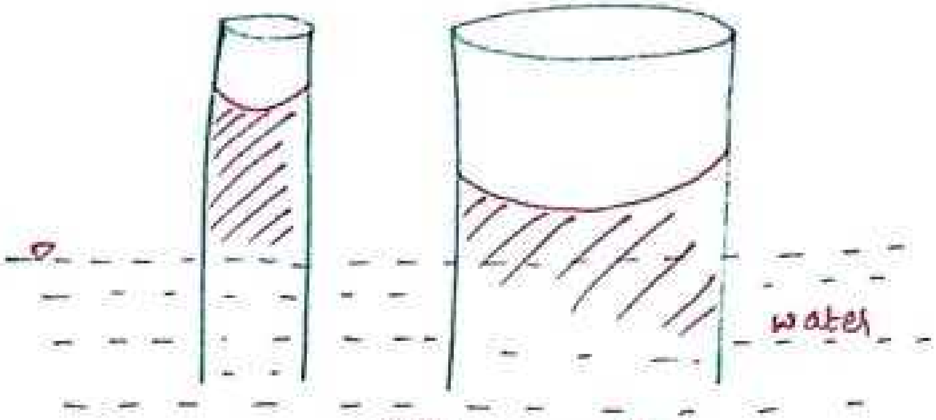
$h = \dfrac{4\sigma\cos\theta(d_1 - d_2)}{w d_2^2}$

$$wt = F_{s_2} - F_{s_1}$$

$$w \times \left(\frac{\pi}{4} d_2^2\right) \times h = \sigma\cos\theta\,\pi\left[d_2 - d_1\right]$$

$$\boxed{h = \frac{4\sigma\cos\theta(d_2 - d_1)}{w d_2^2}}$$

But



दोनों अलग हैं

Left Points :-

* surface tension is due to cohesive forces ——— and capillarity is due to cohesive and adhesive forces.

MOHIT CHOUKSEY

CHAPTER-2

PRESSURE MEASUREMENT

MOHIT CHOUKSEY

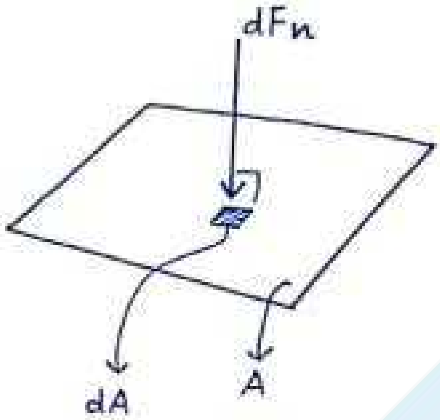
CHAPTER 2

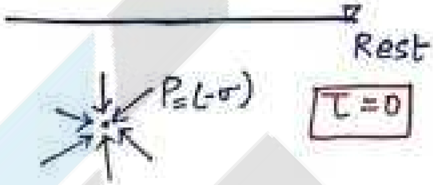
PRESSURE MEASUREMENT

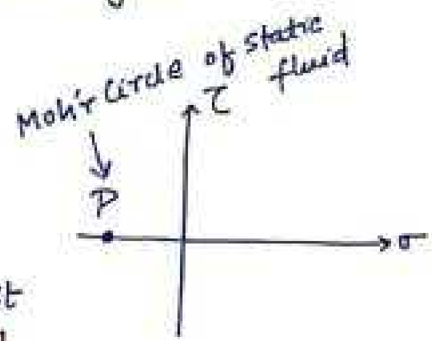
Normal Force Exerted per unit area by the fluid is known as pressure.

Pressure is compressive in nature. As per the Pascal, in the rest, the fluid have no direction that is it have ∞ (infinite) direction.

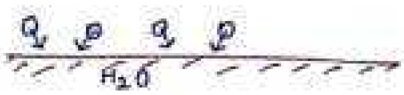
So it is Scalar Quantity. (due to ∞ dir^n of Pressure – any dir^n)







$$\boxed{P = \frac{dF_n}{dA}} \quad \frac{N}{m^2}$$



$F_n = F\sin\theta$

$F_s = F\cos\theta$

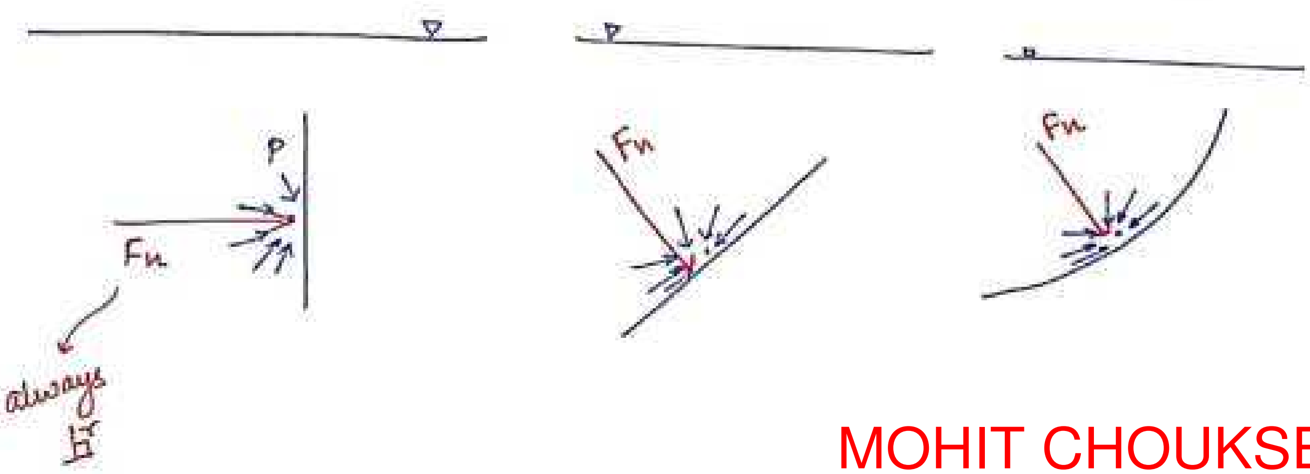
$\tau = 0$

$F_s = 0$

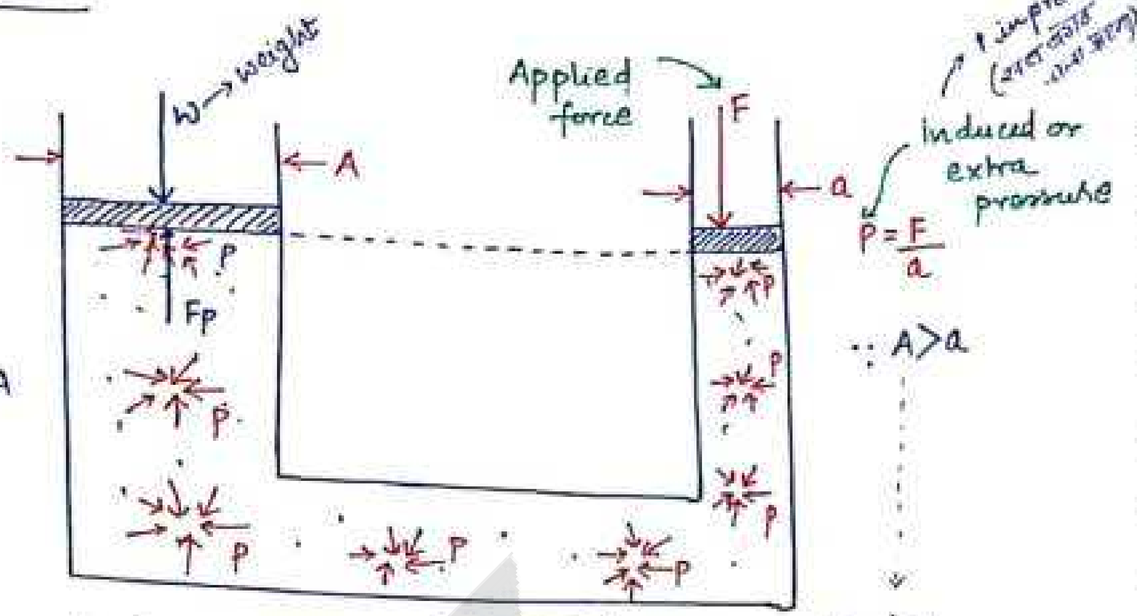
$F \neq 0$

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$



Hydraulic Press



$F_p = P \times A$

$P = \frac{F}{a}$

$\because A > a$

$\therefore F_p = W$

$\Rightarrow P \times A = W$

$\Rightarrow \frac{F}{a} \times A = W$

$\Rightarrow \boxed{\frac{A}{a} = \frac{W}{F}}$

$\because \frac{W}{F} > 1$

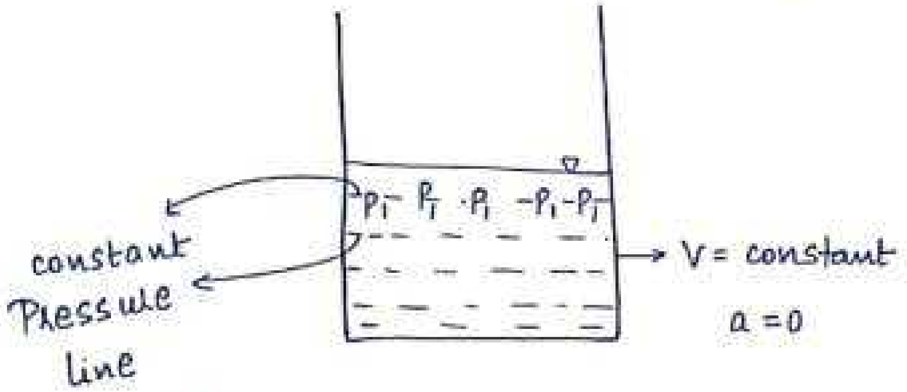
$\Rightarrow \boxed{W > F}$

(viscous effect occurs when fluid in motion)

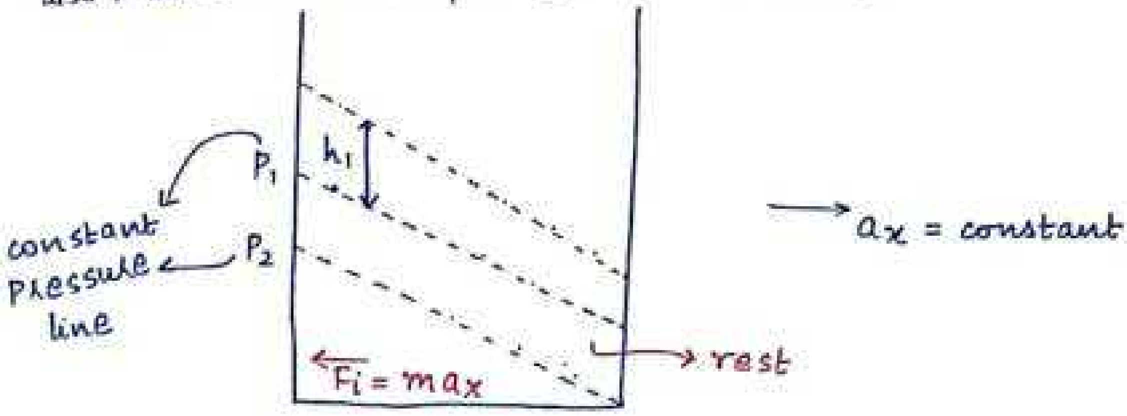
* **Validity of Pascal's law** :- • It is valid for compressible, incompr., viscous, non-viscous fluids but the fluid must be at rest.

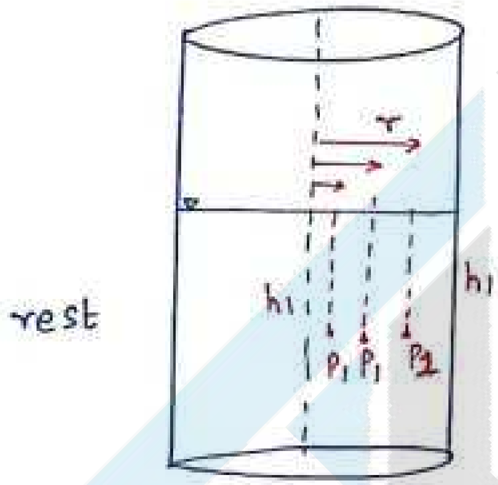
• Fluid in a container, container is moving with constant velocity, uniform acceleration and uniform angular velocity.
(can be treated at Rest)
because relative motion b/w adjacent layer

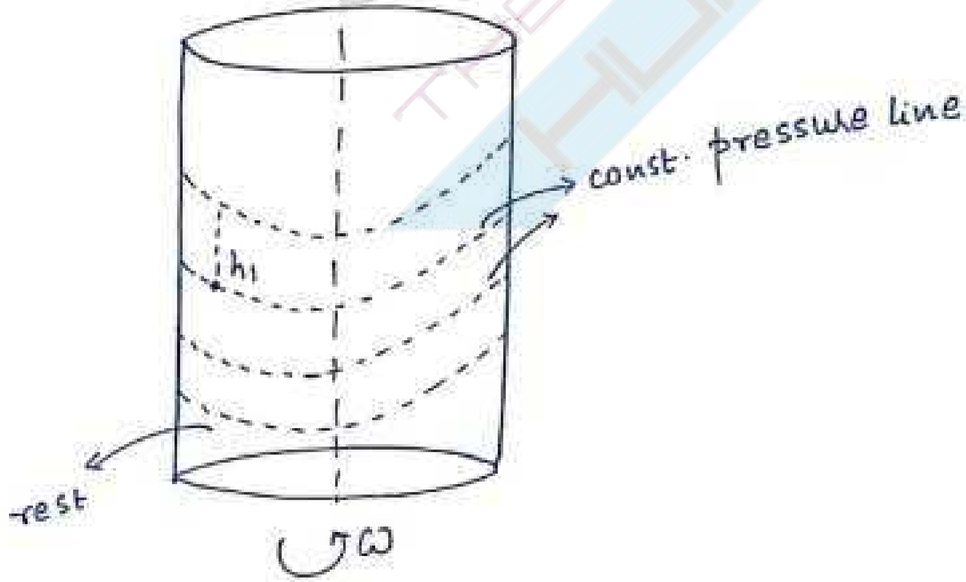
• **Ideal Fluid** :- N.P. → Next Page



* This P in pressure is transmitted to entire section, section, part or molecule of fluid. (3)
also P has no direction at a point (hence scalar not vector)







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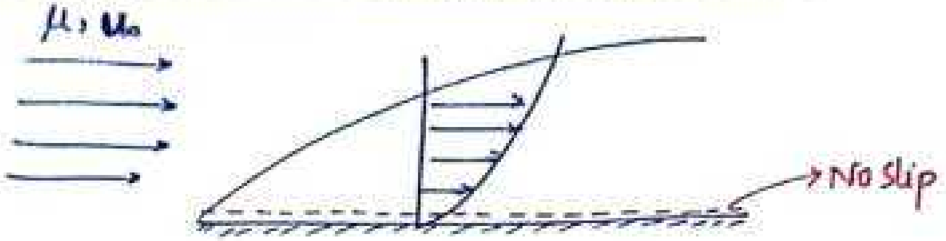
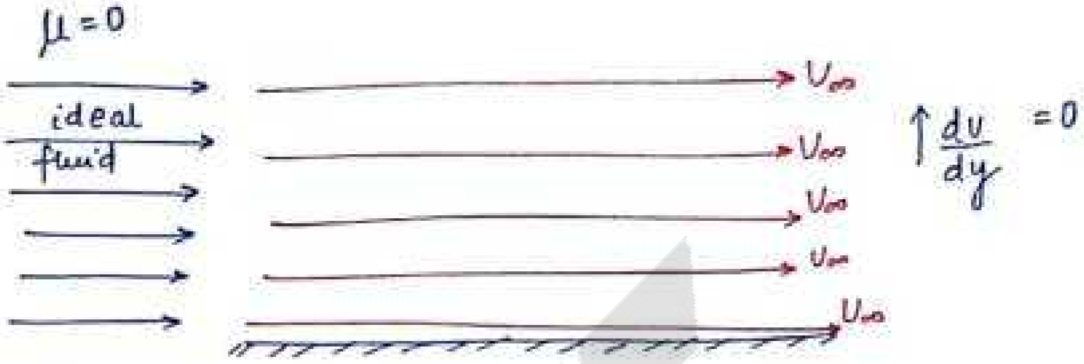




ideal fluid ← just a concept

$\mu = 0$, cohesive force $= 0$, $\sigma = 0$, incompressible

not a gas ∵ gas compressible

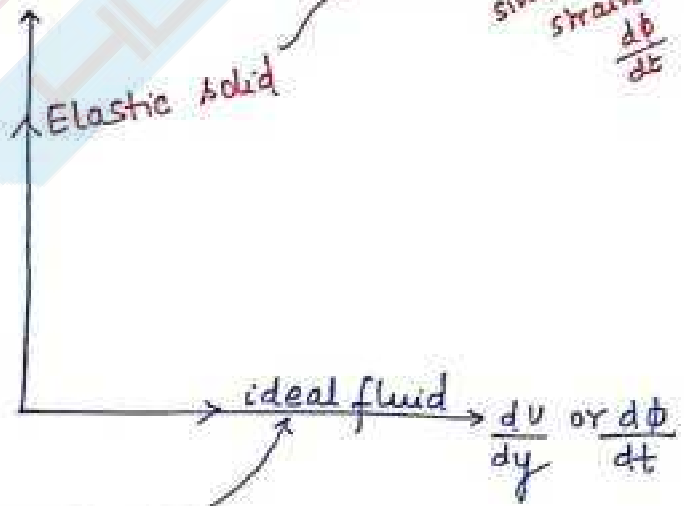
* ideal fluid $\boxed{\mu = 0}$

$$\tau = \mu \frac{du}{dy}$$

$$\tau = 0 \times 0$$

$$\tau = 0$$

solid never becomes liquid since Rate of strain always $\frac{d\phi}{dt}$



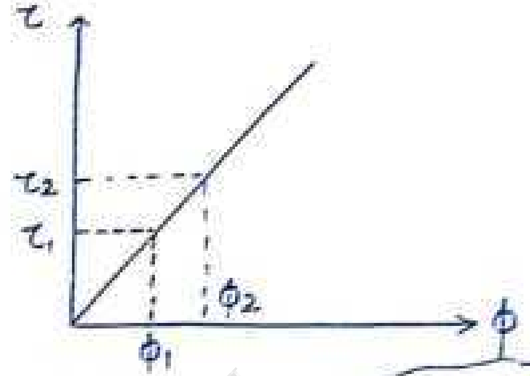
MOHIT CHOUKSEY

*

elastic solid

$\frac{d\phi}{dt}$

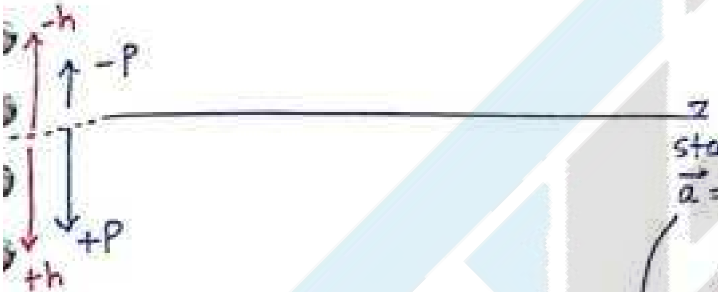
Elastic solid



Capillary में adhesion + cohesion
(side वाले को) (करीब वाले का अपने के बीच)

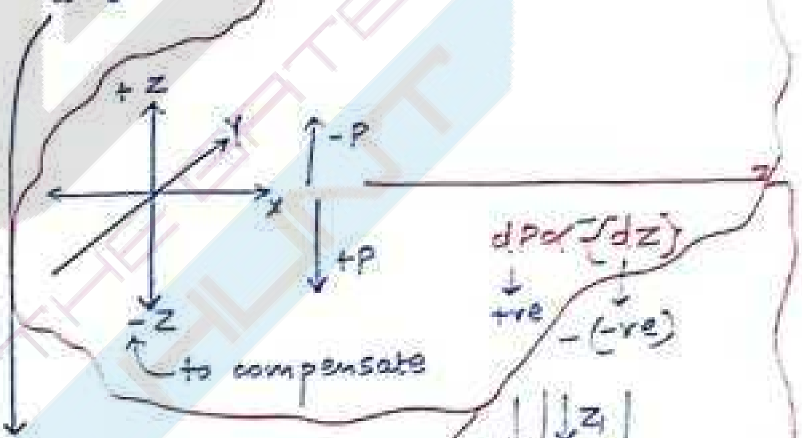
Hydrostatic Law :-

* Our Convention :-

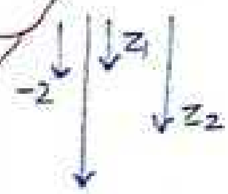


static fluid
$\vec{a} = 0$

In cartesian system



$dP \propto -(dz)$
+ve -(-ve)



$dz = z_2 - z_1$
$= -1$

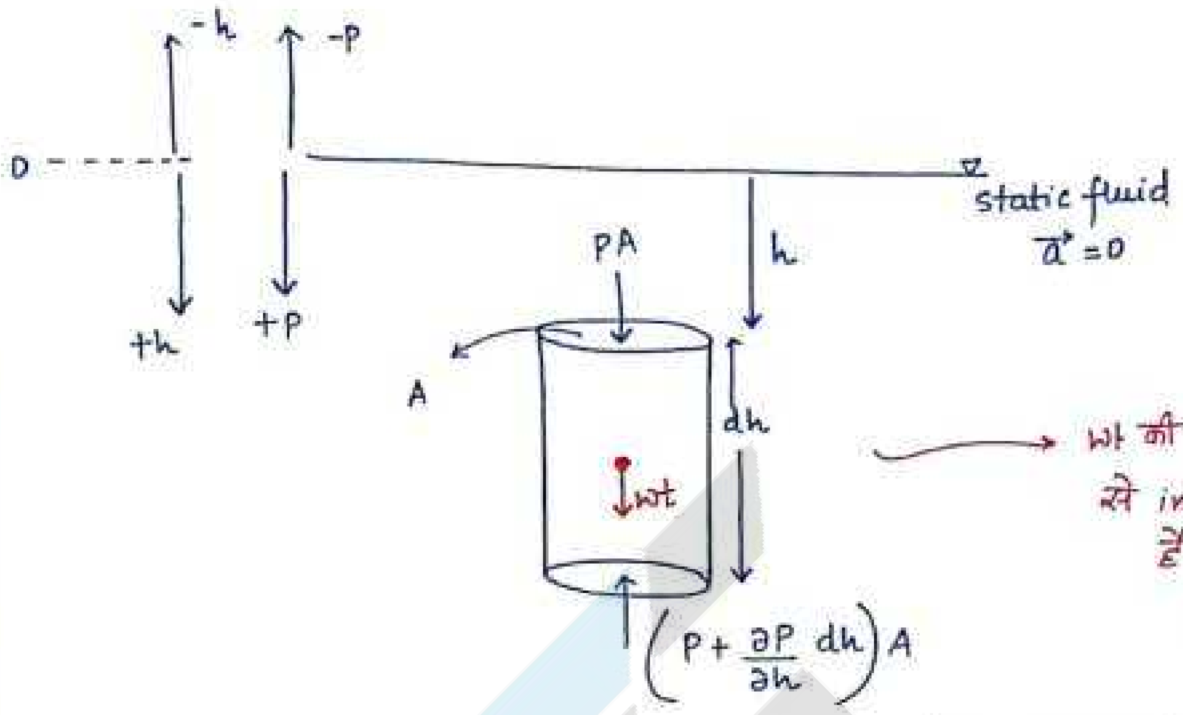
$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

0 =	0	0	0 ✓
0 ≠	0	0	4 ✗
0 =	4	-4	0 ✗

Vertical dir^n में analysis → N.P.

$dP \propto (-dz)$

$\frac{dP}{dz} = -\omega$



wt की वजह से increment है pressure में.

wt की dirn हमेशा ↓ vertical downward
hence 'P' analysis ↓ down dirn में.

$$\Sigma F_{net_z} = m \cancel{a_z}^{\,0}$$

$$\Rightarrow PA + wt - \left(P + \frac{\partial P}{\partial h} dh\right) A = 0$$

$$\Rightarrow \frac{\partial P}{\partial h} \cancel{(A\,dh)} = w \times \underbrace{\cancel{(A\,dh)}}_{\text{volm.}}$$

weight density $w = \rho g$

$$\Rightarrow \boxed{\frac{\partial P}{\partial h} = w} \rightarrow \boxed{\frac{\partial p}{\partial h} = \rho g}$$

Fluid incompressible

$\rho \neq f(P)$ →

$$K = \frac{dP}{(-dV/V)} = \frac{dP}{(d\rho/\rho)} = \rho \frac{dP}{d\rho}$$

$$B = \frac{1}{K} = \frac{1}{\rho} \frac{d\rho}{dP} \qquad \left(\frac{1}{\rho} \neq 0\right)$$

if $B = 0$ so $\frac{d\rho}{dP} = 0$ → $\boxed{\rho \neq f(P)}$

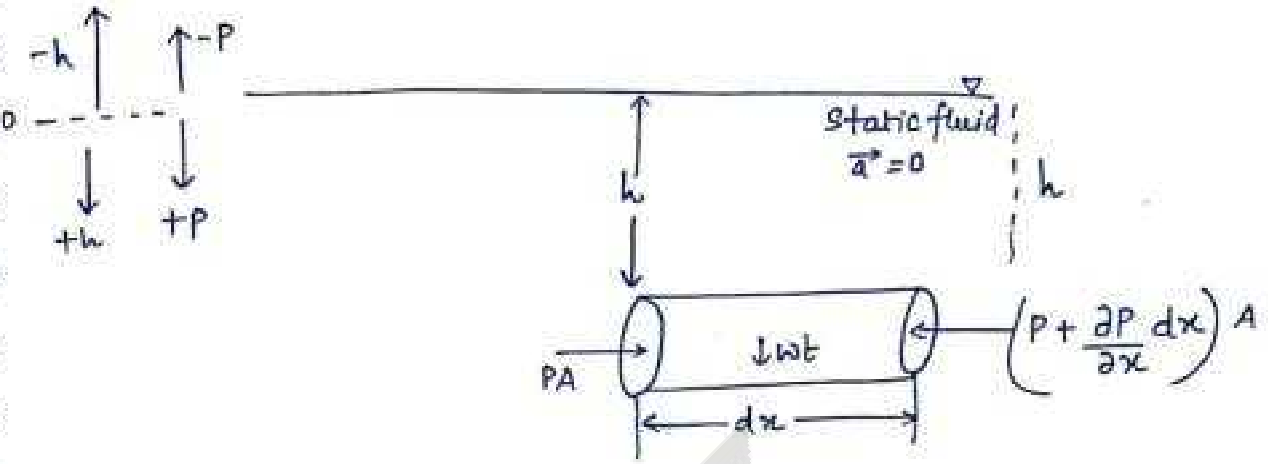
compressibility factor = 0 for incompressible fluid

for incompressible fluid

$$\boxed{dP \propto dh}$$

+ve +ve

NB ① $\xrightarrow{\text{ch-2}}$ d (symbols - - - - meaning) → means x, y, z coordinate



$$\Sigma F_{net_x} = m\cancel{a_x}^{0}$$

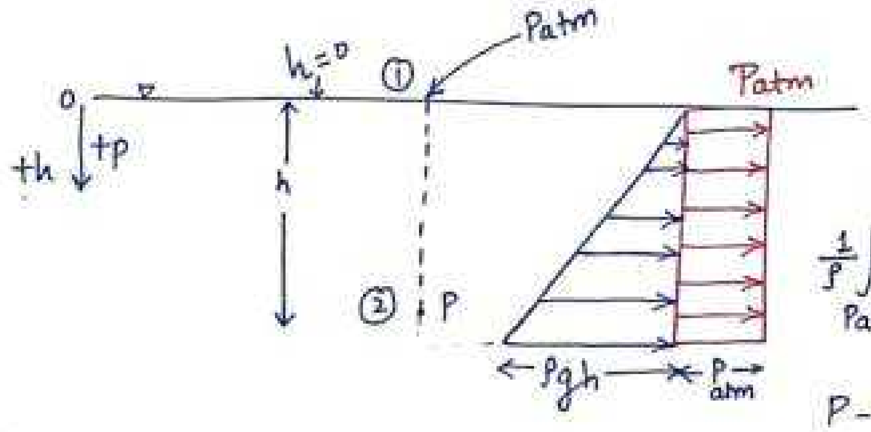
$$PA - \left(P + \frac{\partial P}{\partial x}dx\right)A = 0$$

$$\frac{\partial P}{\partial x}dx\,A = 0$$

$A\,dx \neq 0$ (since area x length cant be equals to 0)

$$\boxed{\frac{\partial P}{\partial x} = 0}$$

NOTE :-



$$\frac{dP}{dh} = \rho g$$

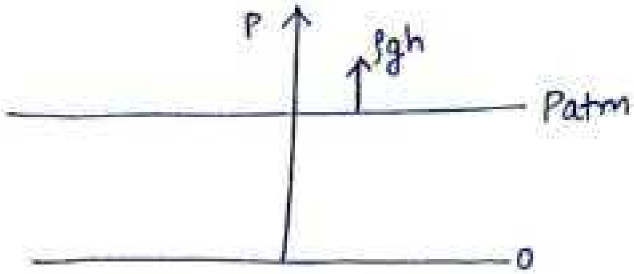
$(\rho \neq P)$

incompressible fluid

$$\frac{1}{\rho}\int_{P_{atm}}^{P} dP = g\int_{0}^{h} dh$$

$$P - P_{atm} = \rho gh$$

$$\boxed{P = P_{atm} + \rho gh}$$ Absolute



linear variation ✓ incomp. fluid **

Patm = 0

$\boxed{P = \rho g h}$ Gauge

$P \propto h$

Now, we are assuming compressible ($\rho = f(P)$)

fluid is compressible:
(Gas) eg: air

$$\frac{dP}{dh} = \rho g$$

$P = \rho R T$
T = constant
(isotherm condition)

$$\rho = \frac{P}{RT}$$

$$\Rightarrow dP = \frac{P}{RT} g \cdot dh$$

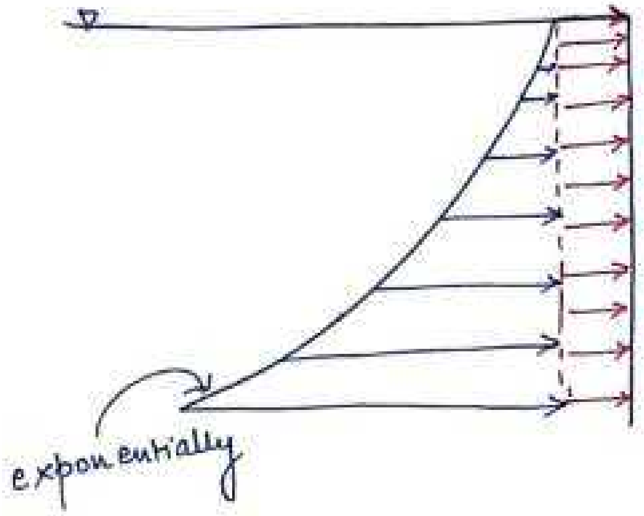
$$\Rightarrow \int_{P_{atm}}^{P} \frac{dP}{P} = \frac{g}{RT} \int_0^h dh$$

$$\Rightarrow \ln\left(\frac{P}{P_{atm}}\right) = \frac{gh}{RT}$$

$$\Rightarrow e^{gh/RT} = \frac{P}{P_{atm}}$$

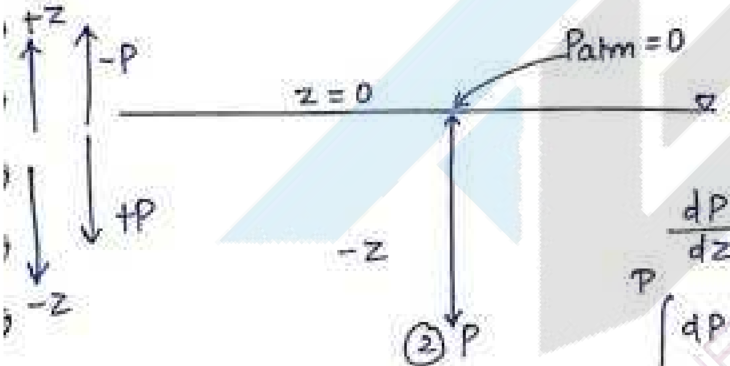
$$\Rightarrow P = P_{atm} e^{gh/RT}$$

$$\Rightarrow \boxed{P = P_{atm} e^{\frac{g}{RT} \cdot h}}$$



for Adiabatic, $P = \rho^{\gamma} C \rightarrow \rho = \left[\frac{C}{P}\right]^{1/\gamma} \rightarrow \rho = \frac{C'}{P^{1/\gamma}}$

cartesian system



$$\frac{dP}{dz} = -\omega$$

$$\int_0^P dP = -\rho g \int_0^{-z} dz$$

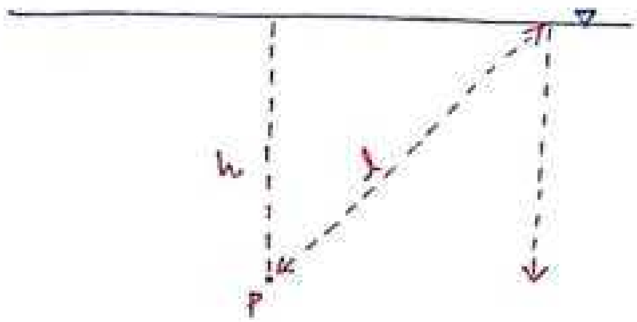
$$\boxed{P = \rho g z}$$

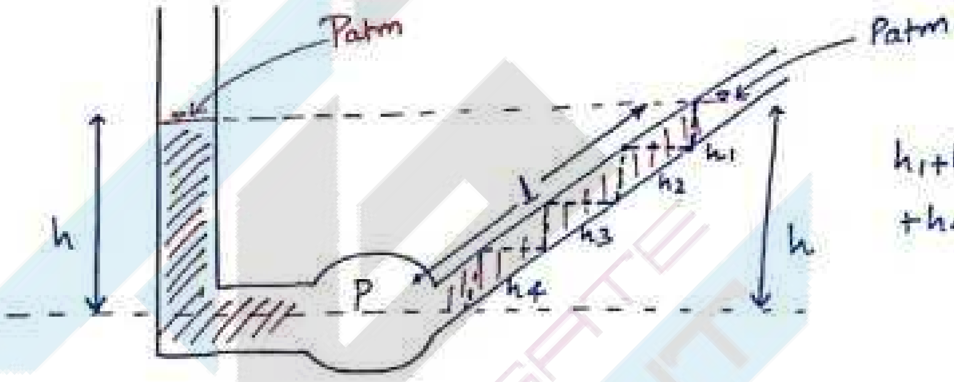
hence, Results same

sirf convention change होनेसे notation change है But same Results आएंगे।

* earth की Toposphere में exponential flow.

* Special Case





$h_1 + h_2 + h_3 + h_4 = h$

$P > P_{atm}$

$P_{atm} + \rho g h = P$

$P_{atm} = \rho g h_1 + \rho g h_2 + \rho g h_3 + \rho g h_4$

$P = P_{atm} + \rho g h$

$\cancel{P = P_{atm} + \rho g l}$

inclined depth/length no use in 'P' measurement

But ∠ length use l

$\boxed{l \sin\theta = h}$

$\sin\theta < 1$

$\boxed{l > h}$

same 'p' is measured in more length → hence 'P' is magnified.

* HYDROSTATIC PARADOX → [झुठलावा / अपनी बात को ही contradict करना
अपनी ही बात का अपवाद]

(11)

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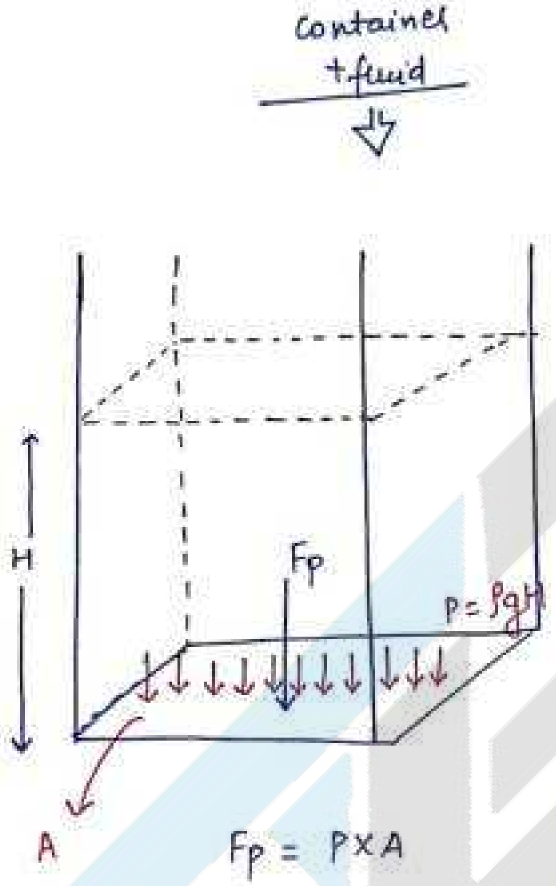
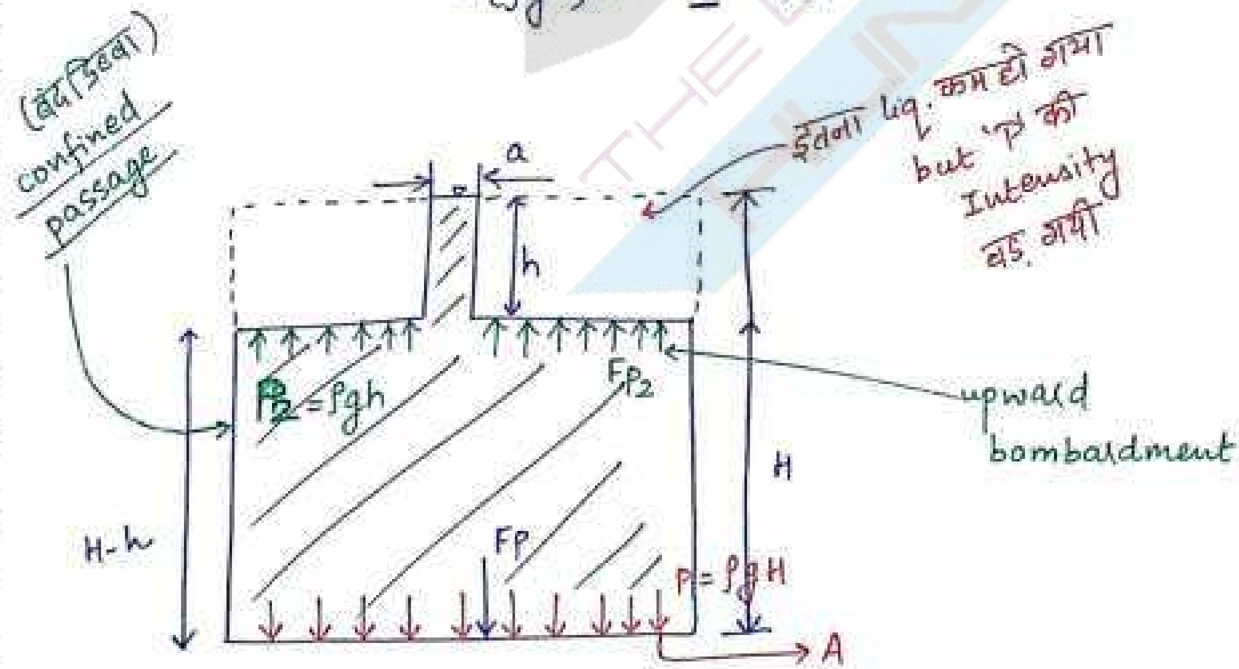


Wt = w × volume (weight density (ρg))

= w × HA (↓)

Fp = P × A   [wH = (ρg) H]

= (ρgH) A = wHA (↓)



Fp = P × A   [P = ρgH = wH]

= wHA (↓)

$$Wt = W \times \text{volume}$$

$$= W\left[A(H-h) + ah\right]$$

$$= WAH - wAh + wah$$

$$\boxed{Wt = WAH - wh(A-a)}$$

$$F_{P_2} = (\rho g)h(A-a)(\uparrow) = \omega h(A-a)(\uparrow)$$

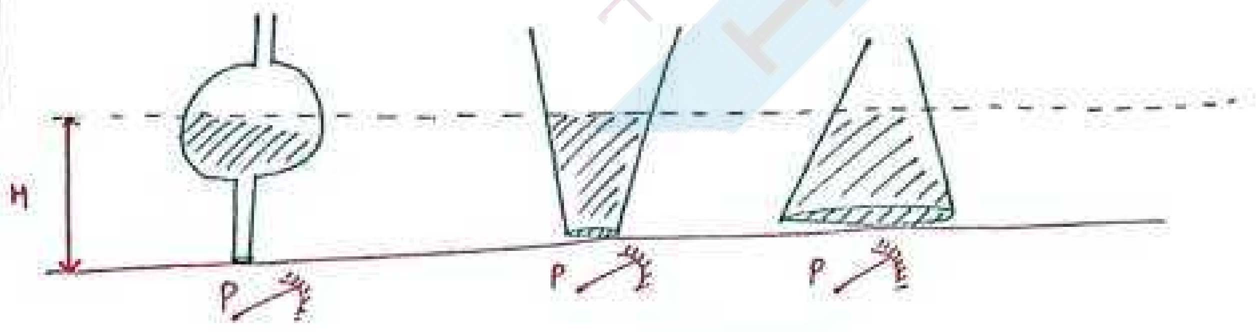
$$(F_{net})_v(\downarrow) = F_{P_1}(\downarrow) - F_{P_2}(\uparrow)$$

Net force by the fluid on the container in vertical direction $= WAH - [\omega h(A-a)]$

$= \text{—''—}$

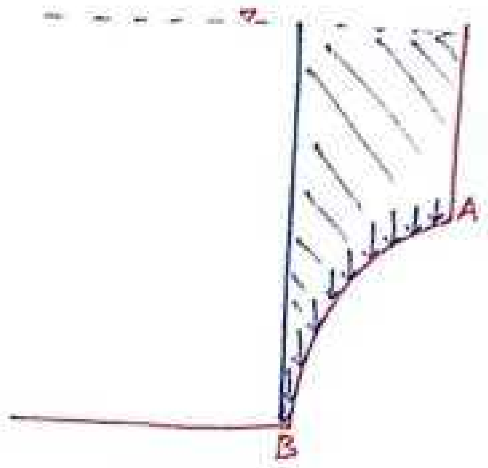
$$(F_{net\,v})\downarrow = \underline{WAH - \omega h(A-a)}$$

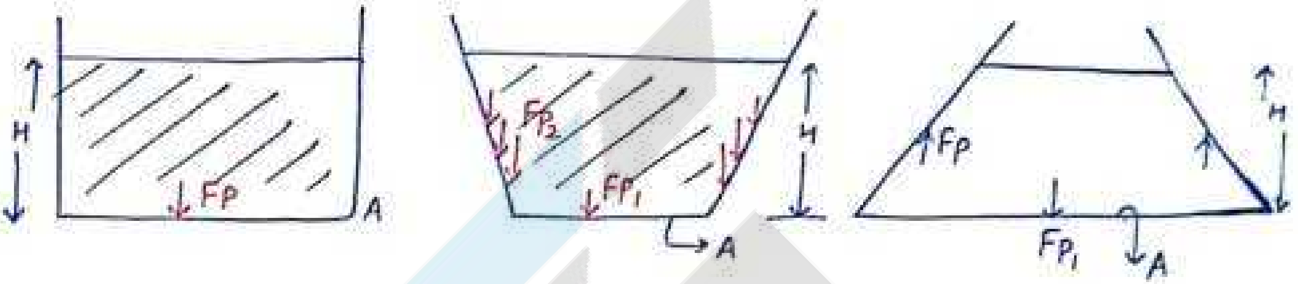
$$\boxed{(F_{net\,v})\downarrow = (Wt)\downarrow}$$



$$\boxed{P = P = P}$$

$$\underline{F_{P_1} < F_{P_2} < F_{P_3}} \quad (\because A_3 > A_2 > A_1)$$





जब P की बात होगी तो A, h देखेंगे
और --,,-- W ——— ,, ——— ,, ——— vertical Downward force देखेंगे।

The hydrostatic pressure force depends only the depth of the liquid in the container, it is independent of weight of liquid.

UNITS OF PRESSURE :-

$P = F/A \Rightarrow (N/m^2)$

$\boxed{N/m^2 = Pa}$

$10^3 \, N/m^2 = kPa$

$10^6 \, N/m^2 = MPa$

$\frac{1 \, kgf}{cm^2} \approx \frac{10 \, N}{10^{-4} \, m^2}$

$\frac{kgf}{cm^2} = 10^{+5} \, \frac{N}{m^2}$

$\boxed{kgf/cm^2 = 1 \, Bar}$

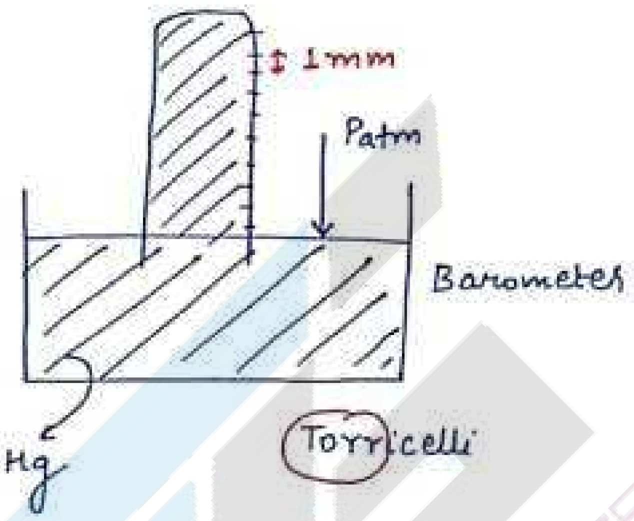
$P_{atm} = 101.325\ kPa$

$= 1.01325 \times 10^5\ Pa$ → बार बार आएगी

$1\ P_{atm} = 1.01325\ bar$ ← Britain

$$\boxed{1\ P_{atm} \approx 1\ Bar}$$

Manometer



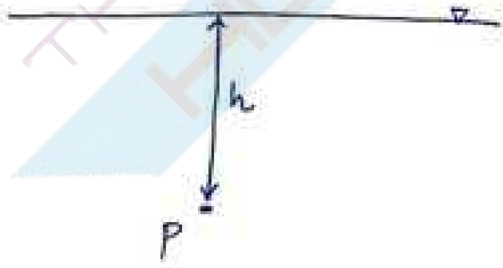
Torricelli

$$\boxed{1\ Torr = 1\ mm\ of\ Hg}$$

$P = \omega h \left(N/m^2 \right)$

fluid का specific weight

also called weight Density



$$\boxed{h = \frac{P}{\omega_l}}$$ → head of liquid column

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$1\ Torr = 10^{-3}\ m$ of Hg $\left(h = \frac{P}{\omega_L}\right)$

$= 10^{-3}\ (13.6 \times 9810) \rightarrow (h \times \omega_{Hg} = P)$

$(h \times S \times \omega_L = P)$

(13.6) $\left[\begin{matrix} 1000 \times 9.81 \\ \rho_{w} g \end{matrix}\right]$

$\boxed{1\ Torr = 133.33\ Pa}$

$S_{Hg} = \frac{\omega_{Hg}}{\omega_L \rightarrow H_2O}$

F.P.S. → Foot, Pounds, Sec

$\boxed{Psi = \frac{Lbf}{(inch)^2}}$

Lbf ⟹ pounds force

1 inch ⟹ 2.54 cm

Lbf = 0.453 kgf

$\boxed{1\ atm = 14.7\ Psi}$ *** (for SSC-JE exams)

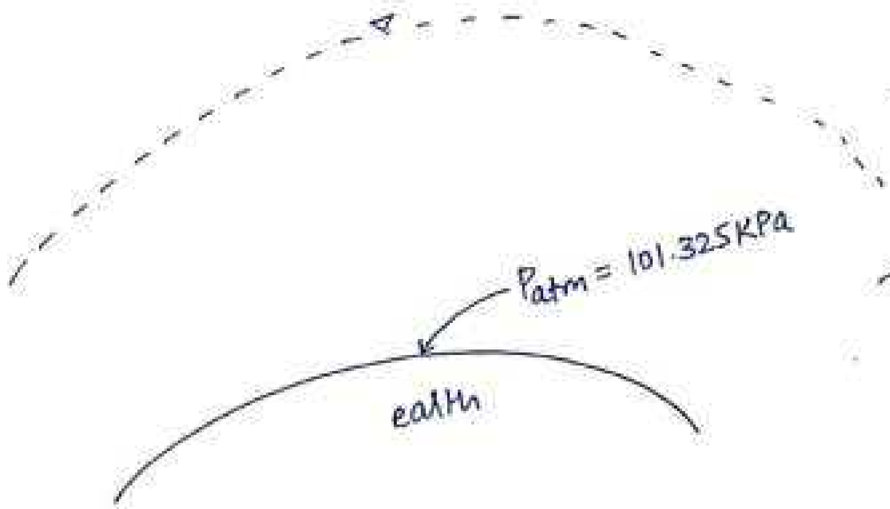
$$Psi = \frac{0.453\ kgf}{(2.54\ cm^2)} = 0.0702\ \frac{Kgf}{cm^2} = 0.0702\ Bar = 0.0702\ atm$$

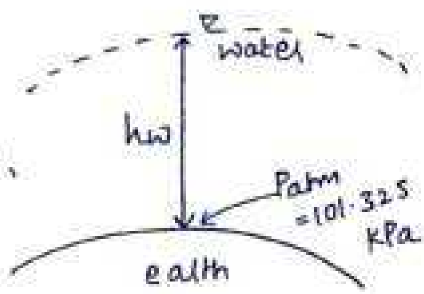
Psi = 0.0702 atm

$\boxed{1\ atm = \frac{1}{0.0702}\ Psi = 14.7\ Psi}$

* ATMOSPHERIC PRESSURE :-

The Pressure exerted by the environmental mass on the earth surface is known as Atmospheric pressure.



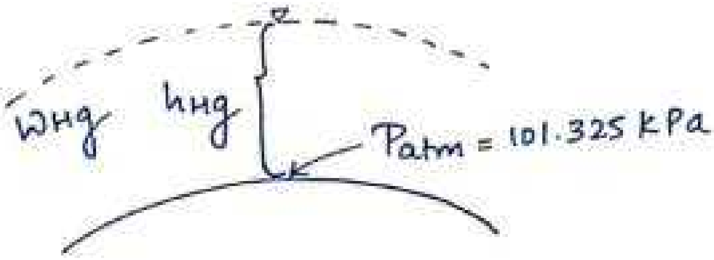


$\boxed{\omega_w = \rho_w g}$

$\Rightarrow \omega_w h_w = P_{atm}$

$9810 \times h_w = 101.325 \times 10^3$

$h_w = 10.3\ m$ of water column

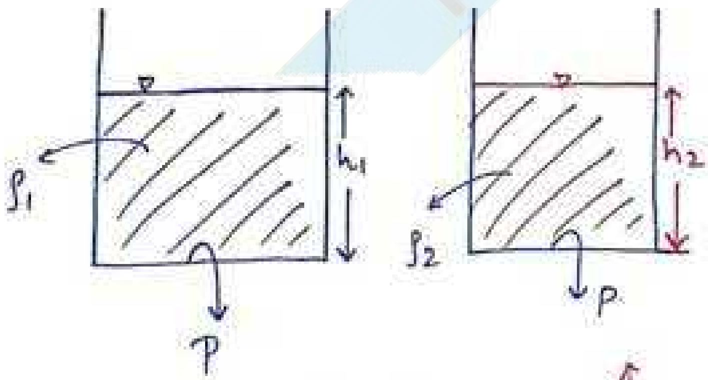


$\omega_{Hg} h_{Hg} = 101.325 \times 10^3$

$h_{Hg} = \dfrac{101.325 \times 10^3}{13.6 \times 9810}$

$\boxed{h_{Hg} = 760\ mm \text{ of } Hg \text{ or } = 76\ cm \text{ of } Hg}$

CONVERSION OF ONE LIQUID COLUMN TO THE ANOTHER:-



for same pressure

$\rho_1 g h_1 = P$ $\qquad$ $\rho_2 g h_2 = P$

$$\boxed{\rho_1 g h_1 = \rho_2 g h_2}$$

$$\boxed{\rho_1 h_1 = \rho_2 h_2}$$

$$\boxed{h_2 = \frac{\rho_1 h_1}{\rho_2}}$$

$$\boxed{w_1 h_1 = w_2 h_2}$$

$$\boxed{h_2 = \frac{w_1 h_1}{w_2}}$$

also, $\rho_1 h_1 = \rho_2 h_2$

$$\frac{\rho_1}{\rho_w} h_1 = \frac{\rho_2}{\rho_w} h_2$$

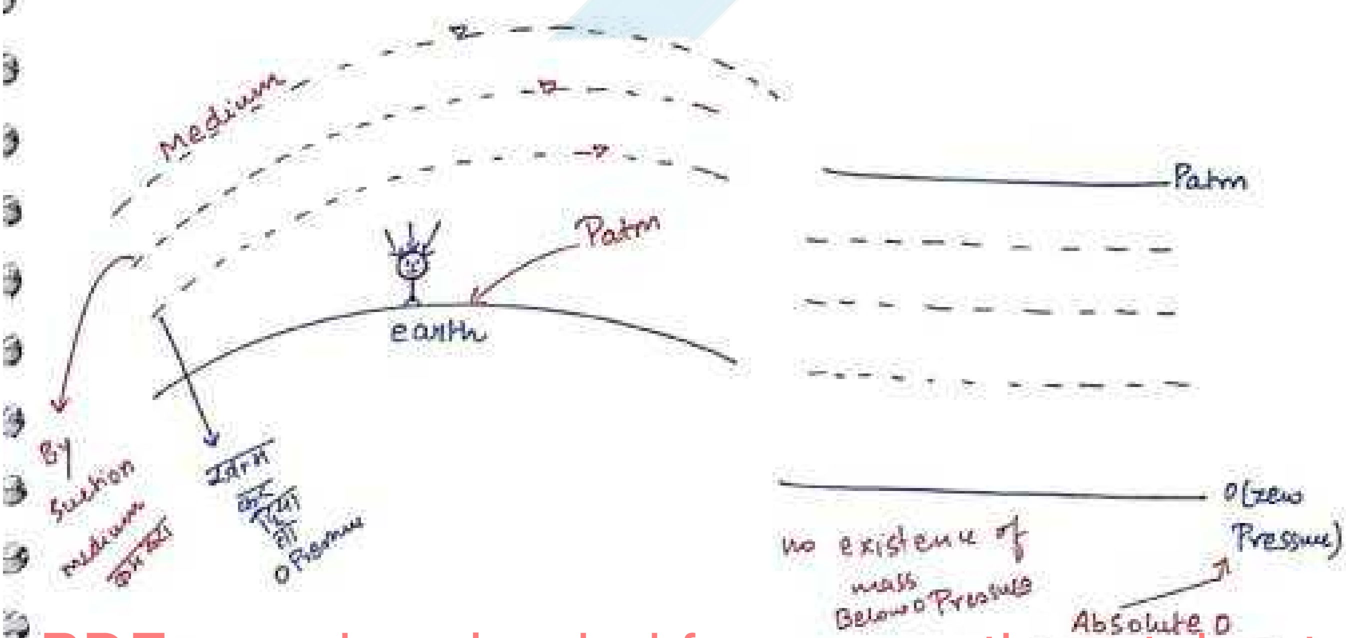
$$\boxed{S_1 h_1 = S_2 h_2}$$

$$\boxed{h_2 = \frac{S_1 h_1}{S_2}}$$

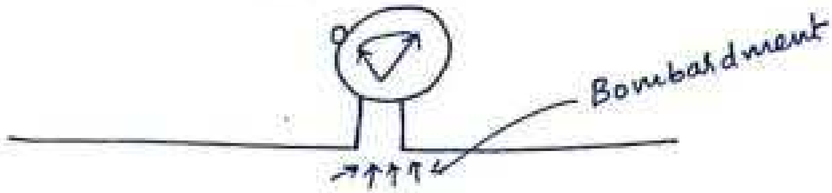
$S_2 \Rightarrow$ water $\boxed{S_2 = 1}$

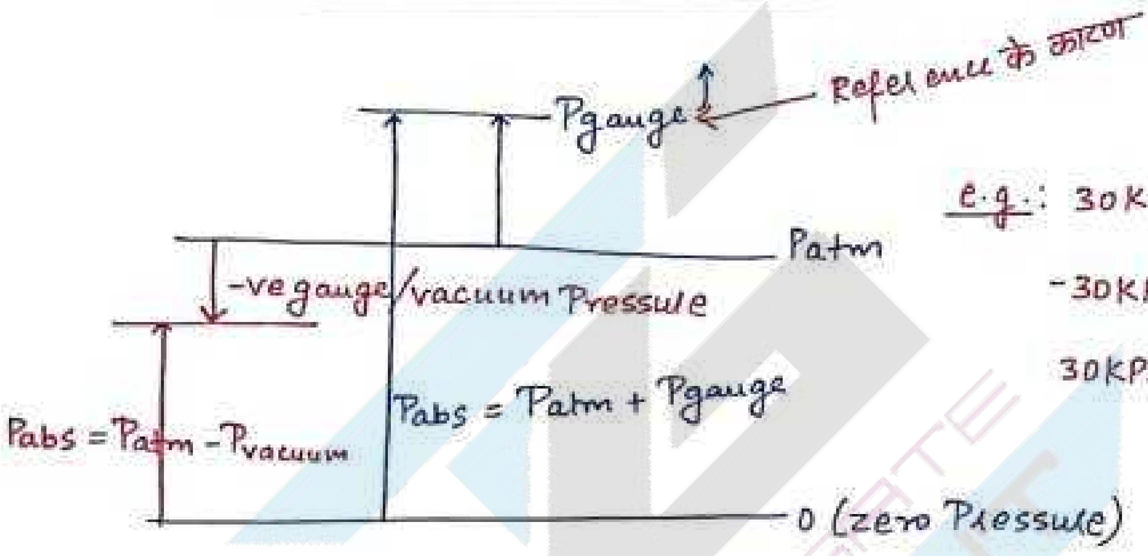
$$\boxed{h_2 = S_1 h_1}$$

ABSOLUTE PRESSURE, GAUGE PRESSURE, NEGATIVE GAUGE PRESSURE OR VACCUUM PRESSURE :-



Bombardment के Reference में
0.

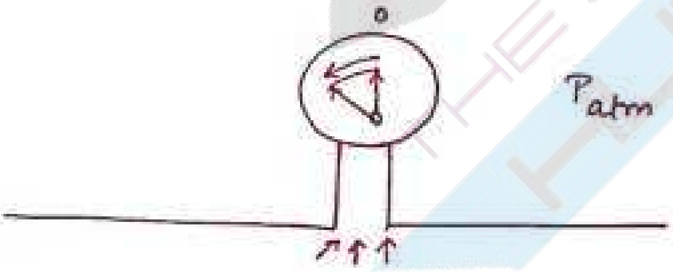




e.g.: 30 KPa Vacuum Pressure

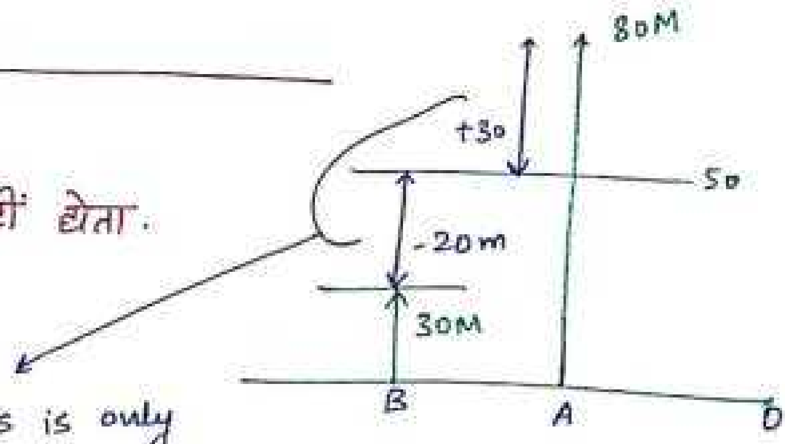
-30 KPa Pressure

30 KPa (-ve) gauge



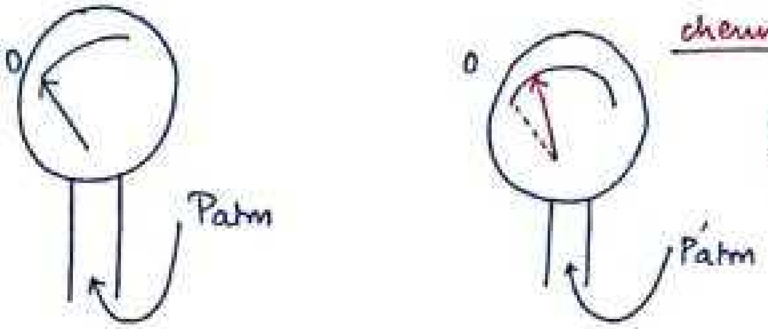
Patm से कितना कम
Bombardment

Abs. Pr. → कभी -ve नहीं होता.



This is only
Reference study

*



chennai

Large Bombardment

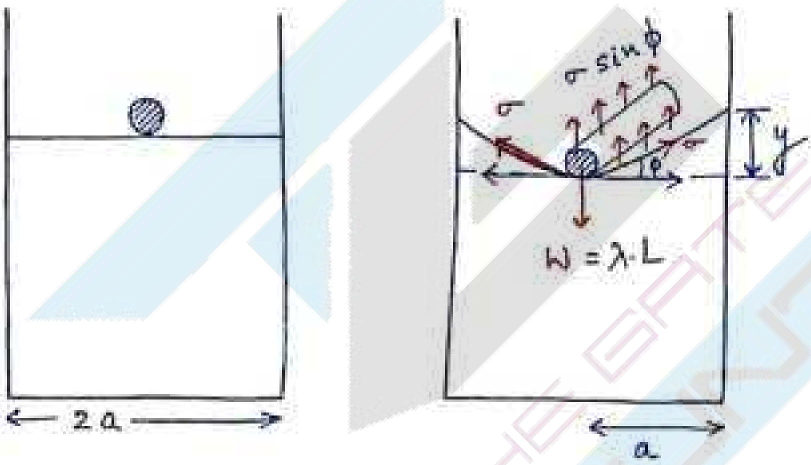
J&K का gauge

$P'_{atm} > P$

Note → There may be +ve gauge pressure, -ve gauge pressure but there cannot be -ve absolute pressure.

Gauges are caliberated as per their local environment.

8/11/2016



$F_s = \sigma \sin\phi . l + \sigma . \sin\phi . l$

$wt = \lambda . l$

$2\sigma \sin\phi . \cancel{l} = \lambda \cancel{l}$

$\sigma = \dfrac{\lambda}{2\sin\phi}$

$\sigma = \dfrac{\lambda}{2(y/a)}$

$\boxed{\sigma = \dfrac{\lambda a}{2y}}$

$\sin\phi = \dfrac{y}{\sqrt{a^2+y^2}}$

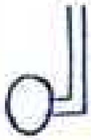
$y <<<< a$

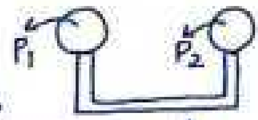
$= \dfrac{y}{a\cancel{\sqrt{1+y^2/a^2}}}$

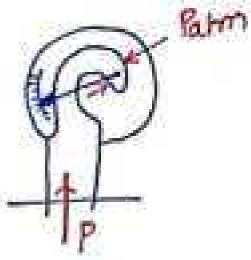
Classification of Pressure measuring instrument :-

Classification

- on the basis of column deflection
  - simple manometer
    - Piezometer
    - U-Tube Manometer
    - Single Column manometer
  - Differential manometer
    - Differential U-Tube manometer
    - Inverted Tube manometer
- on the basis of Elastic properties (Mechanical Gauges) — Not in syll. (X)
  - Bourdon Tube





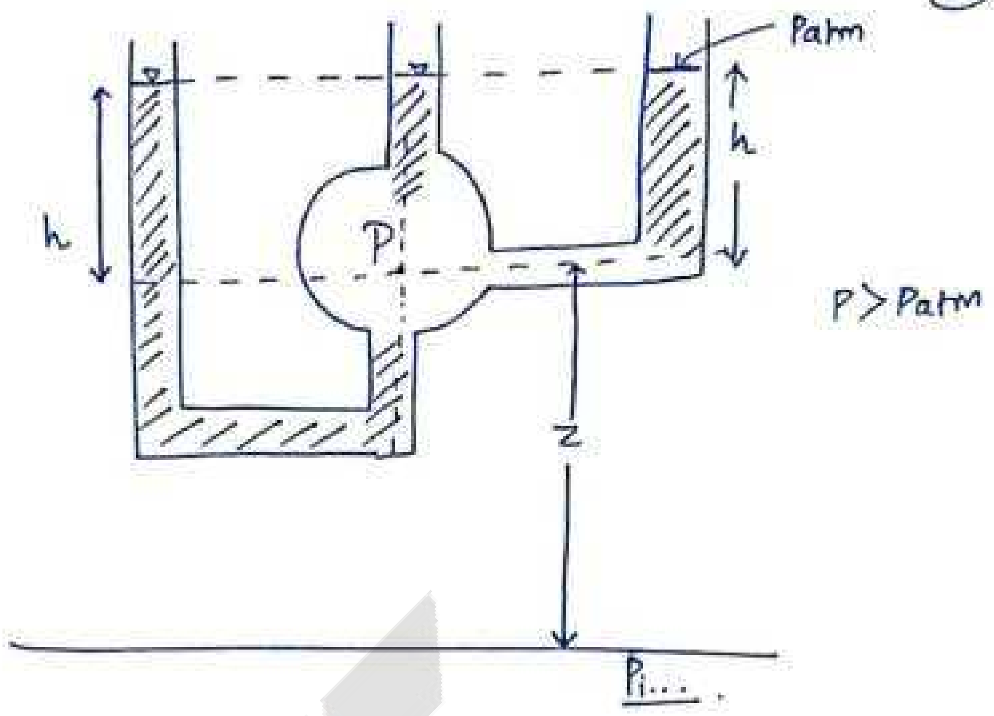


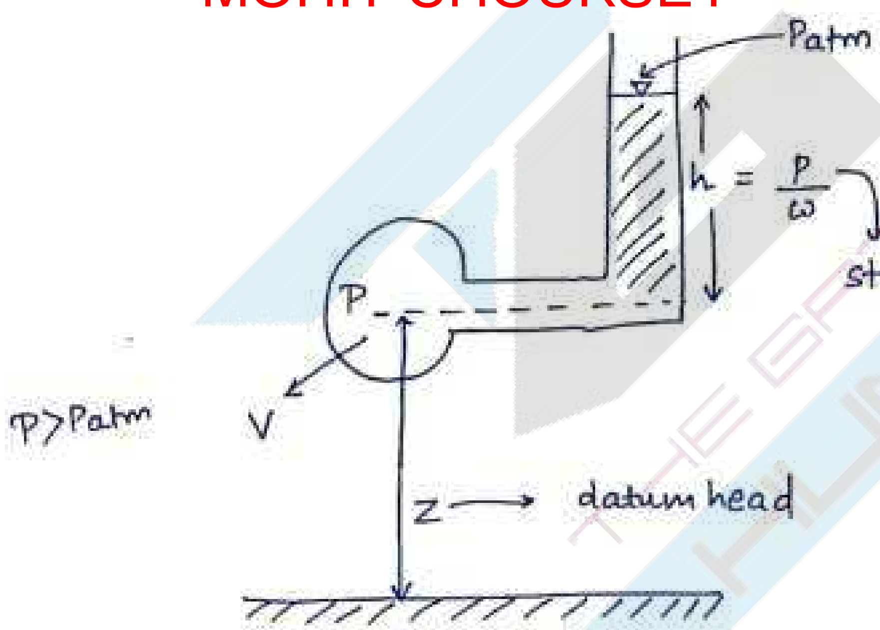
PIEZOMETER :- It is a simple manometer, its both ends are open. one end is connected where pressure is to measure and another end is opened to atmosphere. The disadvantages are :- ① Cannot measure gases pressure.

② — " — " — Vacuum pressure.

③ For high pressure low density fluid gives large deflection, so not suitable for high pressure.

MOHIT CHOUKSEY

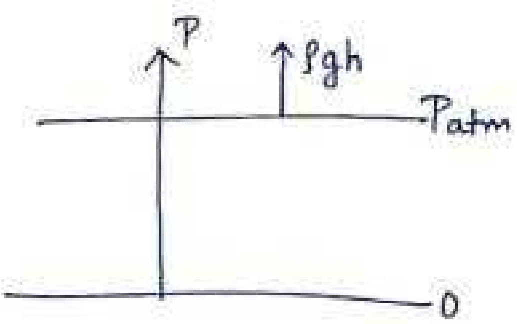




$$P_{atm} + \rho g h = P$$

$$\boxed{P = P_{atm} + \rho g h}$$

Abs



$P_{atm} = 0$

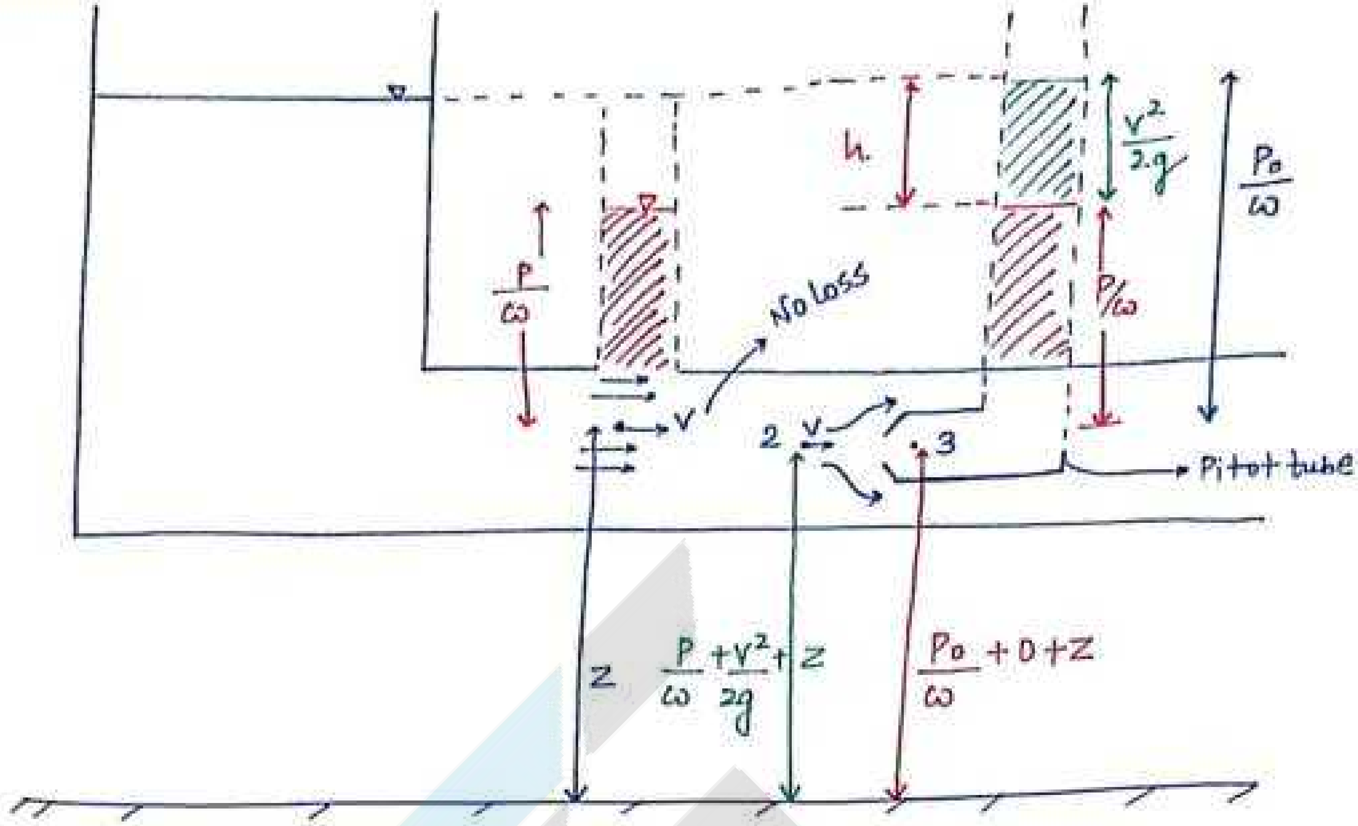
$\boxed{P = wh}$ gauge pressure

$\boxed{P/w = h}$ gauge (head)

Piezometric head = $(P/w + z)$

Piezometer measures gauge-pressure of the Pipeline.

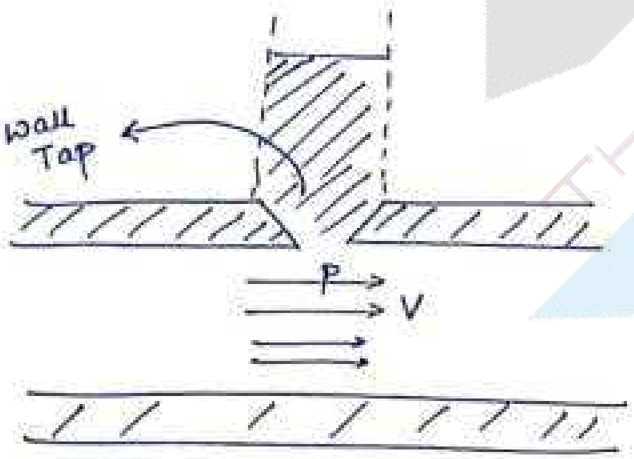
Note:→



$\frac{P_o}{\omega}$ ⟶ stagnation head

$\frac{V^2}{2g} = h$

$\boxed{V_o = \sqrt{2gh}}$



MOHIT CHOUKSEY

VAPOUR PRESSURE

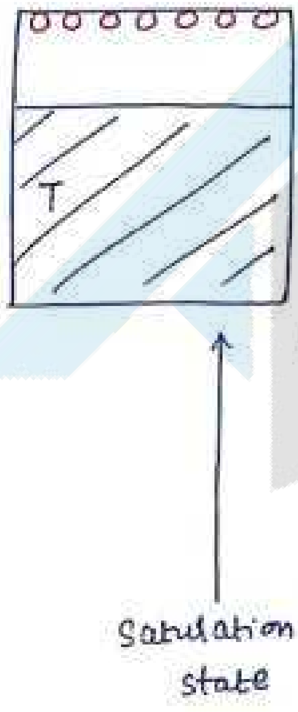
Partial pressure exerted by the saturated vapour condition in close container on liquid surface is known as Vapour Pressure at corresponding temperature.

20°C P_{vw} = 2.5 KPa

100°C P_{vw} = 101 KPa ⟶ 1atm pressure

20°C $P_{v\,gasoline}$ = 30 KPa

20°C $P_{v\,Hg}$ = 0.17 KPa

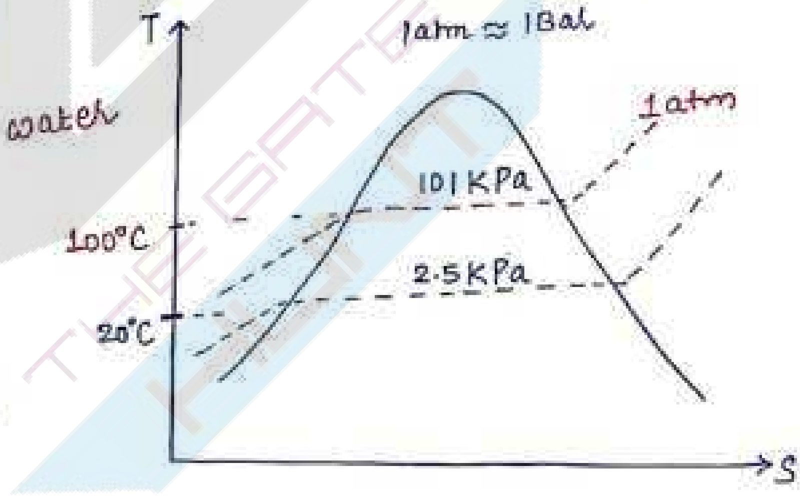


1atm = 101.325 KPa

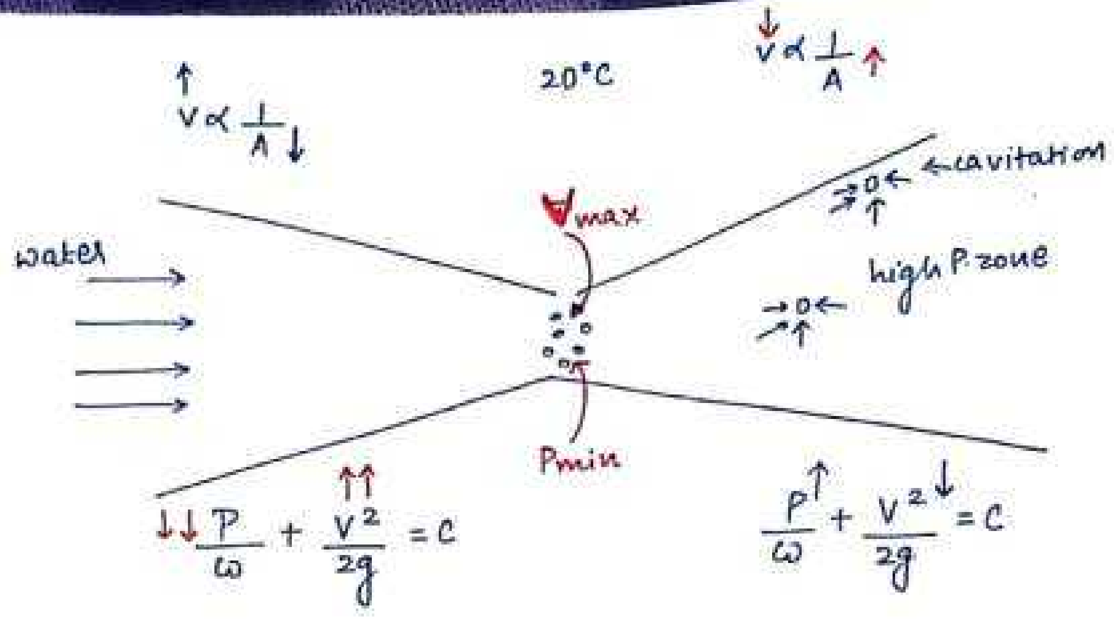
1atm = 1.01325×10^5 Pa

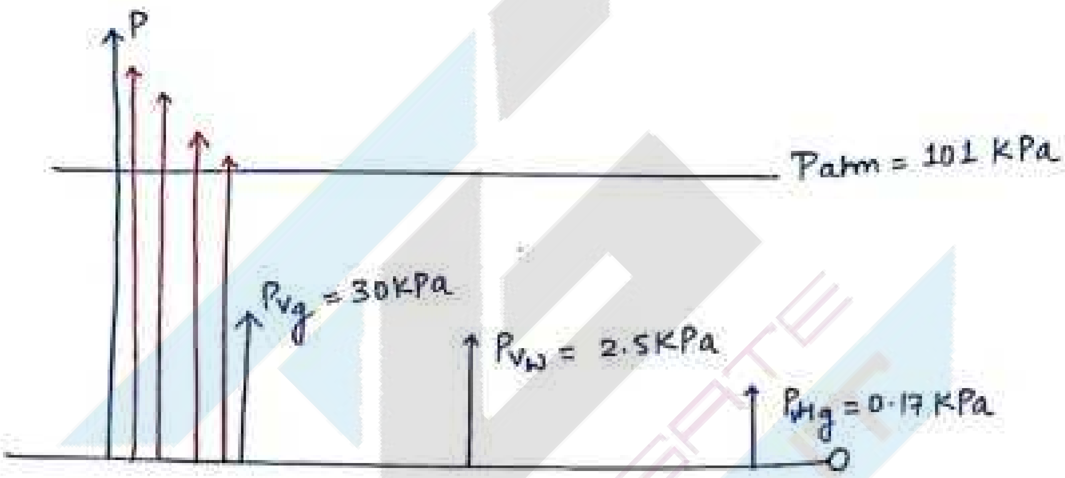
1atm = 1.01325 Bar

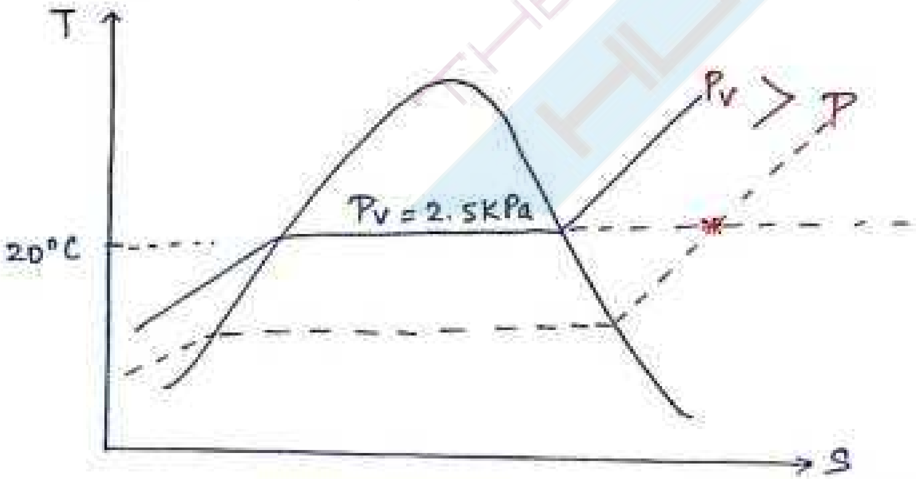
1atm ≈ 1 Bar



MOHIT CHOUKSEY







evaporation occurs when

$P_v > P$ i.e. $P < P_v$
↓
20°C

CAVITATION →

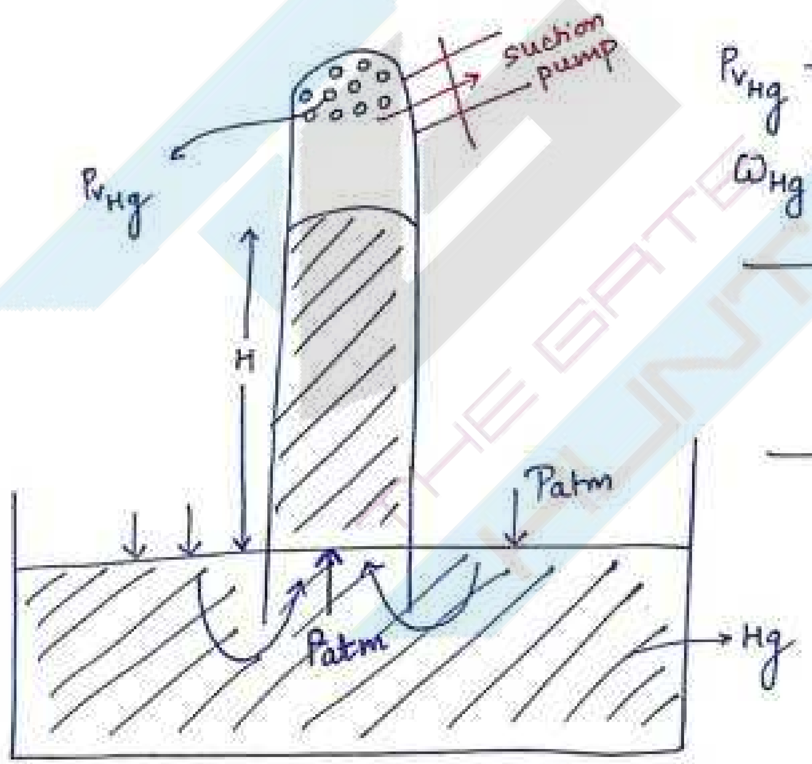
when fluid flows, in the flow if the absolute pressure of the flow becomes less than vapour pressure of the fluid at corresponding temperature, the bubbles are formed & if these bubbles carries in a region of high pressure, the bubble gets collapsed and pitting and cavitation phenomenon takes place at the surface of the machine part.

(Bursting at high pressure)

Eg :- at the suction side of the centrifugal pump and the exit of the Reaction Turbine.

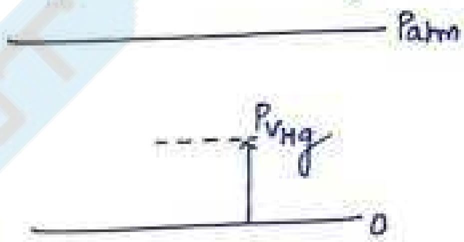
BAROMETER →

It is an instrument which measures local atmospheric pressure.



$$P_{V_{Hg}} + \omega_{Hg} H = P_{atm}$$

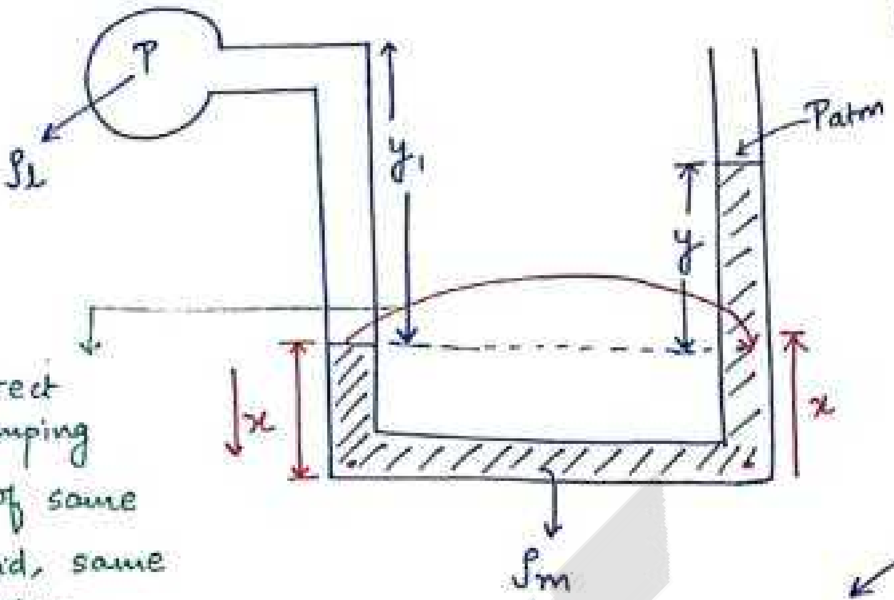
$$\omega_{Hg} H = P_{atm} - \cancel{P_{V_{Hg}}}^{\,0}$$



$$\boxed{\omega_{Hg} H = P_{atm}}$$

Note :→ Hg used in Barometer because of low vapour pressure and high density.

U-TUBE MANOMETER



$$P + \rho_l g y_1 + \cancel{\rho_m g x} - \cancel{\rho_m g x} - \rho_m g y = P_{atm}$$

direct jumping ∵ of same fluid, same datum.

$$\boxed{P = P_{atm} + \rho_m g y - \rho_l g y_1}$$ Absolute

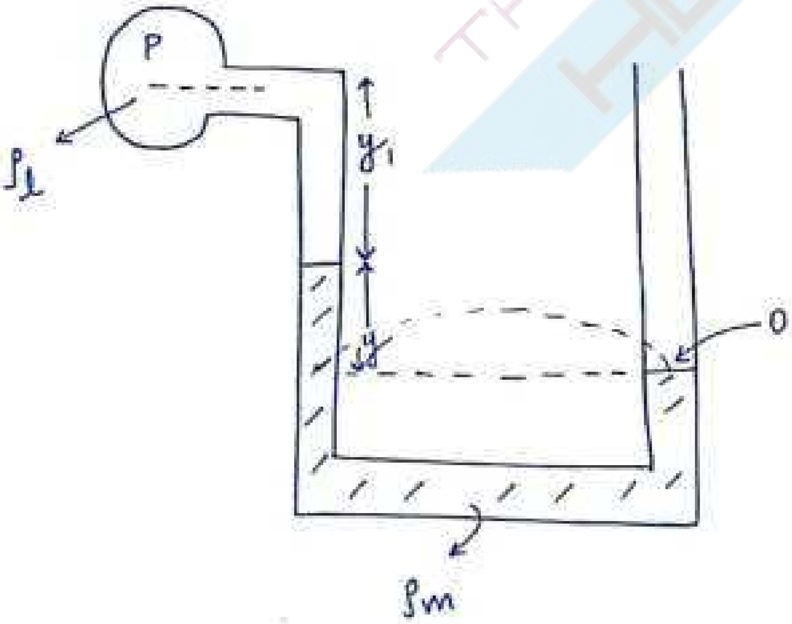
Sign convention

↓ $+P = \rho g h$ ↑ $-P = -\rho g h$

$P_{atm} = 0$

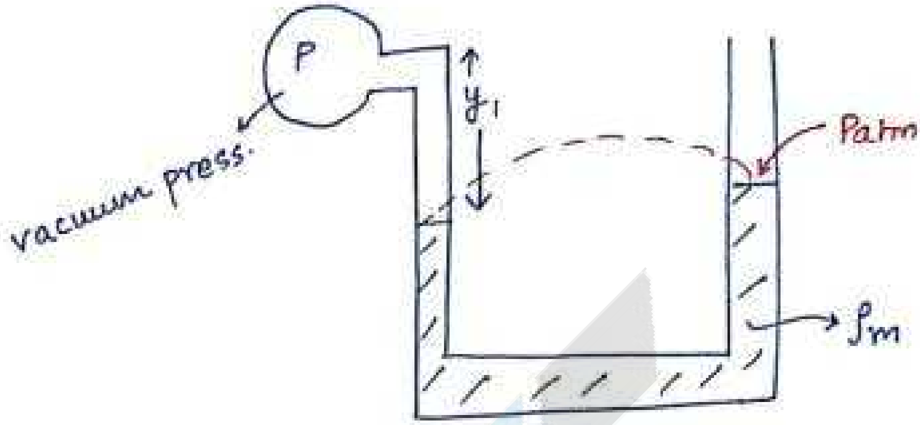
$$\boxed{P = \rho_m g y - \rho_l g y_1}$$ Gauge

$0 + \rho_m g y - \rho_l g y_1 = P$

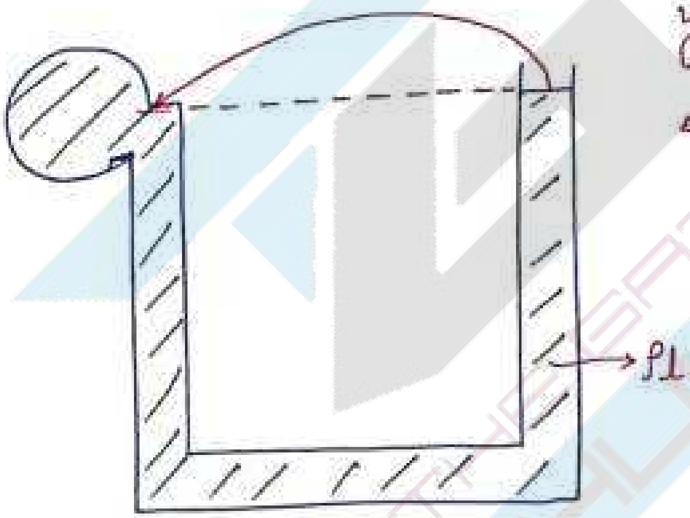


$$0 - \rho_m g y - \rho_l g y_1 = P$$

$$\boxed{P = -\rho_m g y - \rho_l g y_1}$$ -ve gauge



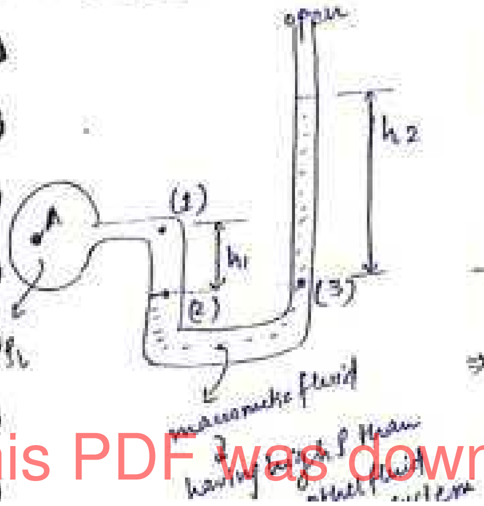
$$\boxed{0 - \rho_l g y_1 = P}$$



$y = 0$

here $\boxed{P = P_{atm}}$

* U-Tube manometer → high pressure can be measured, manometric fluid is different and can be preferred according to application, Pascal law is applied.



$\rho_m \rightarrow$ density of manometric fluid

Pascal Law

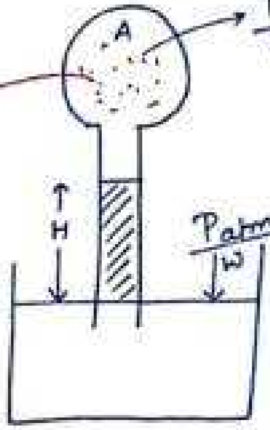
Press. at 2 = Press. at 3
([illegible]) ([illegible])

$$\Rightarrow P_A + \rho_A g h_1 = \rho_m g h_2$$

$$\Rightarrow \boxed{P_A = \rho_m g h_2 - \rho_A g h_1}$$

Workbook

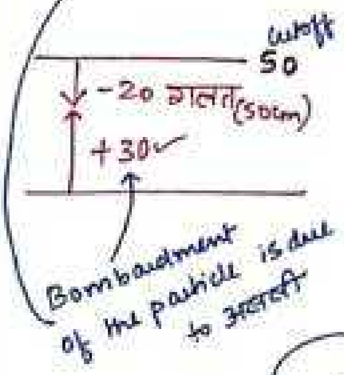
Q12 (CH→2)



$\frac{P_A}{w}$ = 50cm of Hg vacuum

$\frac{P_A}{w} = 26$

$\frac{P_{atm}}{w}$ = 76 cm of Hg



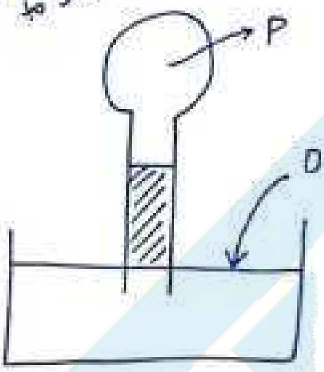
$\frac{P_A}{w} + H = \frac{P_{atm}}{w}$

~~50 + H = 76~~
~~H = 26~~

~~-50 + H = 76~~
~~H = 126~~

$26 + H = 76$

$\boxed{H = 50}$



*

$0 - H = -50$

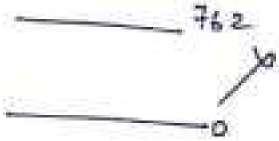
$\boxed{H = 50}$

Q13

P_{atm} = 762 mm of Hg

700 mm of Hg

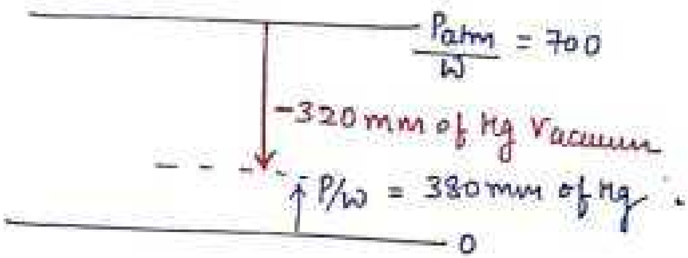
380 mm Hg



(a) ✓

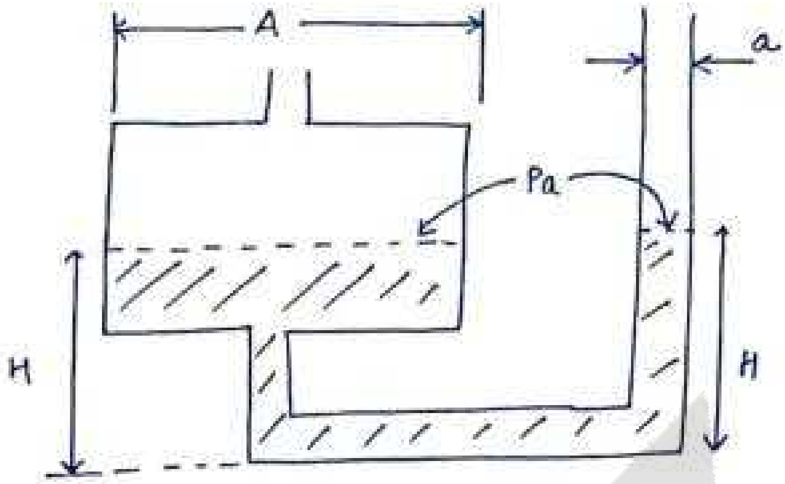
SIR

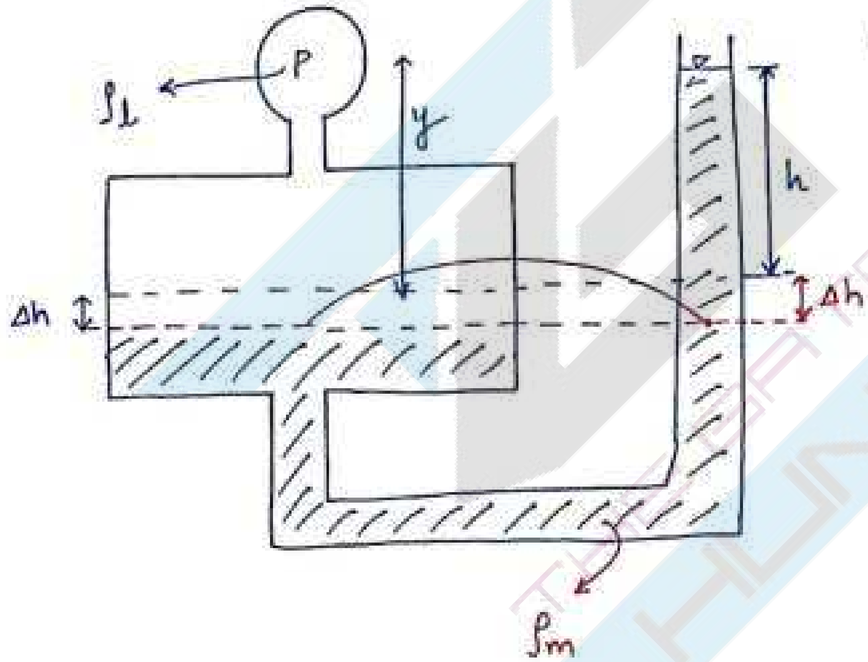
$\frac{P_{atm}}{w}$ = 762 mm of Hg



SINGLE COLUMN MANOMETER :-

→ High sensitivity





$$P + \rho_1 g y + \rho_1 g \Delta h - \rho_m g \Delta h - \rho_m g h = P_{atm}$$

Sign convention
↳ same as hydrostatic law

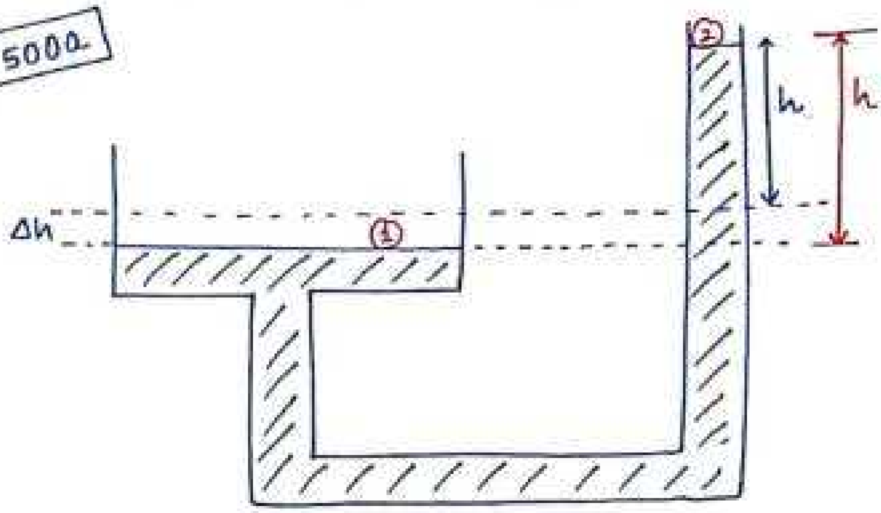
↓ $+\rho g h$ ↑ $-\rho g h$

$$\boxed{A \Delta h = a h}$$

*Small Δh → high h → high sensitivity → This manometer is used for more precise system.

WB (14) this pressure measurement

$\boxed{A = 500a}$



error
$= (\cancel{h}+\Delta h)-\cancel{h}$
$= \Delta h$

% error $= \frac{\Delta h}{h} \times 100$

$= \frac{a}{A} \times 100$

$= \frac{1}{500} \times 100$

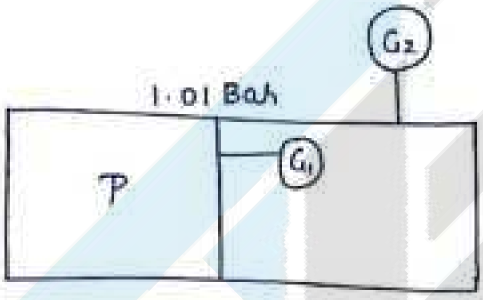
$= 0.2\%$

not 20% ∵ 20paise

$A = 500\,a$ $\quad A\,\Delta h = a\,h$

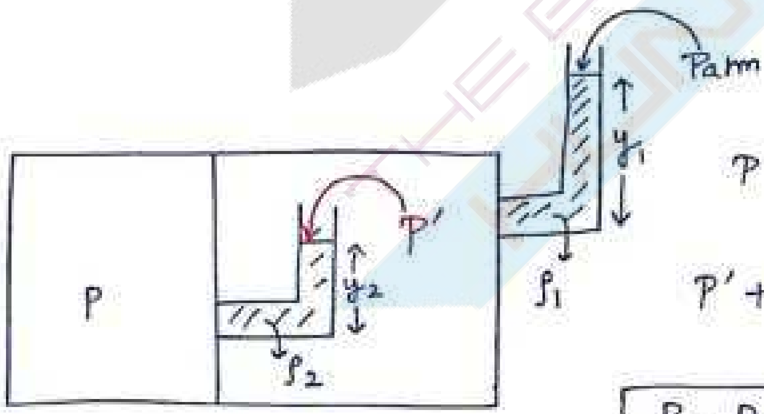
$\frac{\Delta h}{h} = \frac{a}{A}$

WB (19)
IISc



$P_{G_1} = 5\,bar$

GIR

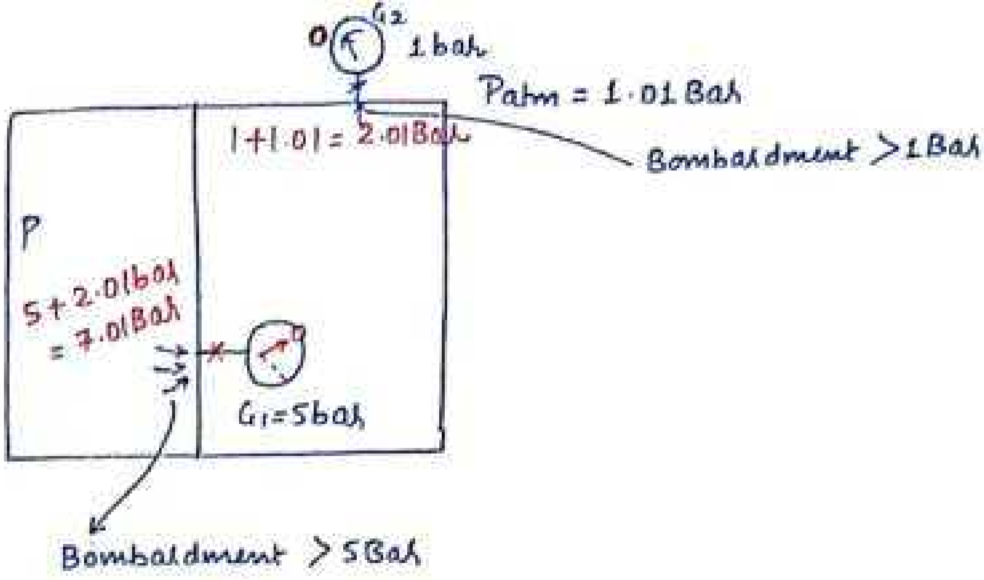


$P' = P_{atm} + \rho_1 g y_1$

$P' + \rho_2 g y_2 = P$

$$\boxed{P = P_{atm} + \rho_1 g y_1 + \rho_2 g y_2}$$

MOHIT CHOUKSEY



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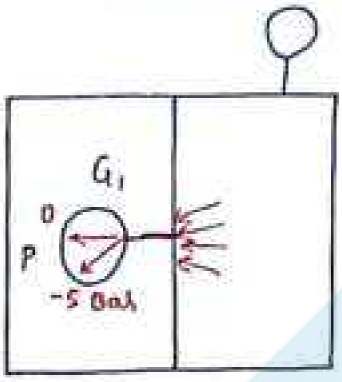


Note :-

ASSUMPTIONS FOR manometer :-

① fluid is static.

② Incompressible fluid.

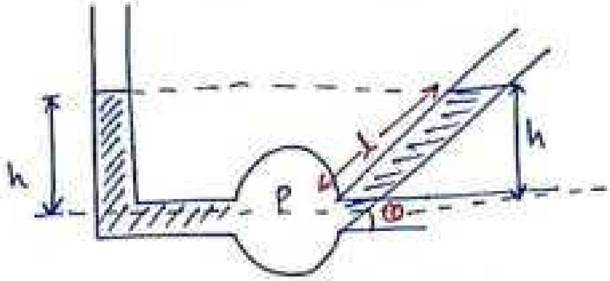
③ Negligible capillarity.

④ Sensitivity of Manometer.

Sensitivity of Manometer.

[" N.P. "]

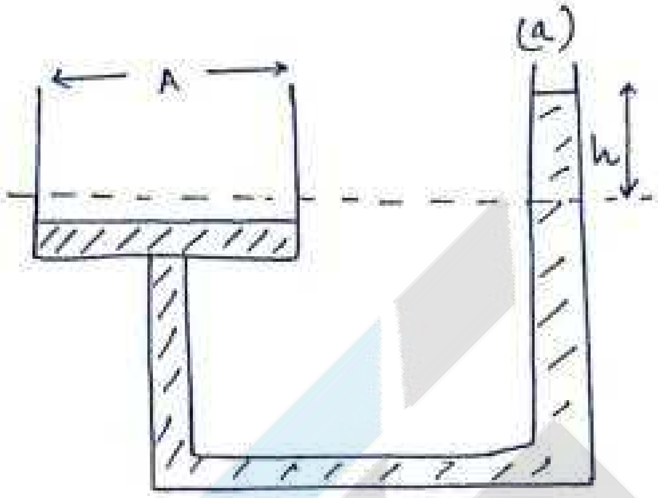
WB → (6)



Sensitivity $= \frac{l}{h}$

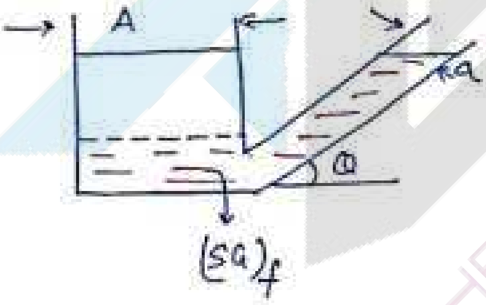
$$\boxed{S = \frac{1}{\sin\theta}}$$

$$\boxed{h = l\sin\theta}$$

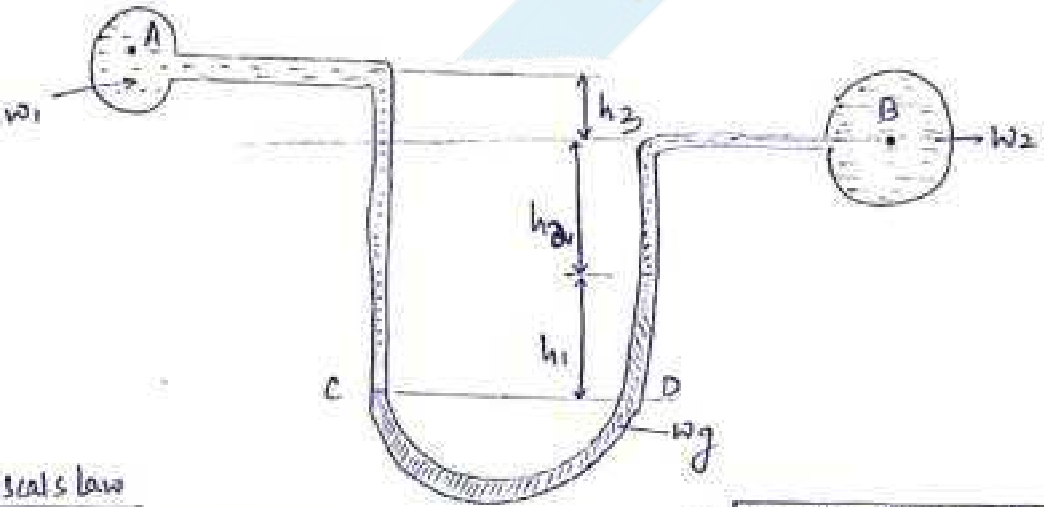


$S \propto \frac{1}{(a/A)}$

$$\boxed{S \propto \frac{1}{(S.G.)_f}}$$



WB → b → compared



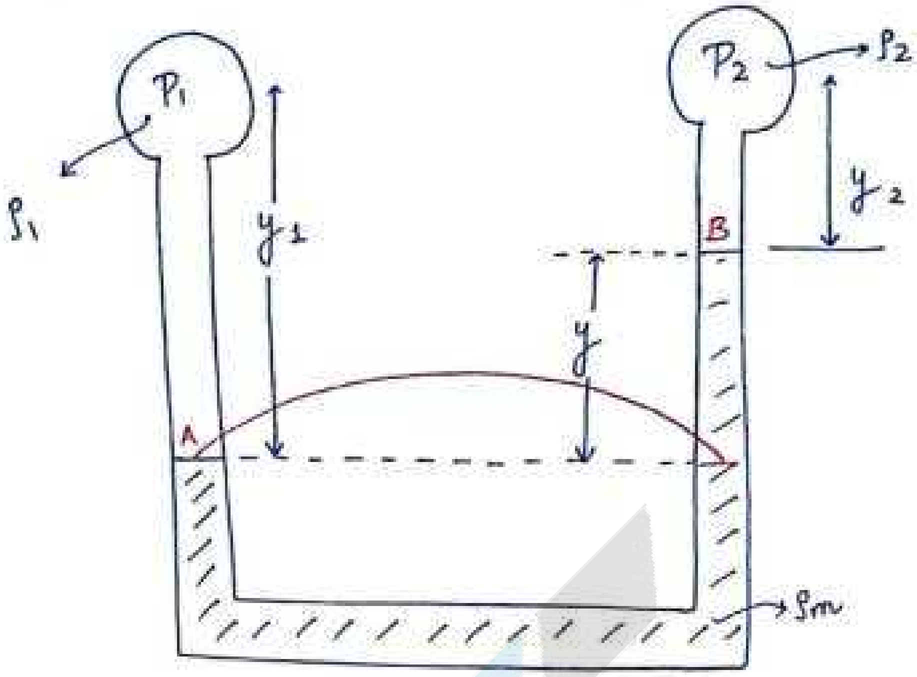
Pascal's law

Press. at C = Press. at D

$\Rightarrow P_A + (\rho_1 g)(h_1 + h_2 + h_3) = P_B + \rho_2 g h_2 + \rho_g g h_1$

$$\Rightarrow \boxed{P_A - P_B = \rho_2 g h_2 + \rho_g g h_1 - \rho_1 g(h_1 + h_2 + h_3)}$$

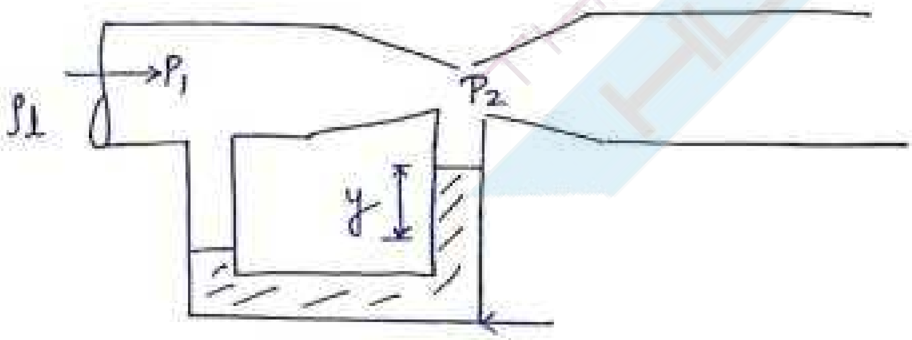
* Differential U-Tube Manometer :-



$$P_1 + \rho_1 g y_1 - \rho_m g y - \rho_2 g y_2 = P_2$$

sign convention
$\downarrow +\rho g h$ $\uparrow -\rho g h$
(ऊपर से नीचे आते वक्त)

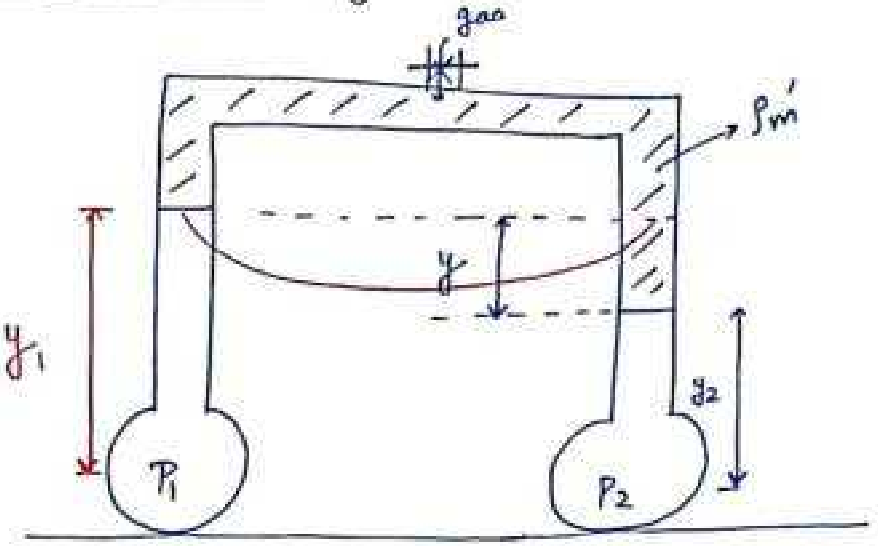
$$\boxed{P_1 - P_2 = \rho_m g y - \underbrace{\rho_1 g y_1 + \rho_2 g y_2}_{-(\text{''} - \text{''})}}$$



→ used to find pressure difference b/w two pipelines.

→ Pressure values in each pipe system cannot be measured.

Invert Tube Manometer → lighter fluid can be used as manometric fluid

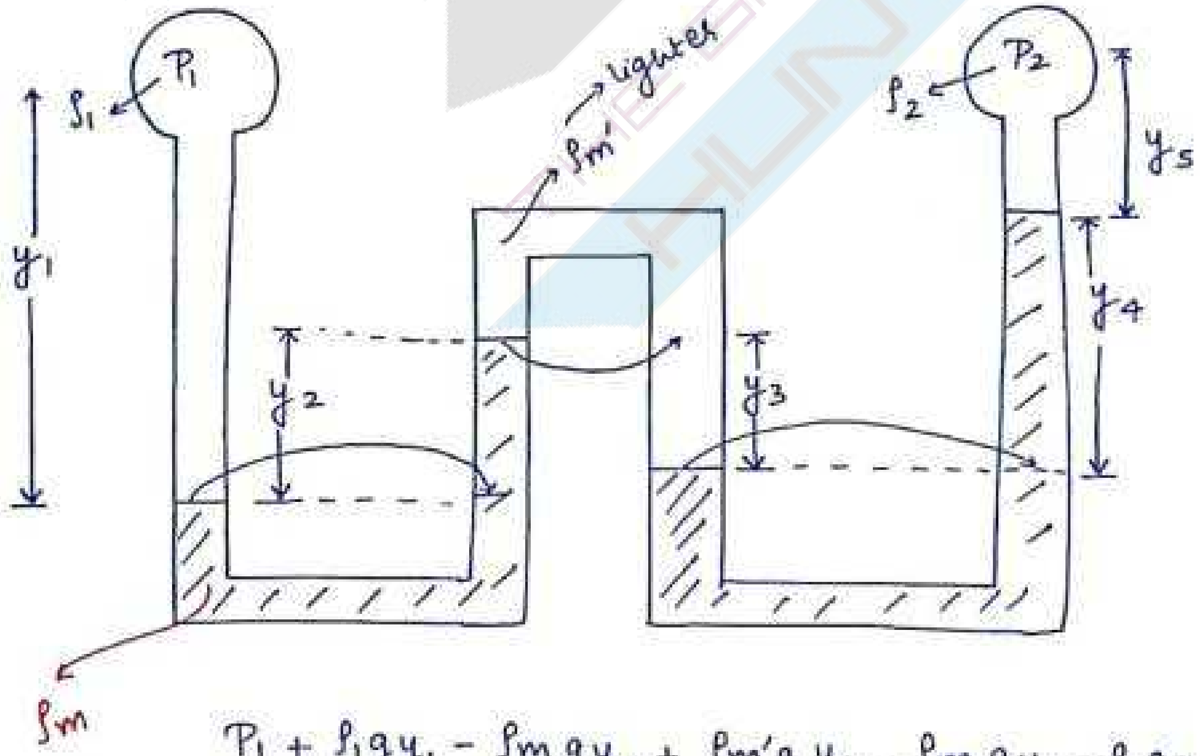


$$P_1 - \rho_1 g y_1 + \rho_m' g y + \rho_2 g y_2 = P_2$$

sign changes

$$\boxed{P_1 - P_2 = -\rho_m' g y + \rho_1 g y_1 - \rho_2 g y_2}$$

Note :- ① Multi U-Tube manometers are used for high pressure differences.



$$P_1 + \rho_1 g y_1 - \rho_m g y_2 + \rho_m' g y_3 - \rho_m g y_4 - \rho_2 g y_5 = P_2$$

$P_1 - P_2 = ?$ ↳ always Mod

1 → d   3 → c   5 → c   7 → a   9 → b   11 → b   13 → a   15 → c   17 → d
2 → b   4 → c   6 → b   8 → c   10 → c   12 → b   14 → c   16 → c   18 → good HW

19 → d   48 → 38.68 KPa   47 → −3.57 KPa   (35)
46 → 1.11 KPa

MOHIT CHOUKSEY

CHAPTER-3

HYDROSTATICS

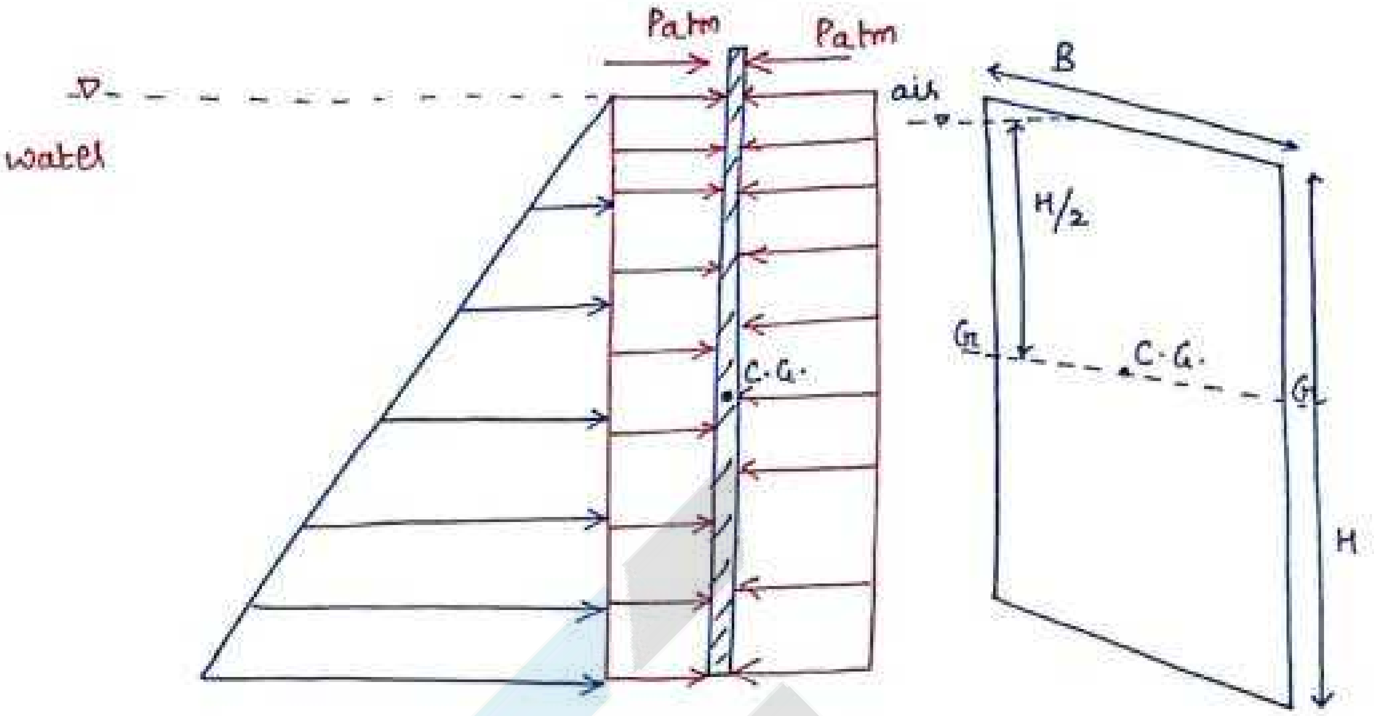
MOHIT CHOUKSEY

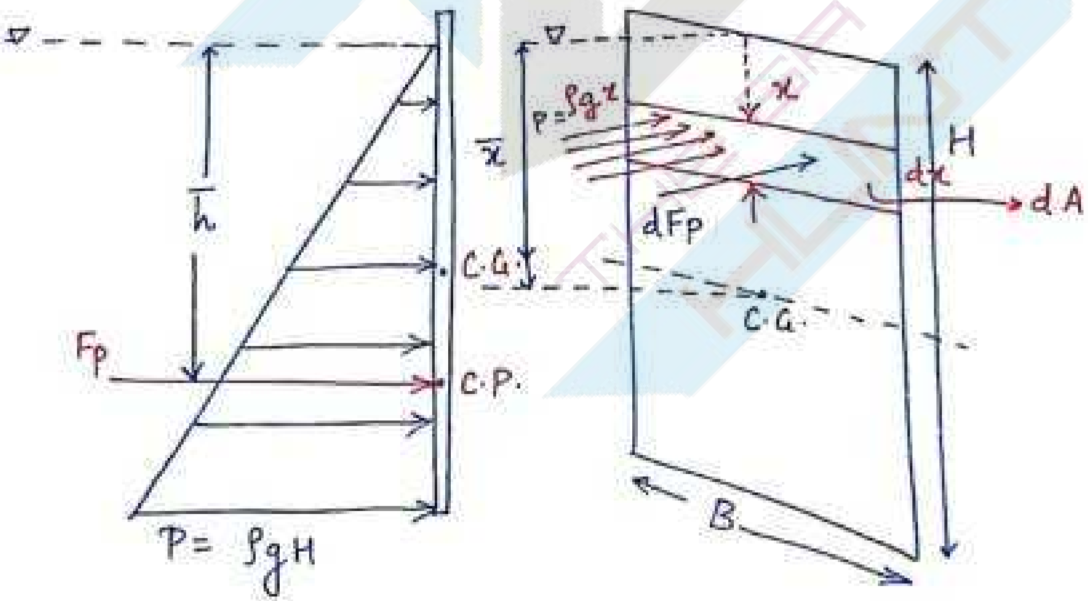
Fluid Statics

- Fluid statics deals with problems associated with fluid at Rest.
- Fluid is liquid - Hydrostatics.
- Fluid is gas - aerostatic.
- Since the fluid is at rest there is no tangential forces and we deal with only normal forces in fluid statics.
- Applications :- Forces acting on floating or submerged Body, design of dams and water storage tanks.

CHAPTER-3

HYDROSTATS / HYDROSTATIC FORCES





$F_p = P \times A$

Inclined or not uniform Profile hence CG and CP are at different locations.

$$\int dF_p = \int P\,dA$$

$$F_p = \int \rho g x\,dA$$

$$F_p = \rho g \int x\,dA$$

$$= \omega \bar{x} A \qquad \left\{ \int x\,dA = \bar{x} A \right\}.$$

$$\boxed{F_p = \omega A \bar{x}} = \rho g A \bar{x}$$

$\bar{x}$ = vertical depth of centroid of flat surface from free surface

<u>Hydrostatic force / Total pressure force</u> :- It is the net force ~~accepted~~ exerted by the fluid on the body. It is defined by

$$\boxed{F_p = \omega A \bar{x}}$$

for flat surface, the expression $\omega A \bar{x}$ is independent of orientation of the flat surface.

<u>COP (Centre of Pressure)</u> → The Point of application of net hydrostatic pressure force exerted by the fluid. it is measured in vertical depth from free surface

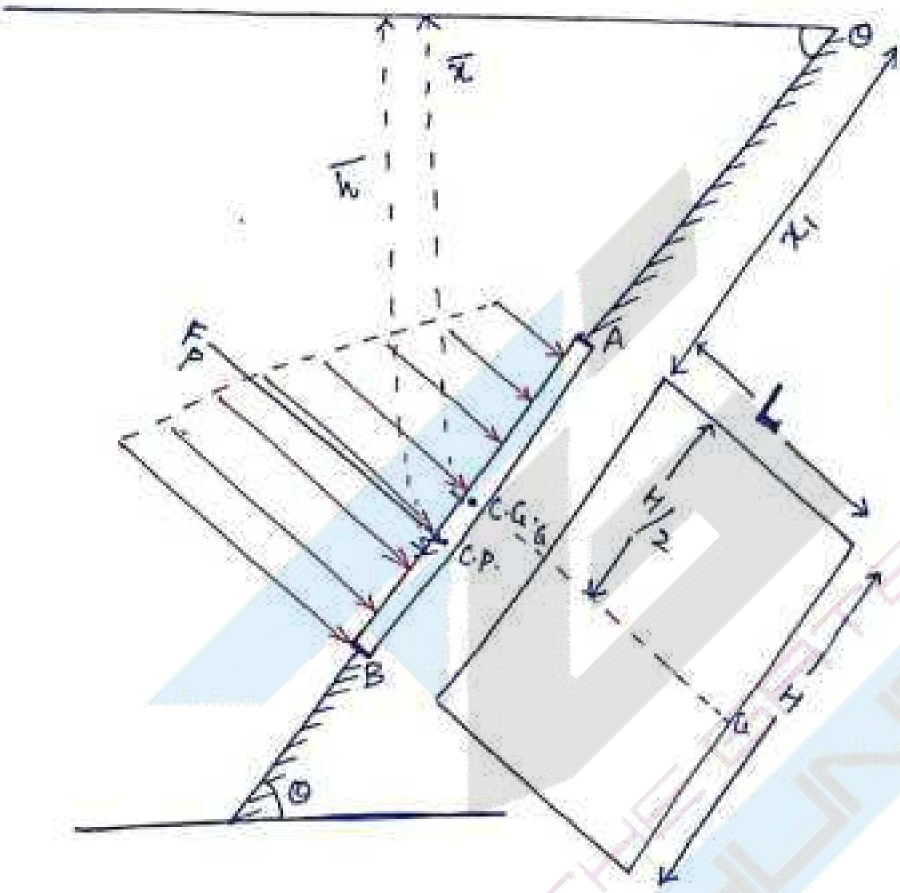
$$\left(F_p \times \bar{h} = \int dF_p \cdot x \right)$$

$$\boxed{\bar{h} = \frac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}}$$

I_{GG} = moment of inertia of the Body about centroidal axis parallel to free surface. (41)

where θ = inclination angle from the free surface or from the horizontal surface.

* Inclined Plate :-



$$\boxed{F_P = \omega A \bar{x}}$$

$$\boxed{\bar{h} = \frac{I_{GG}\sin^2\theta}{A\bar{x}} + \bar{x}}$$

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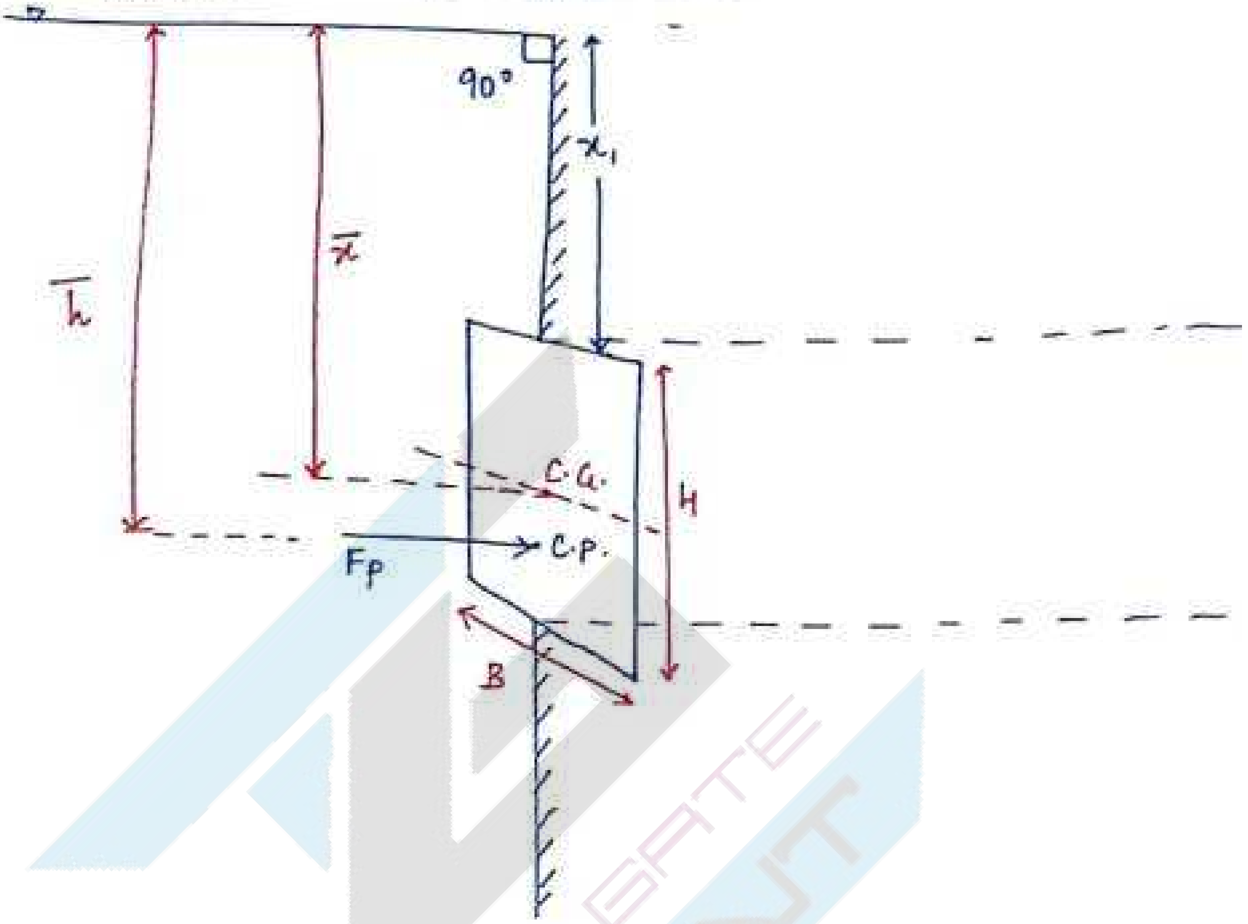
* Vertical Plate :-

$\theta = 90^\circ$

$\sin 90^\circ = 1$

$$\boxed{\bar{h} = \frac{I_{GG}}{A\bar{x}} + \bar{x}}$$



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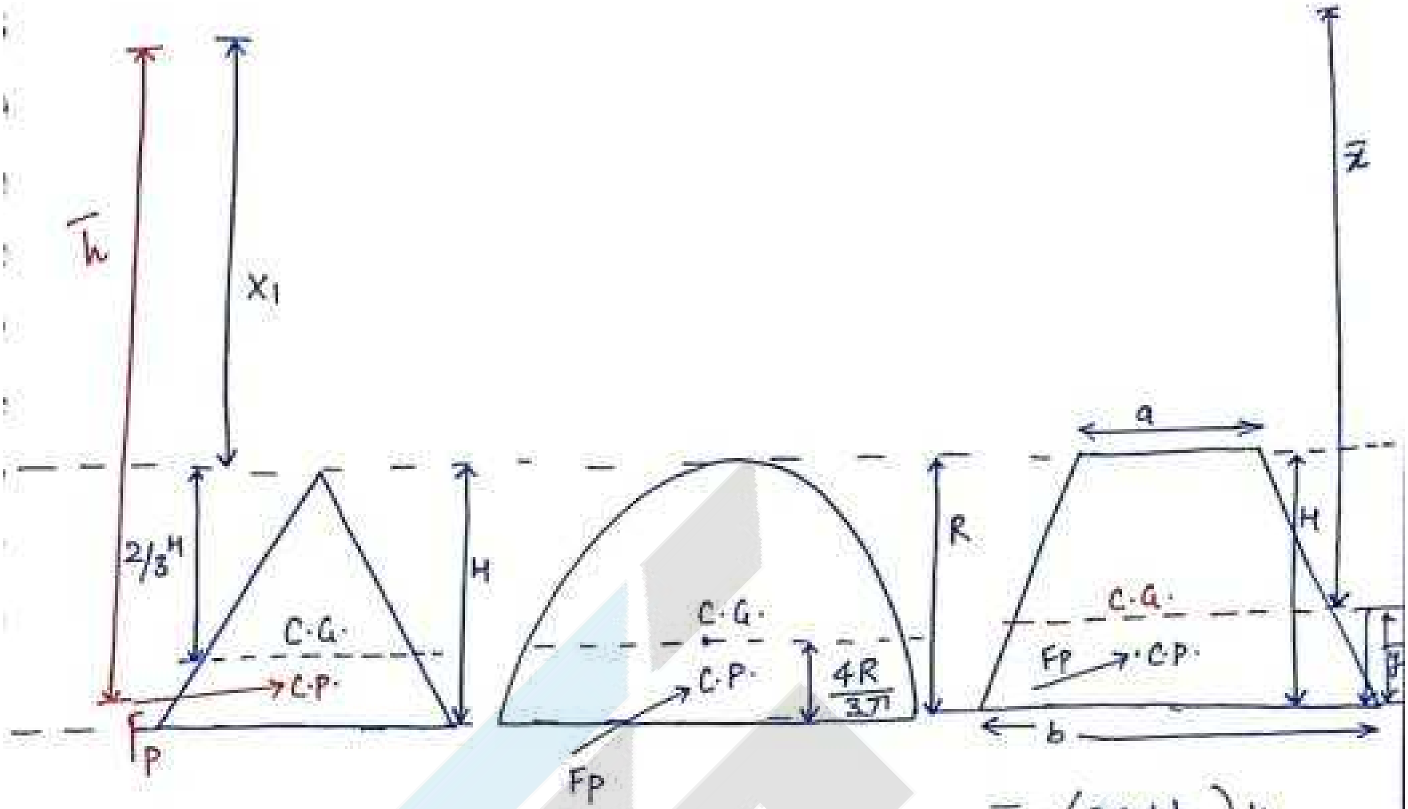
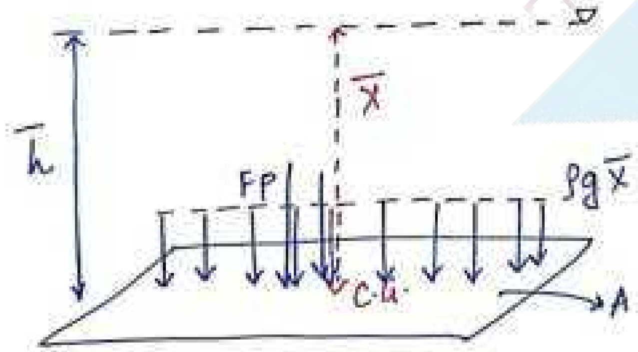


$$\bar{y} = \left(\frac{2a+b}{a+b}\right)\frac{H}{3}$$

Horizontal flat plate

$\theta = 0^{\circ}$ $\sin\theta = 0$ $\boxed{\bar{h} = \bar{x}}$

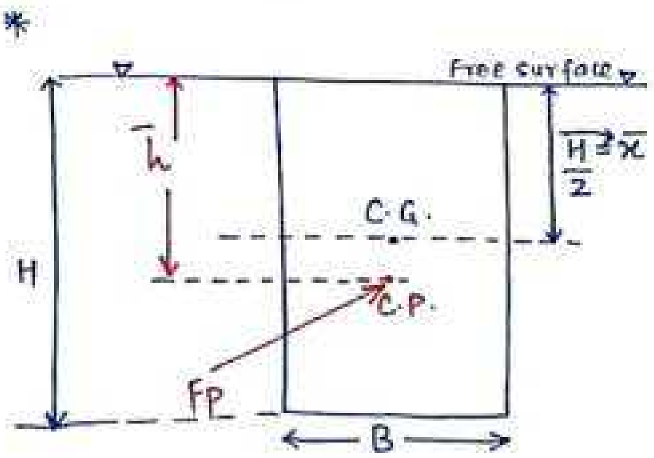


$$F_p = P \times A$$

$$= \rho g \bar{x} \times A$$

$$\boxed{F_p = \omega A \bar{x}}$$

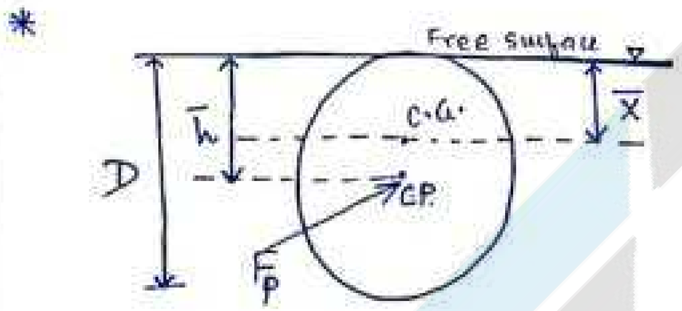
$$\boxed{\bar{h} = \bar{x}}$$



$$\bar{h} = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

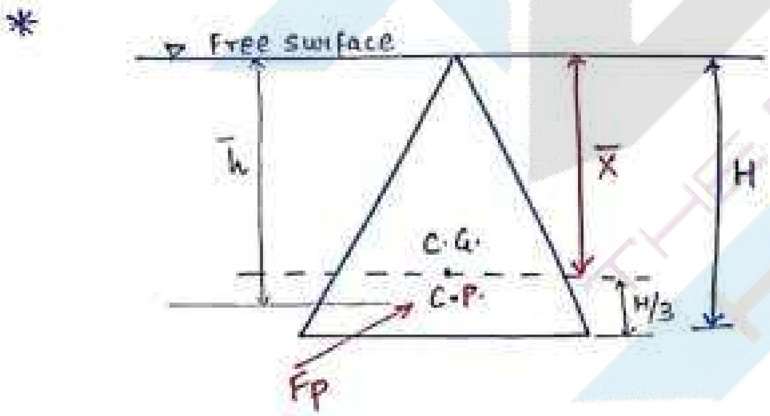
$$\bar{h} = \frac{(BH^3/12)}{(B \times H)\cdot \frac{H}{2}} + H/2$$

$$\boxed{\bar{h} = 2/3\, H}$$



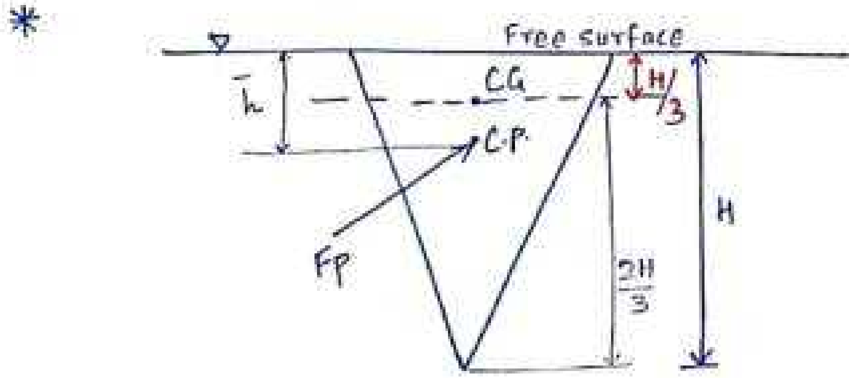
$$\boxed{\bar{x} = D/2}$$

$$\boxed{\bar{h} = \frac{5D}{8}}$$



$$\boxed{\bar{h} = \frac{3H}{4}}$$

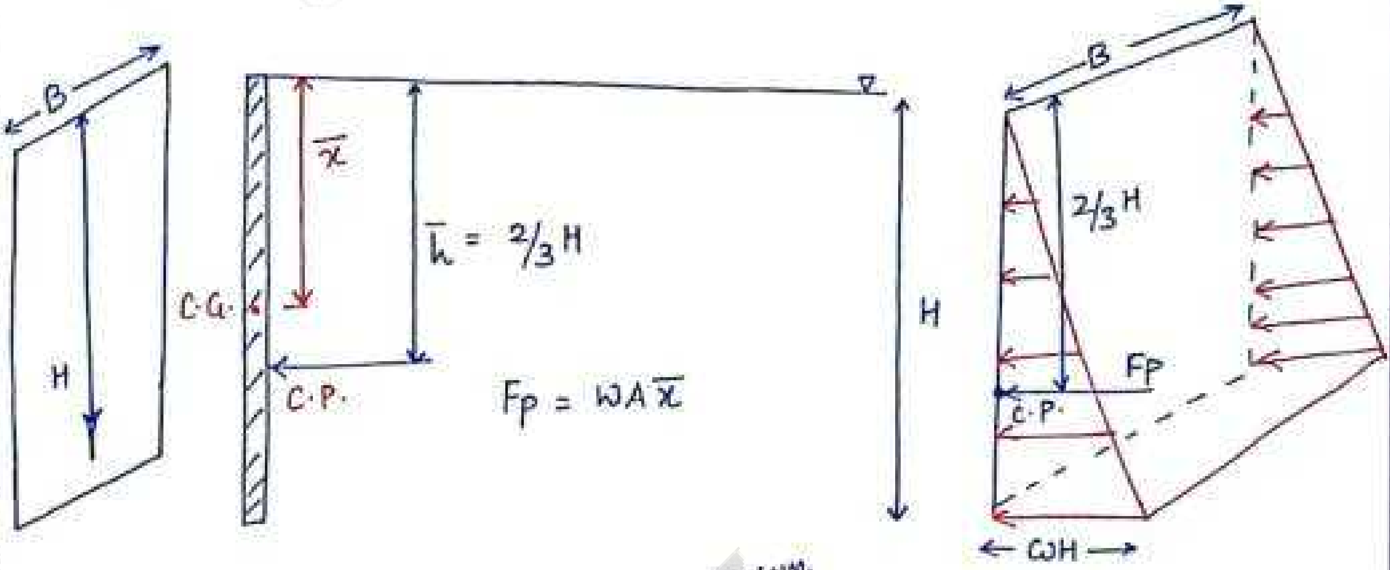
$$\boxed{\bar{x} = \frac{2}{3} H}$$



$$\boxed{\bar{h} = \frac{H}{2}}$$

$$\boxed{\bar{x} = H/3}$$

* Pressure Diagram



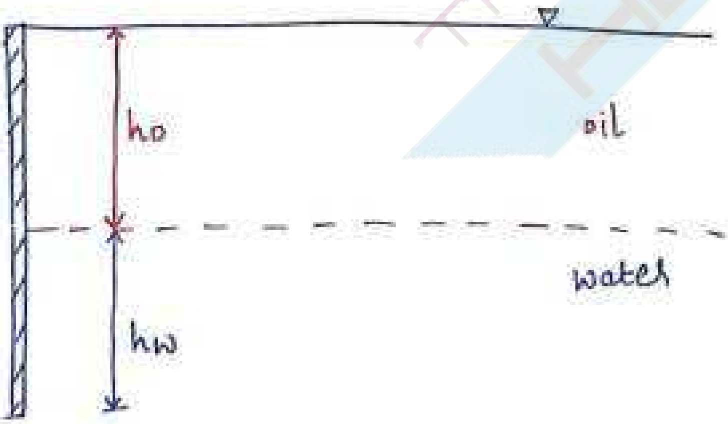
$$\text{vol of P.D.} = \tfrac{1}{2}(wH \times H).B$$

(P.D. → Pressure diagram)

$$= \omega\,(H\times B).\frac{H}{2}$$

$$= \omega A\bar{x}$$

$$\boxed{\text{vol. of PD} = F_P}$$ → Total hydrostatic force or Total pressure Force



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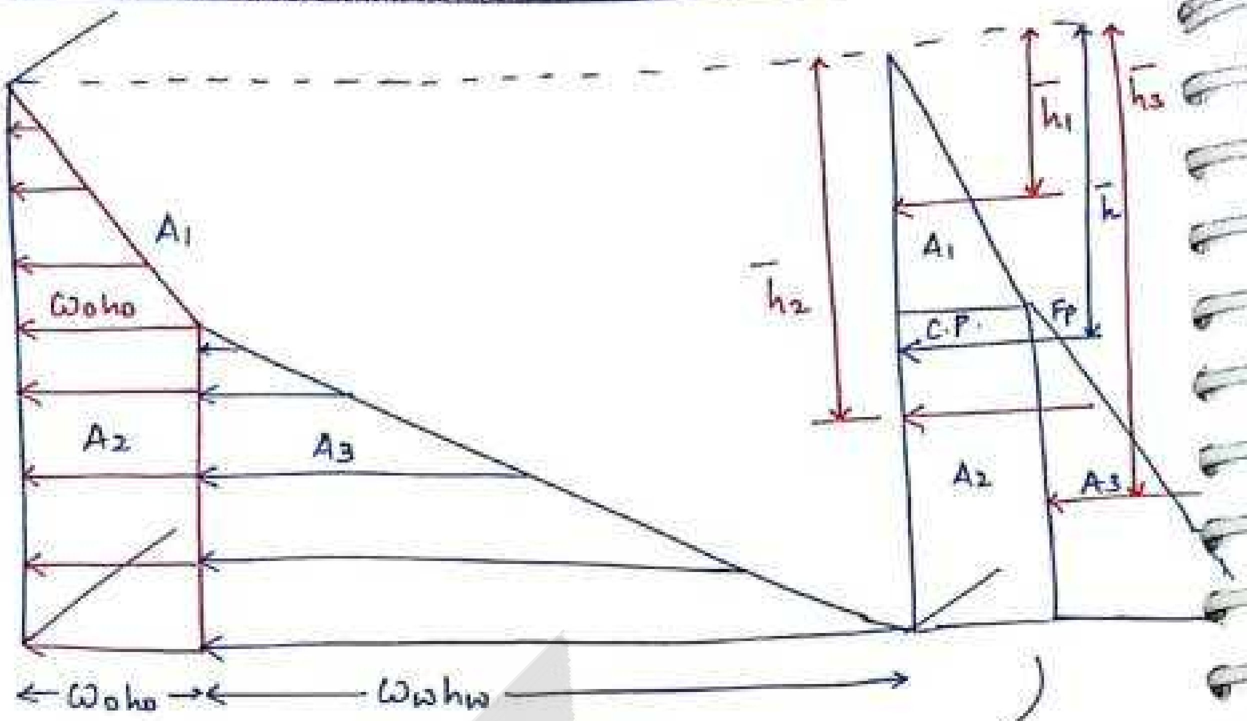

$$\boxed{\text{Vol. of P.D.} = F_P}$$

$$\therefore \boxed{A\bar{h} = A_1\bar{h}_1 + A_2\bar{h}_2 + A_3\bar{h}_3}$$

Note → • The volm. of pressure diagram is the total hydrostatic pressure force and the C.G. or centroid of pressure diagram is centre of Pressure.

The expression $\boxed{wA\bar{x}}$ is valid for single fluid, so for two or more immiscible fluids, the pressure diagram method should be used to find the hydrostatic forces

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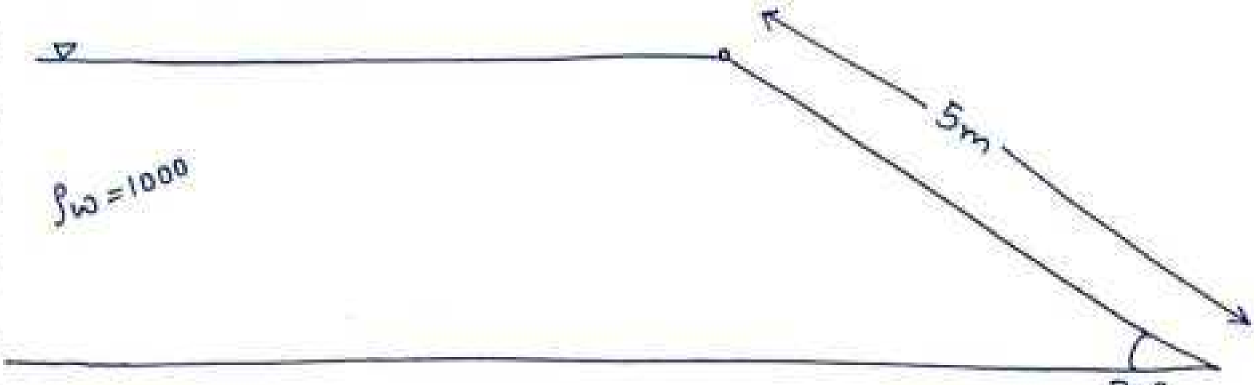
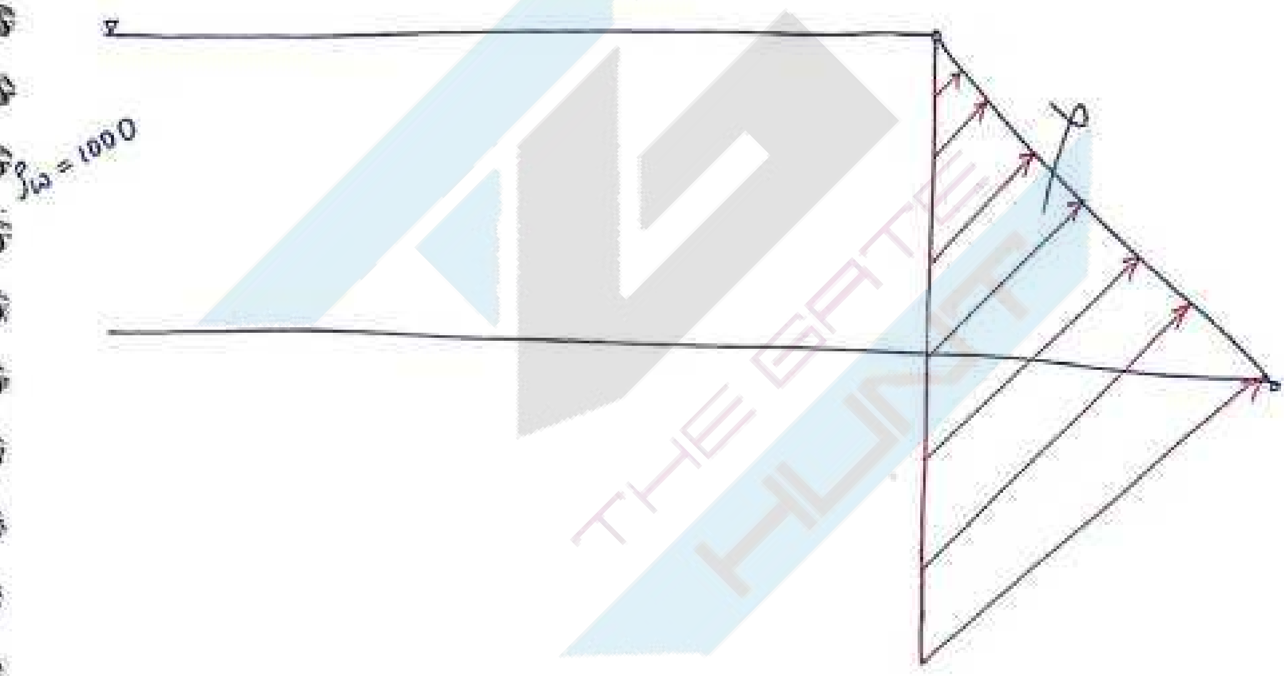
Pg 15 (26) WB

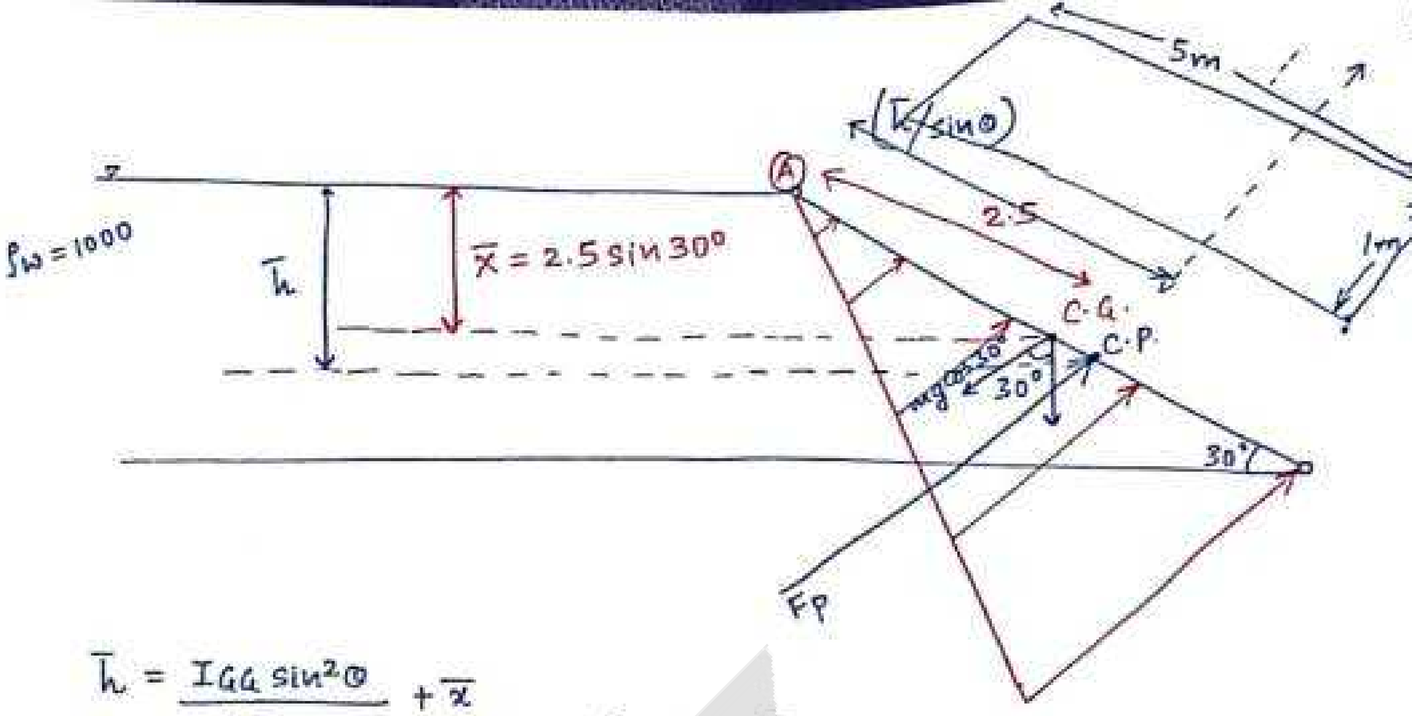


$F_p = \omega A \bar{x}$ $\bar{h} = \frac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}$

SIR



N.P.



$$\bar{h} = \frac{I_{GG}\sin^2\theta}{A\bar{x}} + \bar{x}$$

$\bar{h} = ?$

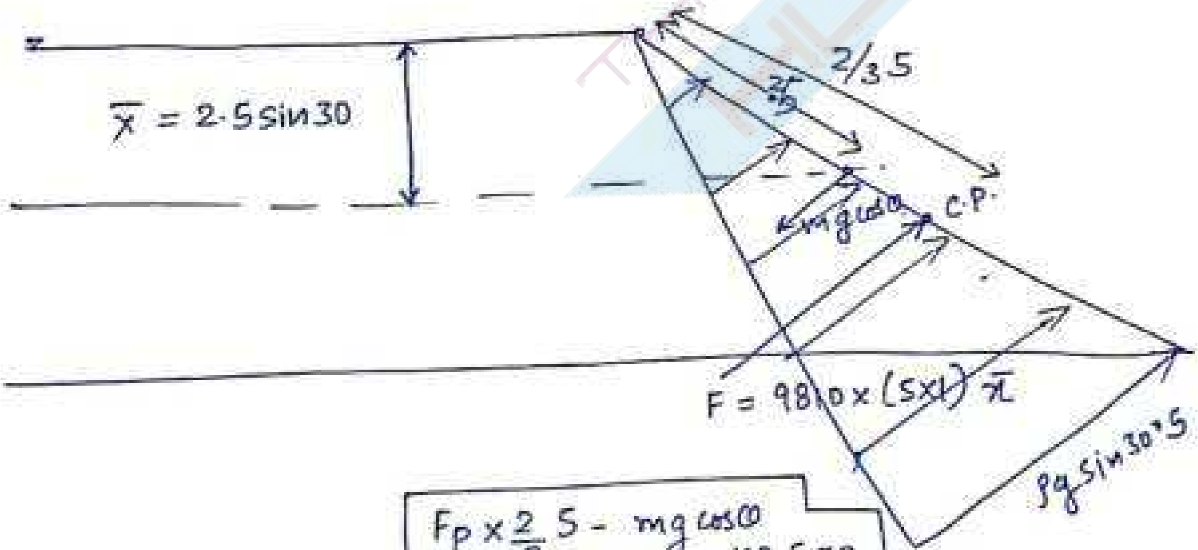
$F_p = \omega A \bar{x}$

$F_p = 9810 \times (5 \times 1)\, 2.5 \sin 30^\circ$

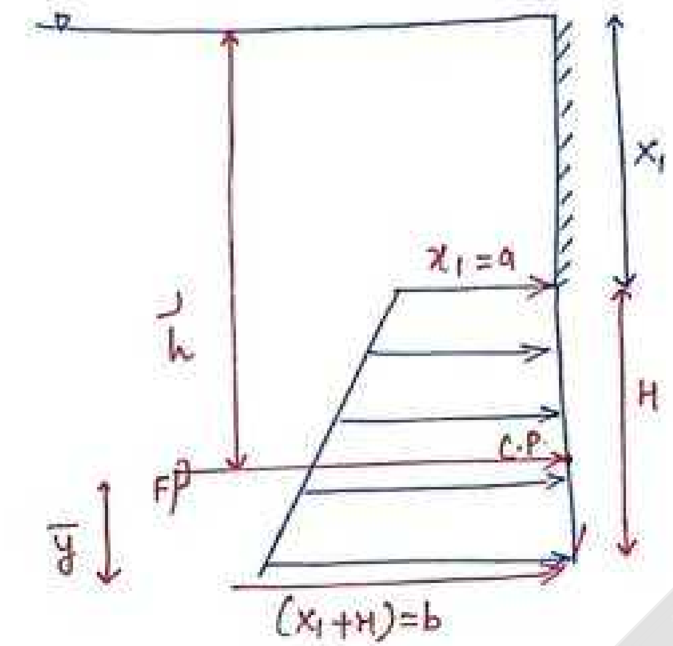
$F_p = 61312.5$ N

$\Sigma M_A = 0$

$$F_p \times \frac{\bar{h}}{\sin\theta} - mg\cos\theta \times 2.5 = 0$$



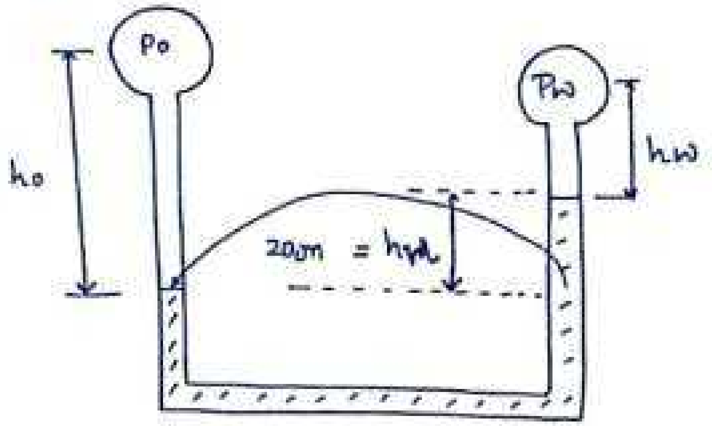
$$\boxed{F_p \times \frac{2}{3}5 - mg\cos\theta \times 2.5 = 0}$$



$$\bar{y} = \left(\frac{2a+b}{a+b}\right)\frac{H}{3}$$

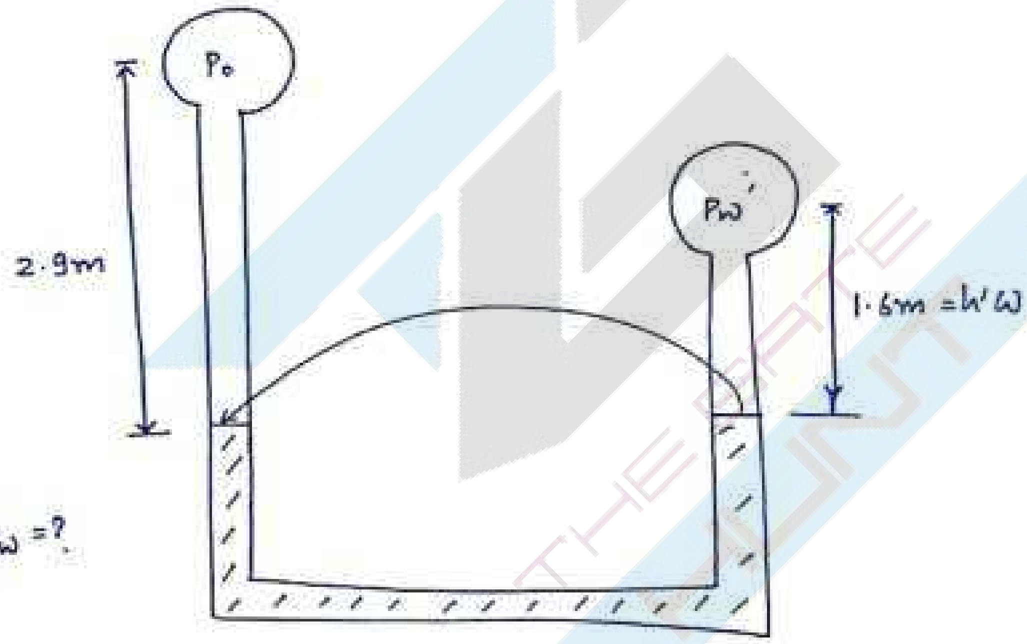
9/11/2016

HW Qn (3)



$$P_w + \rho_w g h_w + \rho_m g h_m = P_o \quad \text{---(i)}$$

? ?



$P_w' - P_w = ?$

$$P_w' + \rho_w g h_w' - \rho_o g h_o' = P_o \quad \text{---(ii)}$$

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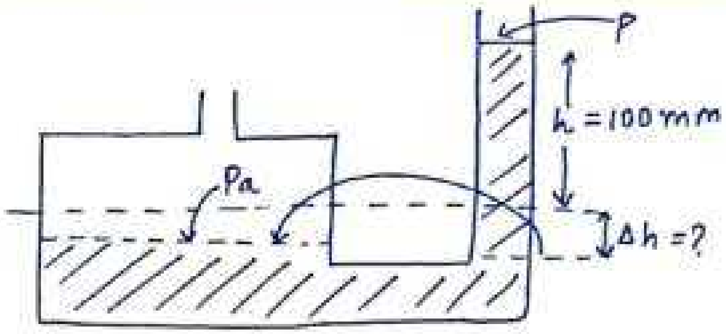
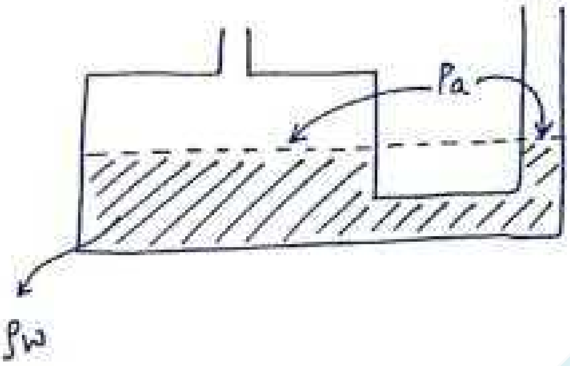
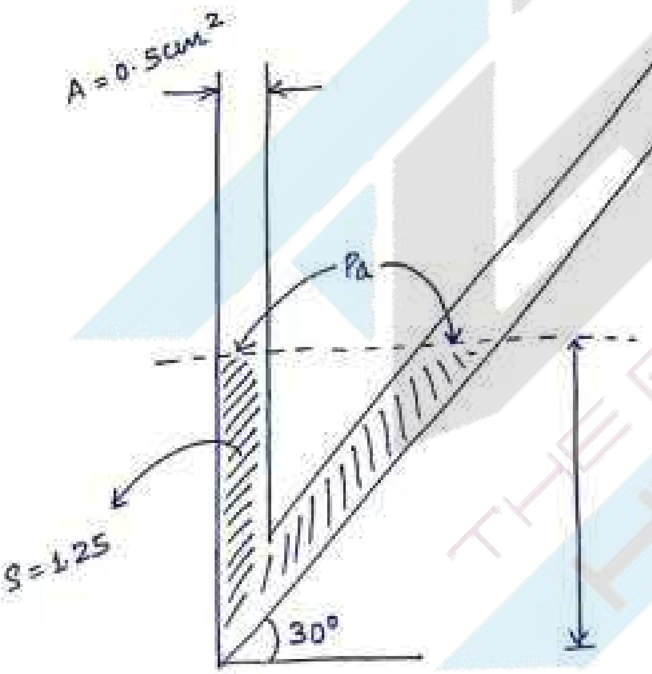


$$P + \rho_m g(h+\Delta h) = P_a$$

$$P - P_a = -\rho_m g(h+\Delta h)$$

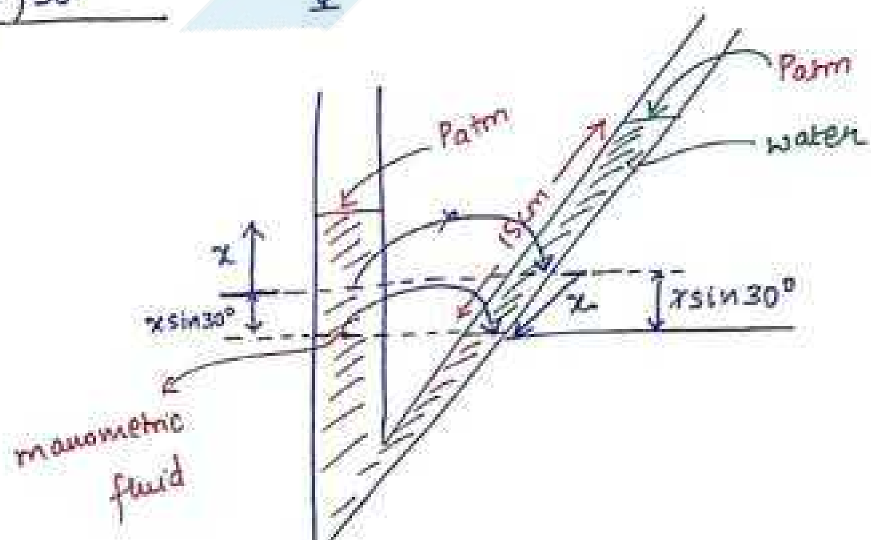


18



$V_w = 7.5\,cm^3$

$l_w = \frac{7.5}{0.5} = 15\,cm$

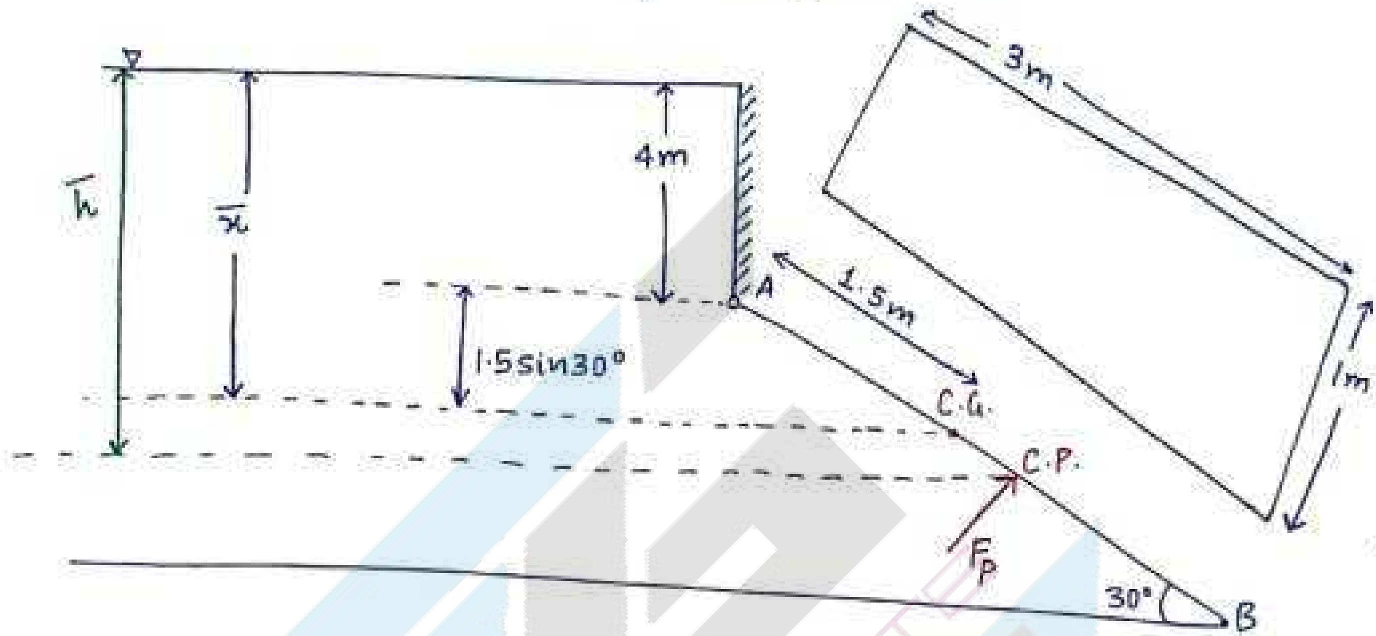


$$P_{atm} + \rho_m g x + \rho_m g x \sin 30^\circ - \rho_w g\, 15 \sin 30^\circ = P_{atm}$$

$$\boxed{x = 4\,cm}$$

Current Topic continues....

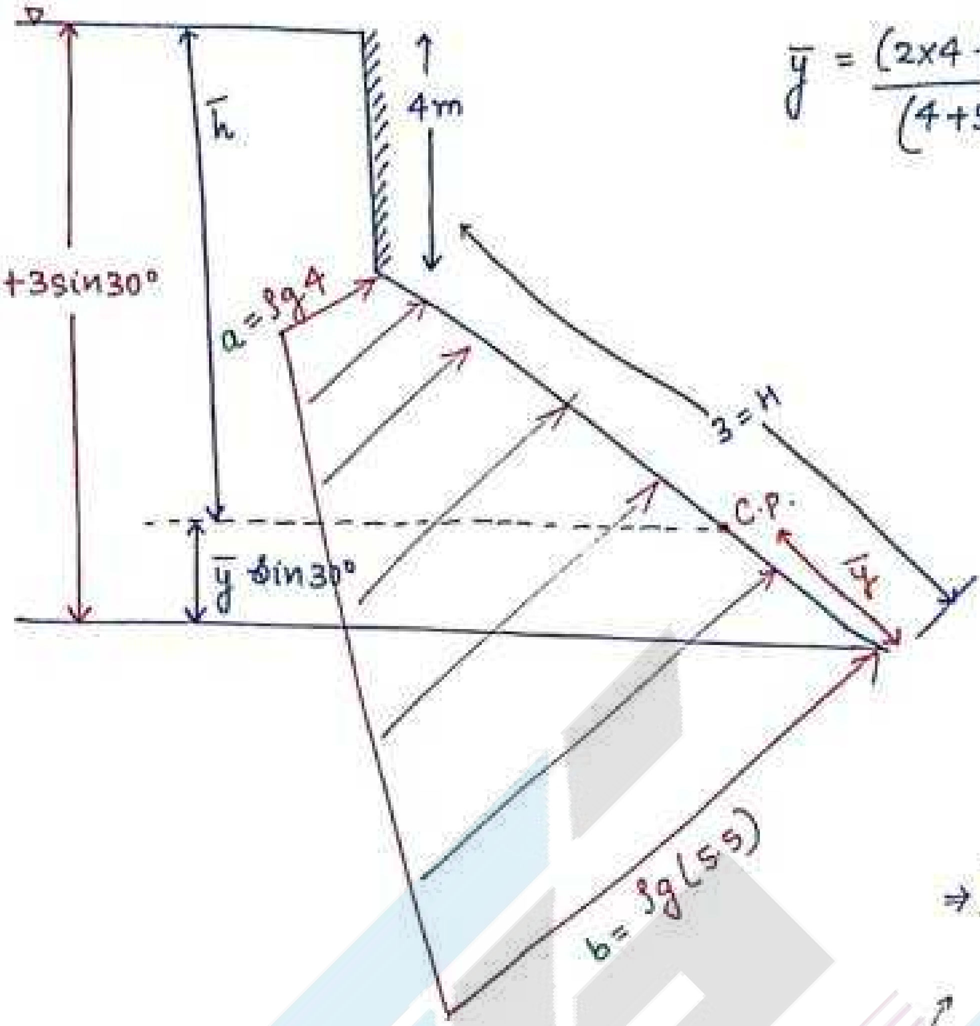
Q> Find the hydrostatic force ~~of~~ g on gate AB as well as the location of the hydrostatic force.



$$F_p = wA\bar{x}$$
$$= 9810 \times (3\times1) \times [4 + 0.75]$$
$$F_p = 139.79\,KN$$

$$\bar{h} = \frac{I_{GG}\sin^2\theta}{A\bar{x}} + \bar{x}$$
$$\bar{h} = \frac{(1\times 3^3/12)\sin^2 30^\circ}{(3\times1)\,4.75} + 4.75$$
$$\bar{h} = 4.789\,m$$

$$\bar{y} = \frac{(2\times4+5.5)}{(4+5.5)} \times \frac{3}{3}$$

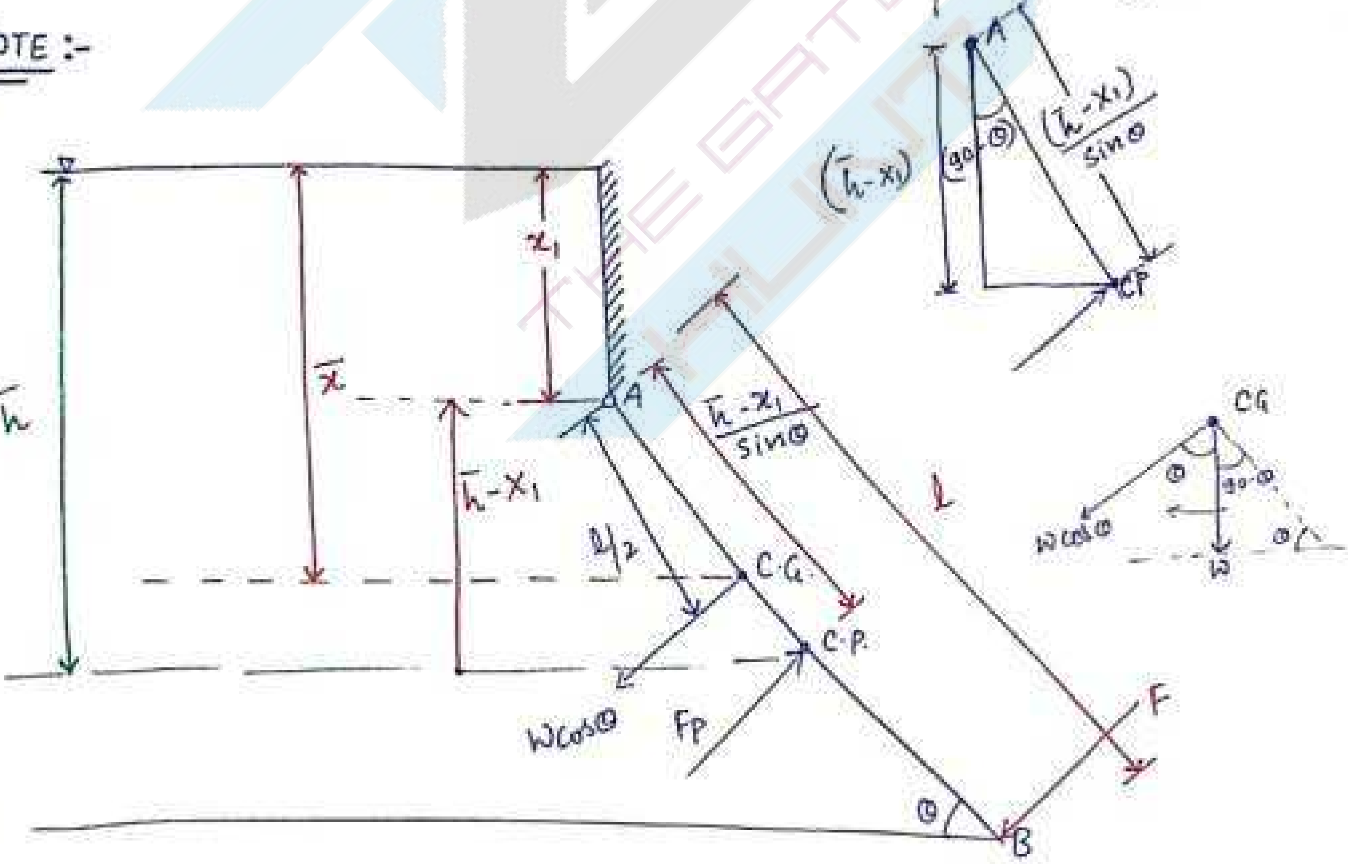


$$\rightarrow \frac{\bar{h}-x_1}{\sin\theta}\cos(90-\theta)$$

$$\Downarrow$$

$$\frac{(\bar{h}-x_1)}{\cancel{\sin\theta}}\cancel{\sin\theta}$$

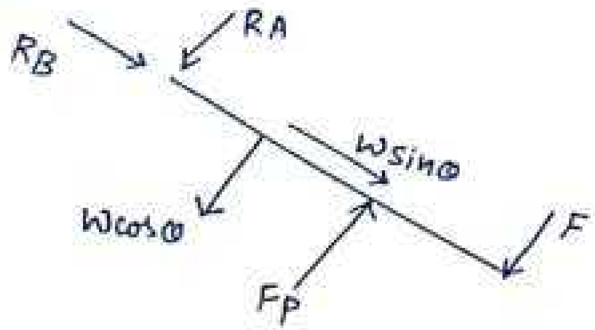
NOTE :-



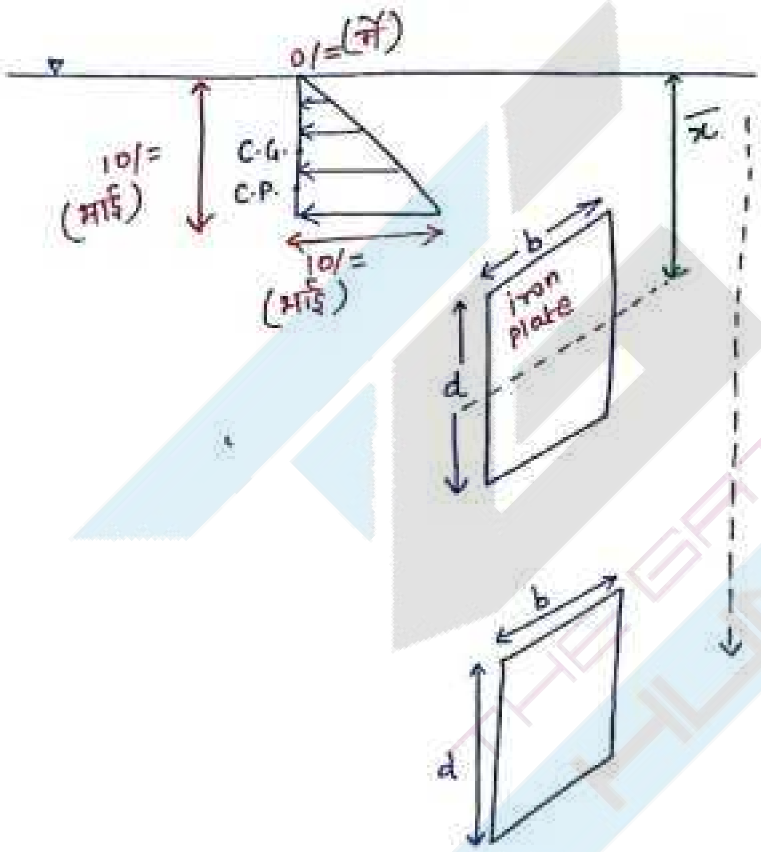
$$F_p = \omega A\bar{x}$$

$\Sigma M_A = 0$

Hinge Reaction



Note :- (Hydraulic analysis) vertical Flat Plate



const

$$\bar{h} = \left[\frac{I_{G} \sin^2\theta}{A\bar{x}}\right] + \bar{x}$$

doesn't change = constant

$A = b \times d$

$\boxed{\bar{h} \approx \bar{x}}$

COP shifting towards C.G.

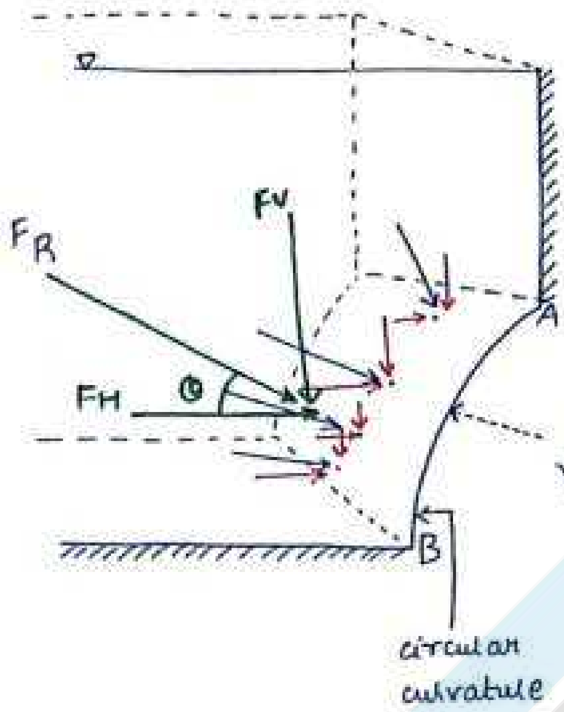
WB Q20 (a) ✓ (c) × never coincides

vertical plane But in horizontal plane है $(\bar{h} = \bar{x})$. ✓

NOTE :- As the depth of immersion of vertical plate increases, the pressure diagram on the surface becomes uniform. so Centre of Pressure shifts towards Centre of Gravity but will never coincide.

Hydrostatic forces on Curved Surfaces.

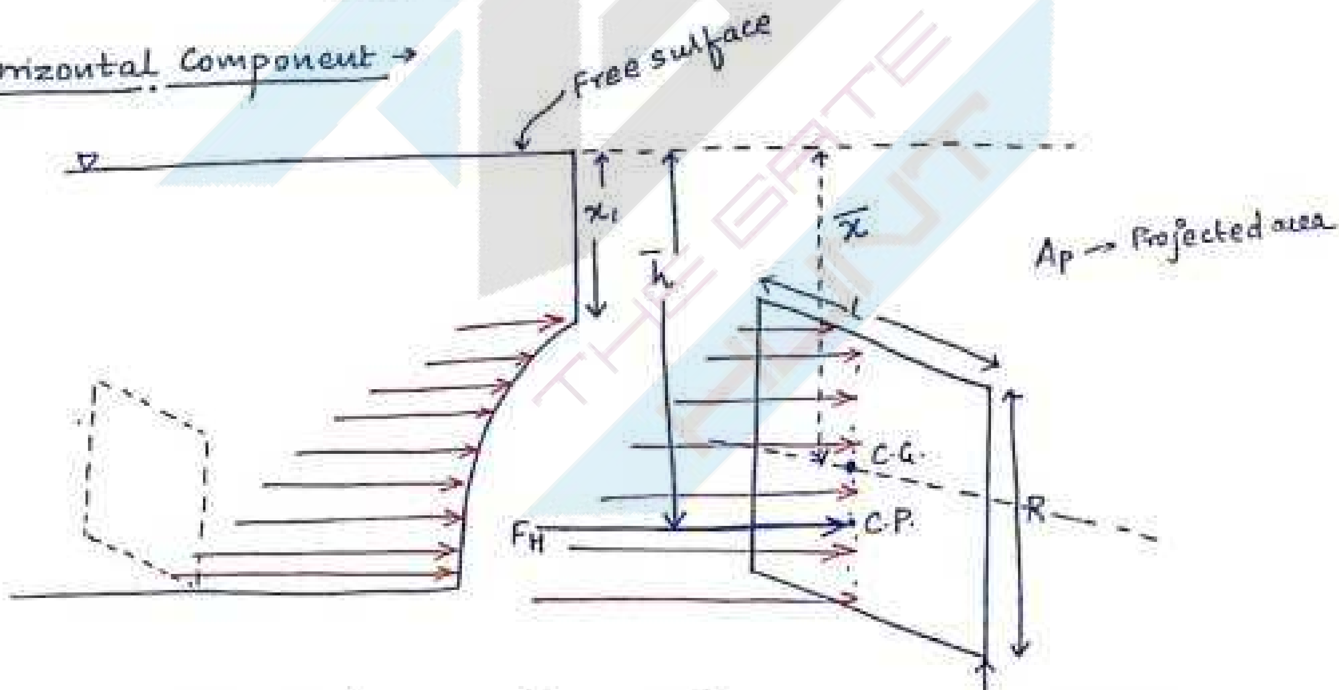
Hydrostatic forces on curved surfaces are in different orientation. The resultant force can be obtained by horizontal and the vertical component of hydrostatic force



$$\sqrt{F_H^2 + F_V^2} = F_R$$

$$\tan\theta = \frac{F_V}{F_H}$$

• Horizontal Component →

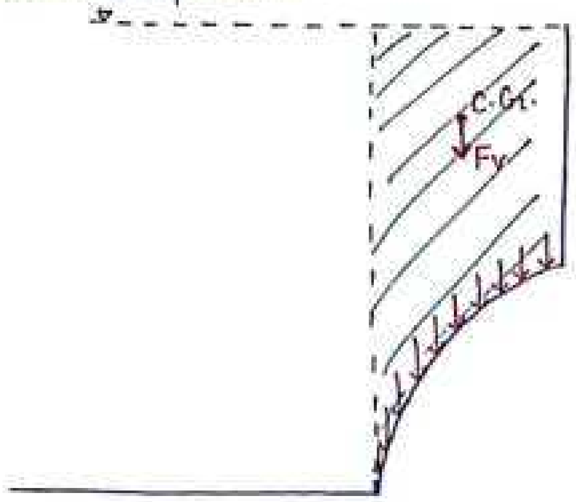


The horizontal component is the hydrostatic pressure force on projected area of curved surface in vertical plane. its point of application is centre of pressure of corresponding area.

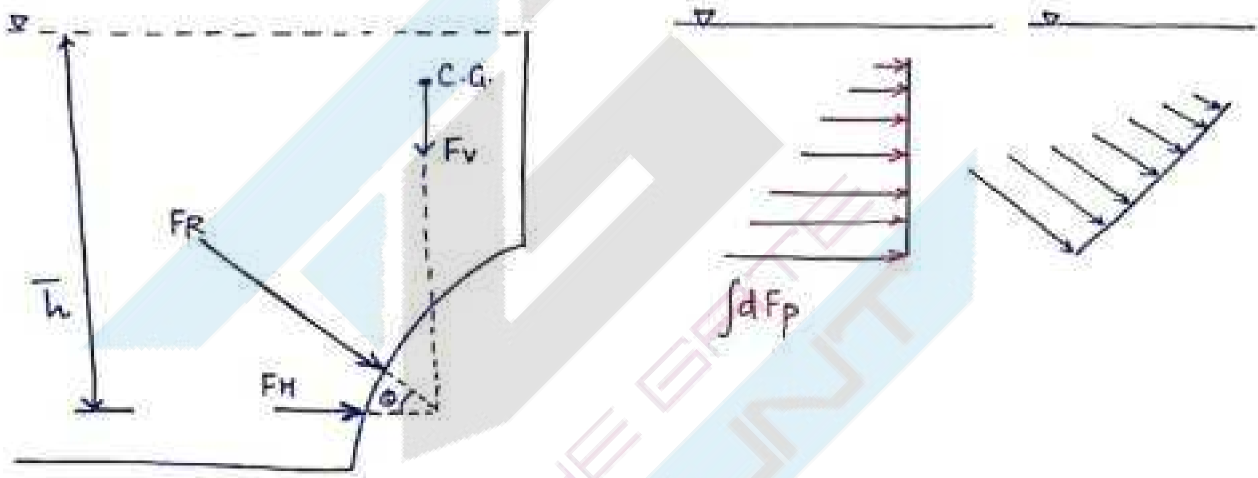
$$F_H = \omega A_p \bar{x}$$

$$\bar{x} = \frac{R}{2} + x_1$$

*Vertical Component :-



It is the weight of the liquid column above the curved surface upto the free surface and it will act at the C.G. of corresponding volume



Q Find the magnitude of Total hydrostatic forces acting on curved surface of a Dam which is shaped according to the Relation $y = \frac{x^2}{9}$ as shown in figure, the height of water retained by the Dam is 10m and the width of the Dam is 1m.

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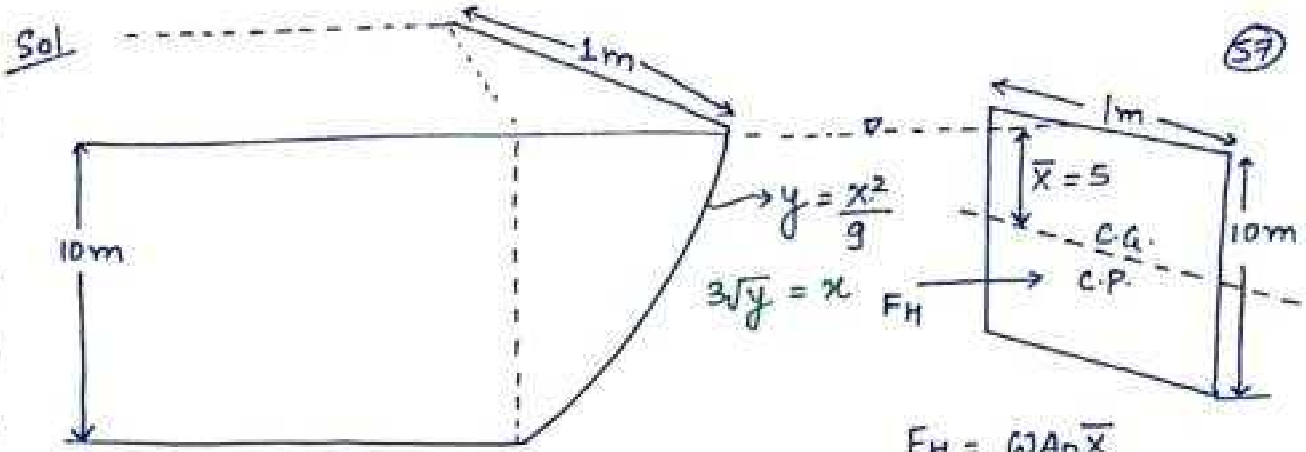
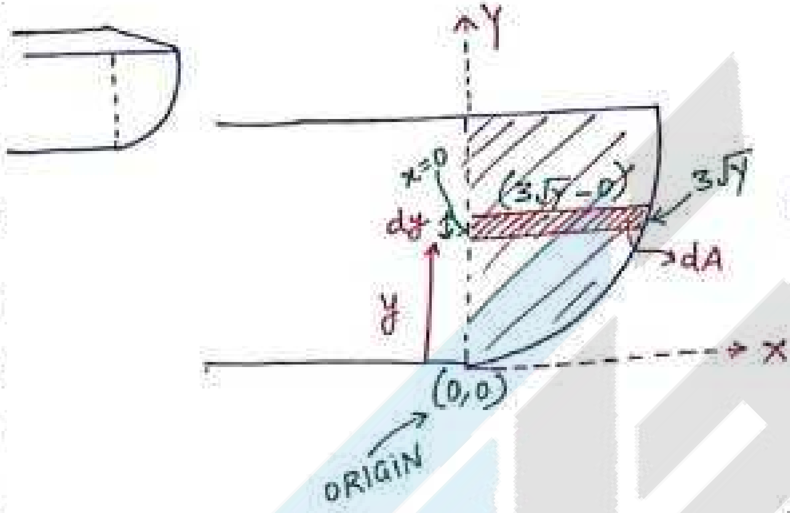
Sol



$F_H = \omega A_p \bar{x}$

$= 9810 \times (10 \times 1) \times 5$

$F_H = 490.5$ KN



$F_V = W \times \text{vol}^m$.

$F_V = W \times A \times 1 = 9810 \times 2 \times 10^{3/2}$

$F_V = 620.4$ KN

$dA = (x - 0)\, dy$

$\int dA = \int_0^{10} (3\sqrt{y} - 0)\, dy$

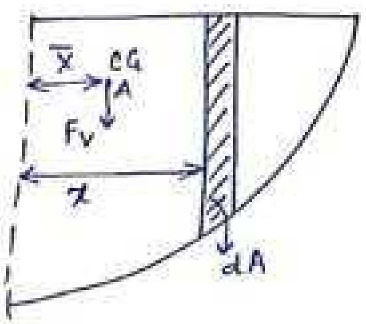
$A = 2 \times 10^{3/2}$

$F_R = \sqrt{\hat{F_H}^2 + \hat{F_V}^2}$

$= \sqrt{F_H^2 + F_V^2}$

$\boxed{F_R = 790.9 \text{ KN}}$

*



$A\bar{x} = \int x\, dA$ (SOM concepts)

$\bar{x} = ?$

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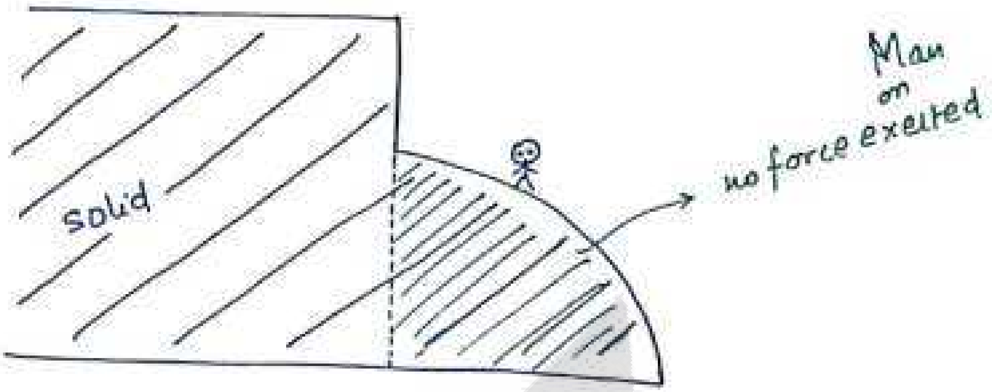
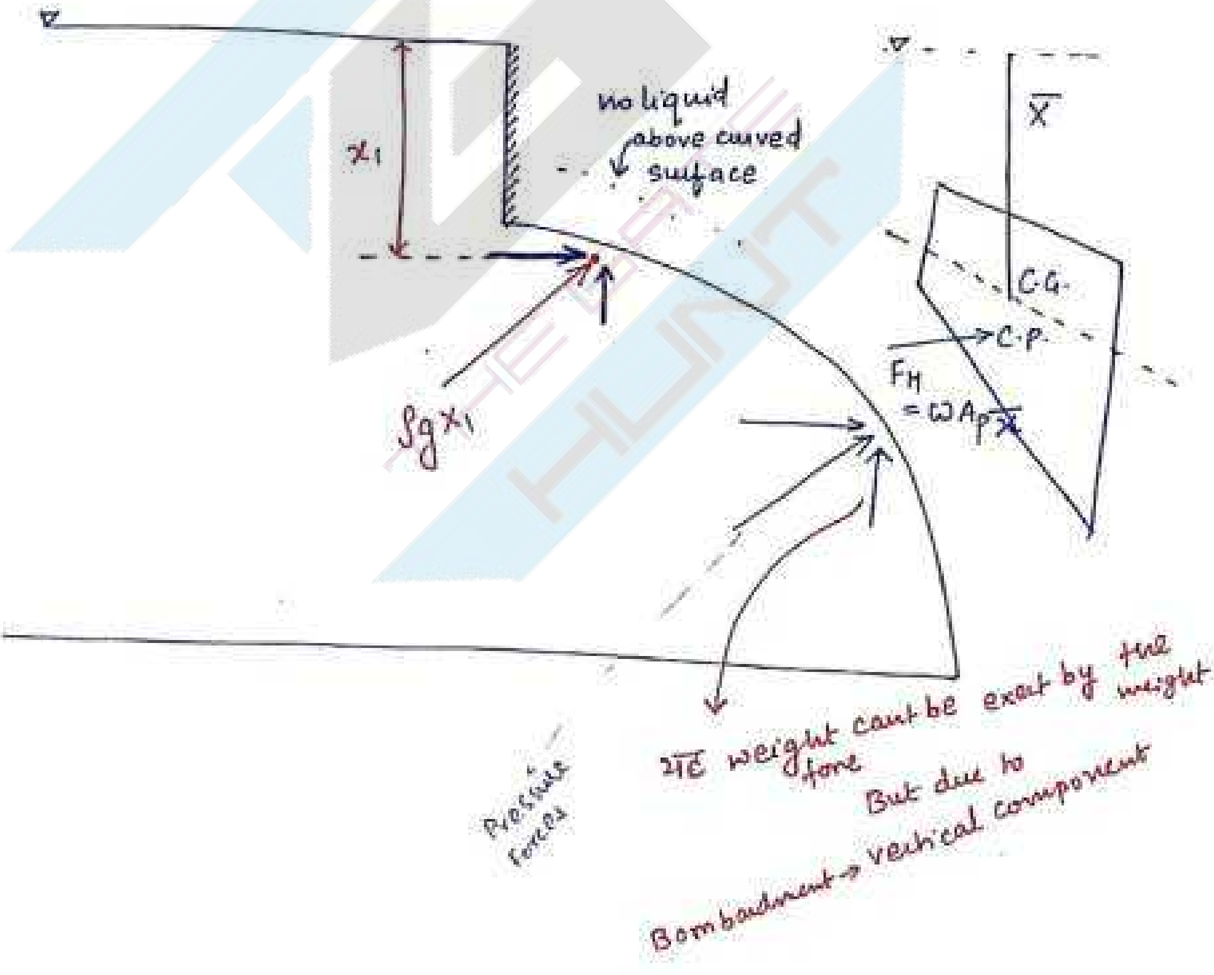
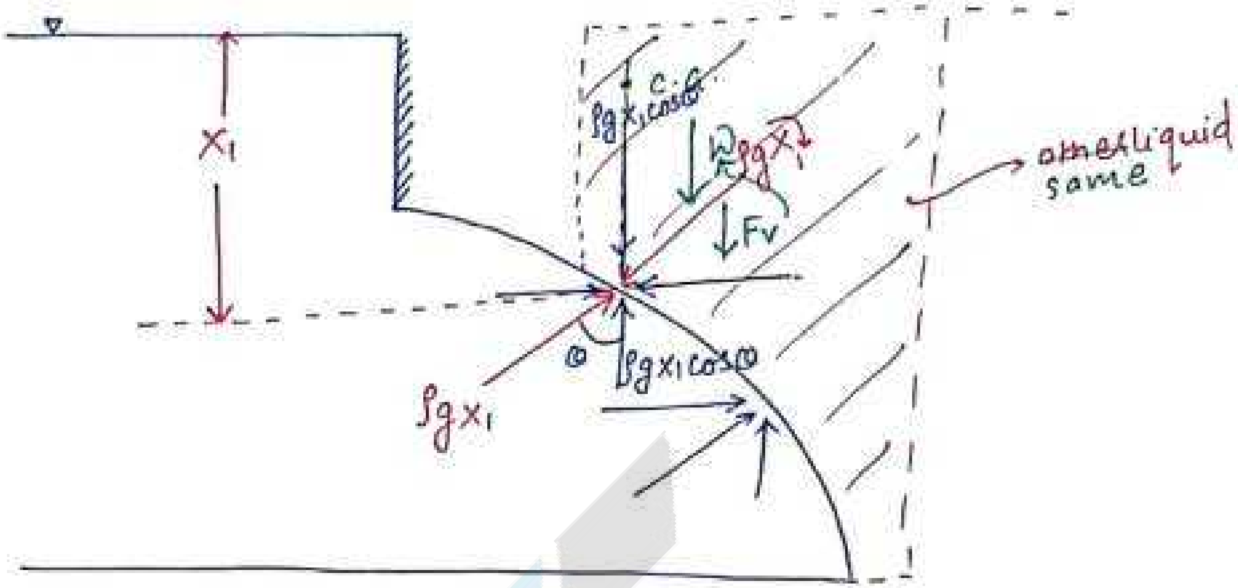
Note :→ ① If there is no liquid above the curved surface.

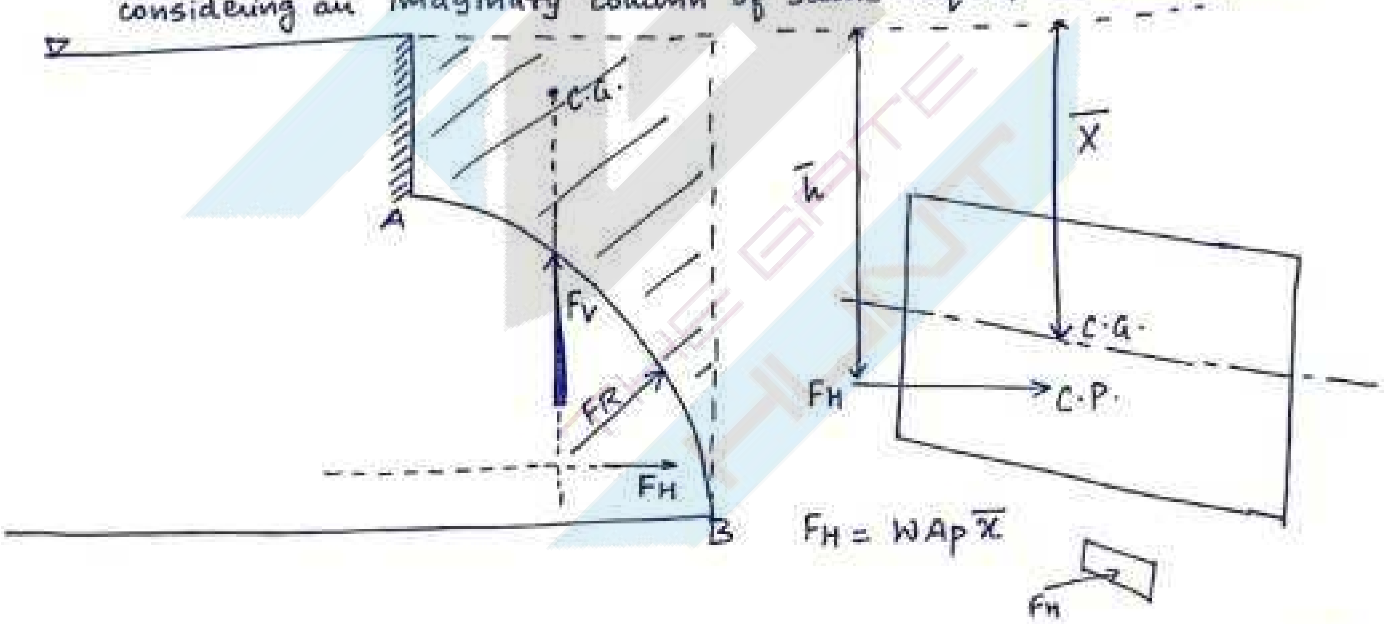




Hence, Analogy comes → Assume liquid ~~another~~ same (59)



modified diagram :→ If there is no liquid above the curved surface, the vertical component can be obtained by considering an imaginary column of same liquid as same free



surface. The weight of an imaginary column is the vertical component. The dir^n. of vertical component is upward and line of action is same as weight of an imaginary column.

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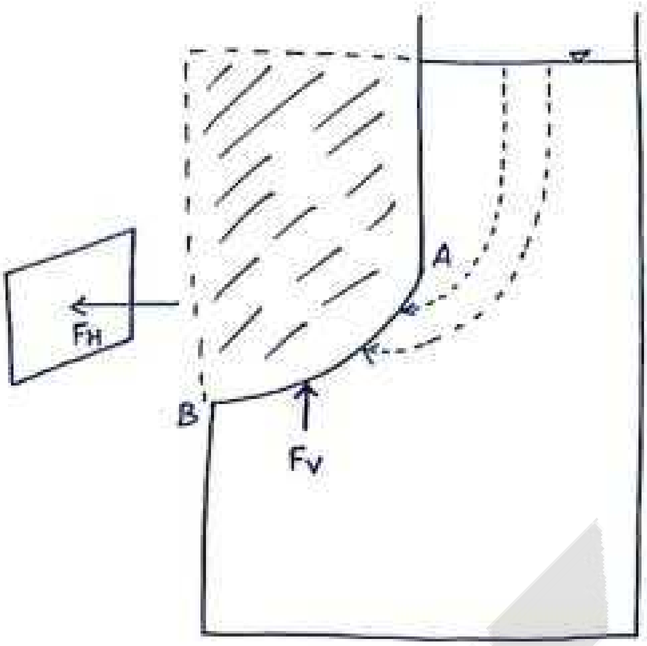
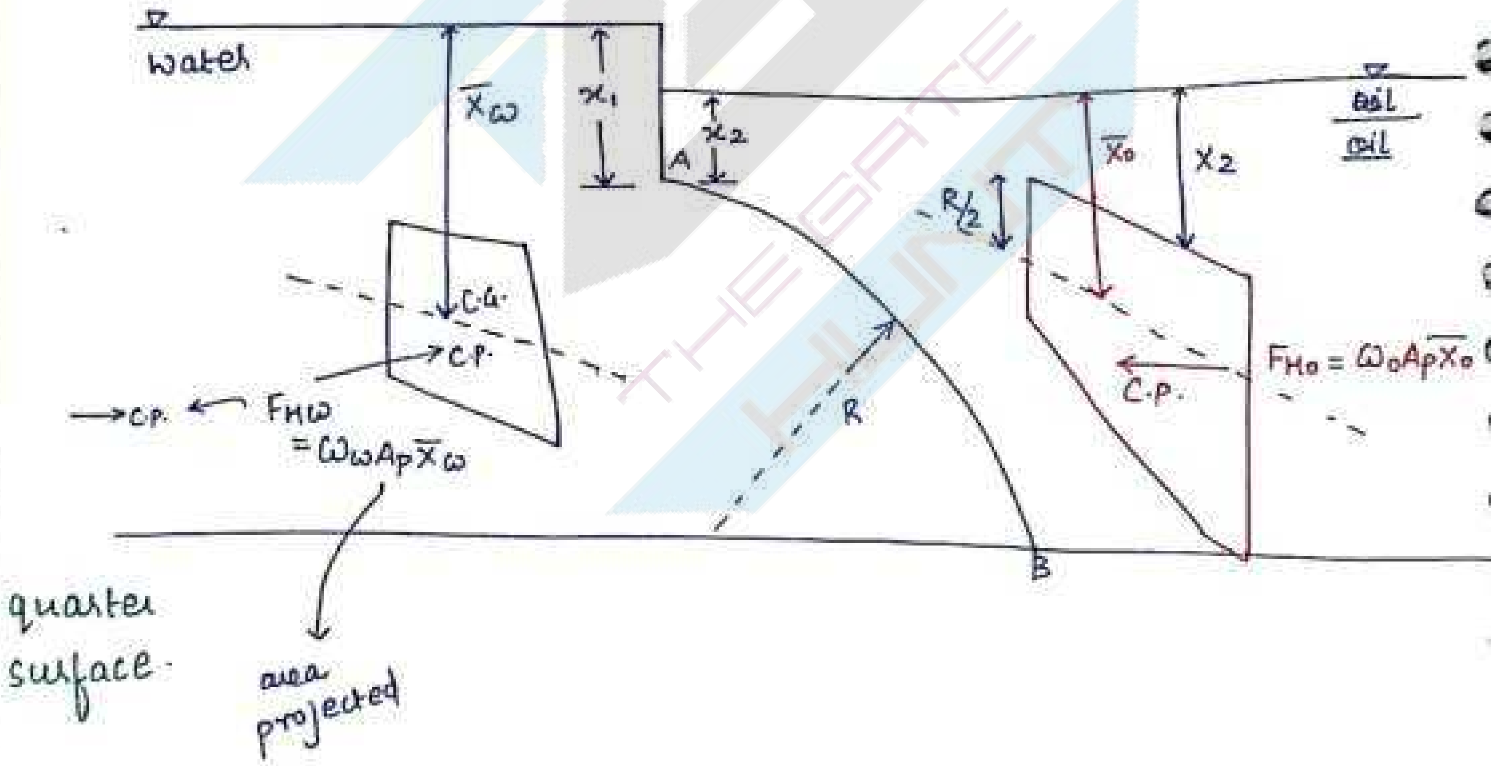
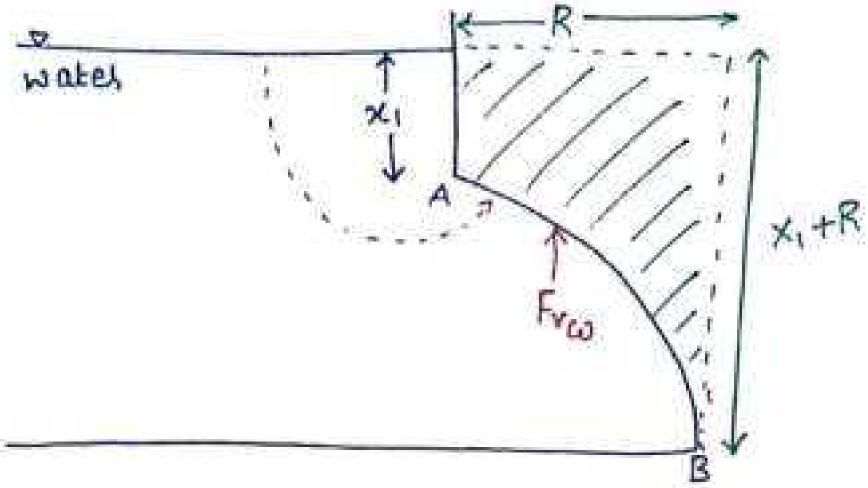
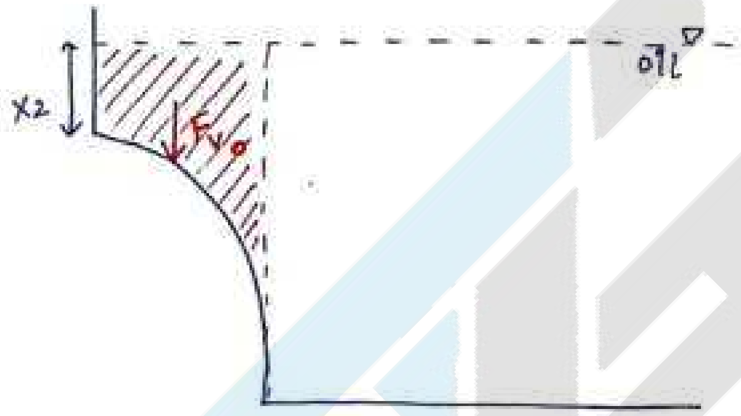
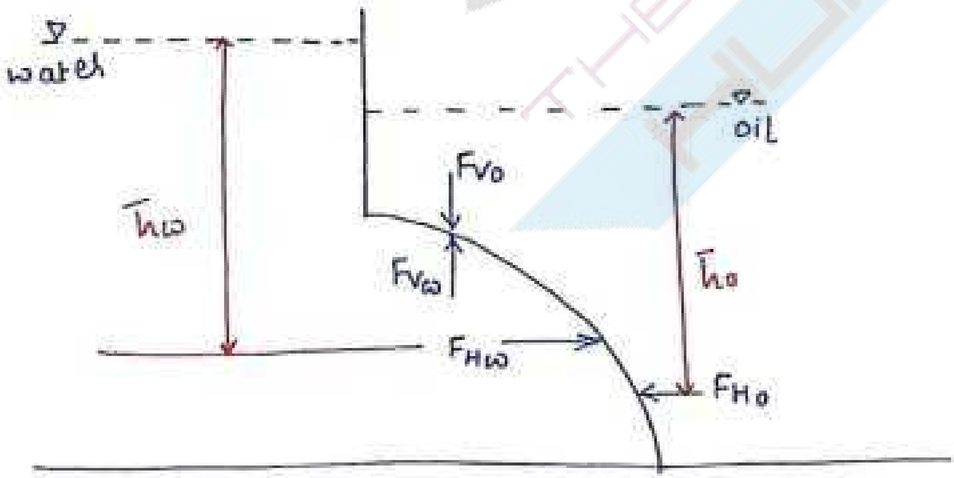

A
B
F_H
F_V

NOTE :- ②

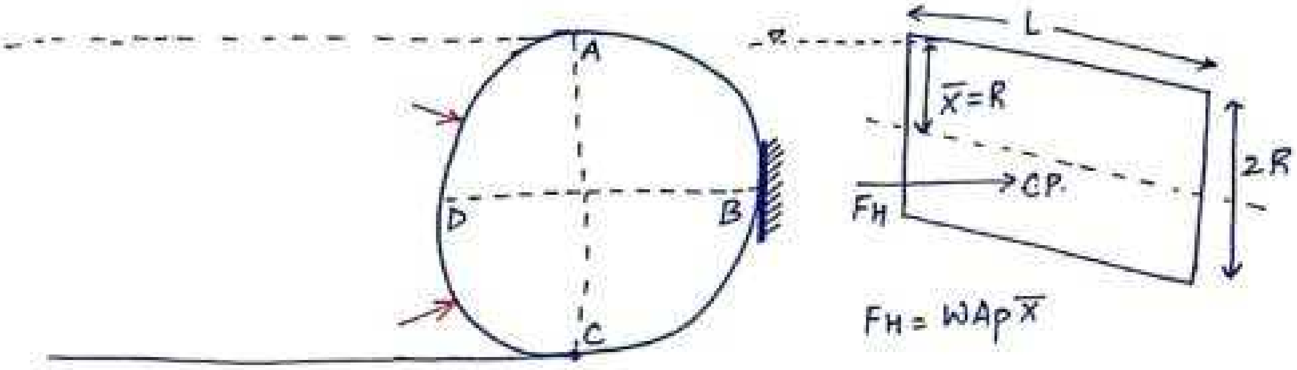

water
$\bar{x}_\omega$
x_1
x_2
A
$\bar{X}_o$
X_2
oil
R/2
C.G.
C.P.
C.P.
$F_{H\omega} = \omega_\omega A_p \bar{x}_\omega$
$F_{Ho} = \omega_o A_p \bar{X}_o$
C.P.
R
B
quarter surface
area projected


water
R
x_1
A
B
x_1+R
$F_{V\omega}$

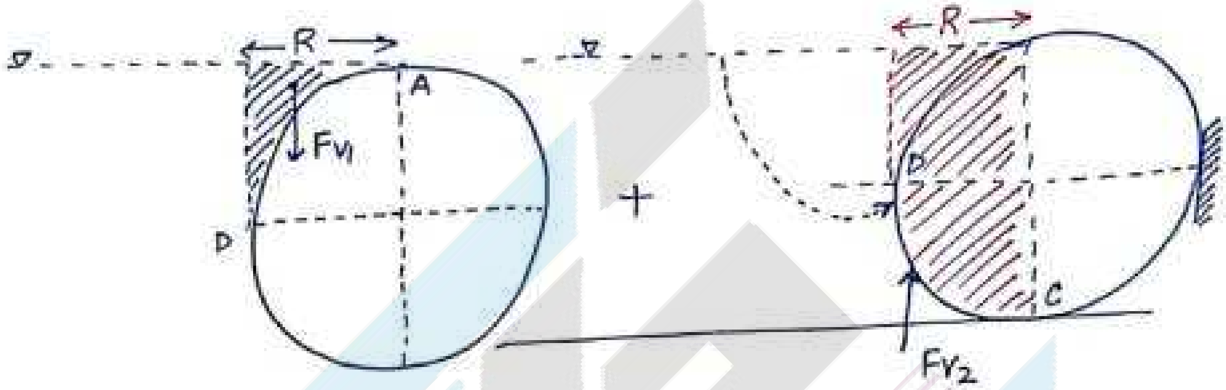

x_2
oil
F_{Vo}


water
oil
$\bar{h}_w$
$\bar{h}_o$
F_{Vo}
$F_{V\omega}$
$F_{H\omega}$
F_{Ho}

NOTE :- 3



$F_H = \omega A_p \bar{x}$



$F_{V_1} = \omega \times vol$

$= \omega \times \left[R^2 - \frac{\pi}{4}R^2\right] \cdot L \; (\downarrow)$

$F_{V_2} = \omega \times vol$

$= \omega \times \left[R^2 + \frac{\pi R^2}{4}\right] \cdot L$

$(\uparrow)$

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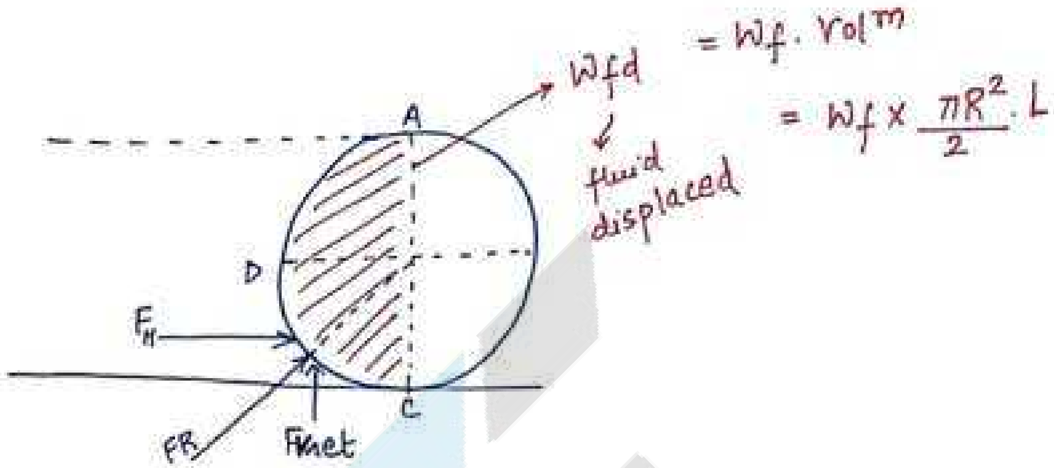
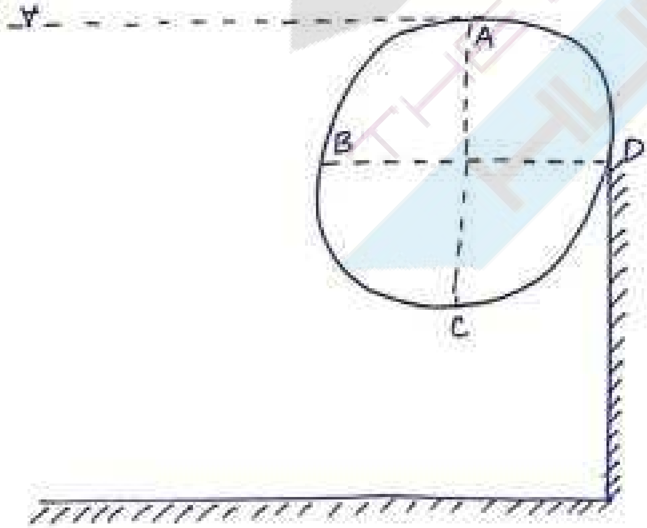


=

$$F_{net_V} = F_{V_2}(\uparrow) - F_{V_1}(\downarrow)$$

$$= w\left(\frac{\pi}{2} R^2 \cdot L\right)(\uparrow)$$

NOTE :- HW ④



?

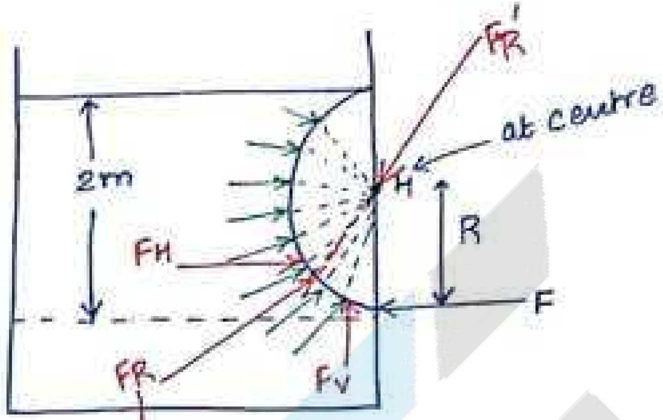
MOHIT CHOUKSEY

WB (24) $F_R = WA\bar{x}$ (not a curved surface)

$\hookrightarrow F_H \rightarrow$ no need

Since forces are in different orientations, then F_H, F_V, F_R calculate karenge.

(28)



Resultant passes through centre

$\Sigma M = 0$

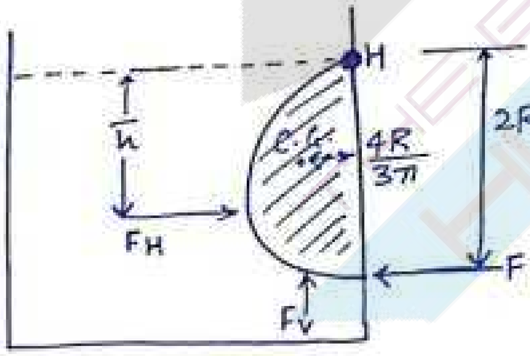
$F \times R = 0$

$R \neq 0$

$\boxed{F = 0}$

at equilibrium

Future Qn →



$F \times 2R + F_V \times \frac{4R}{3\pi} = F_H \times \bar{h}$

$F = ?$

$\bar{h} = \frac{2}{3}(2R)$

Rectangular plate का distance free surface से

Homework

TS Pg 20 (HW)

CHAPTER 4

BUOYANCY & FLOATATION

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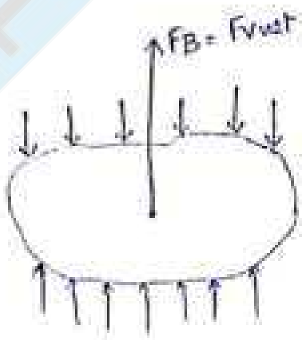


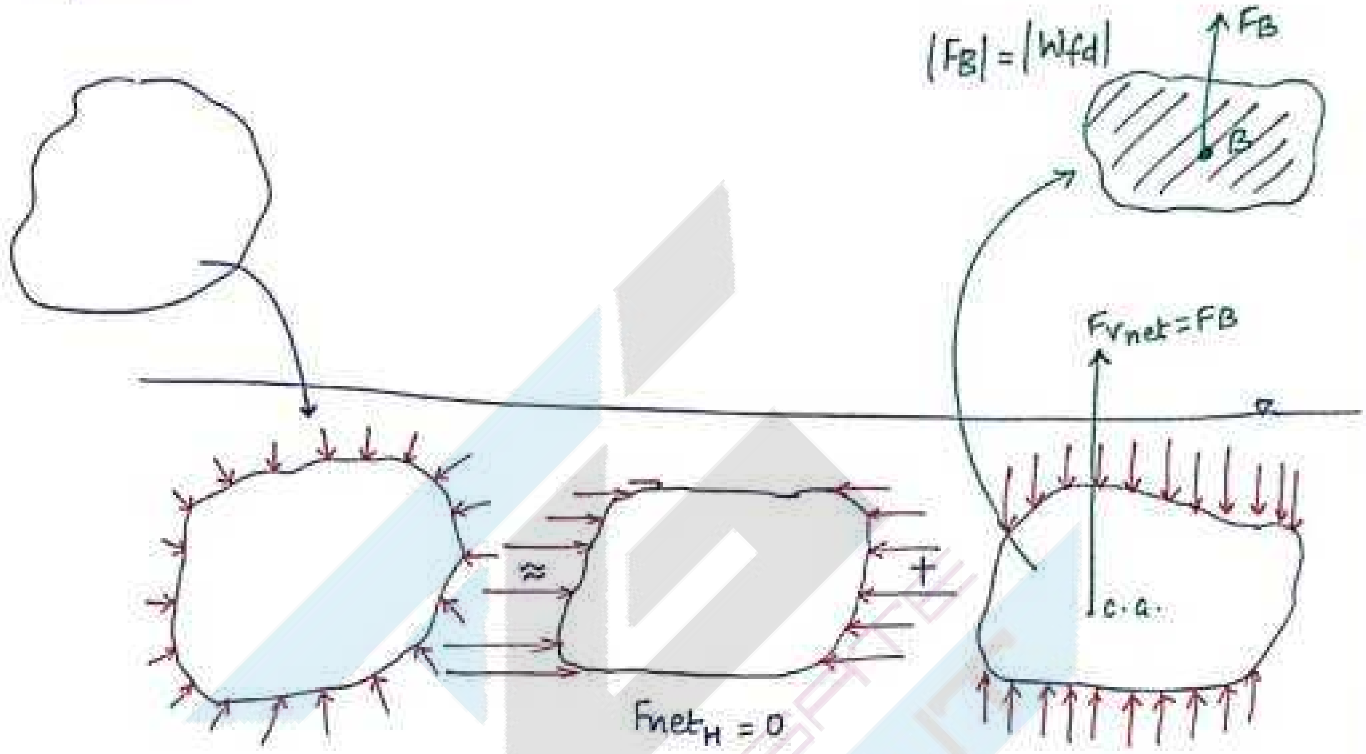
upward force > downward force

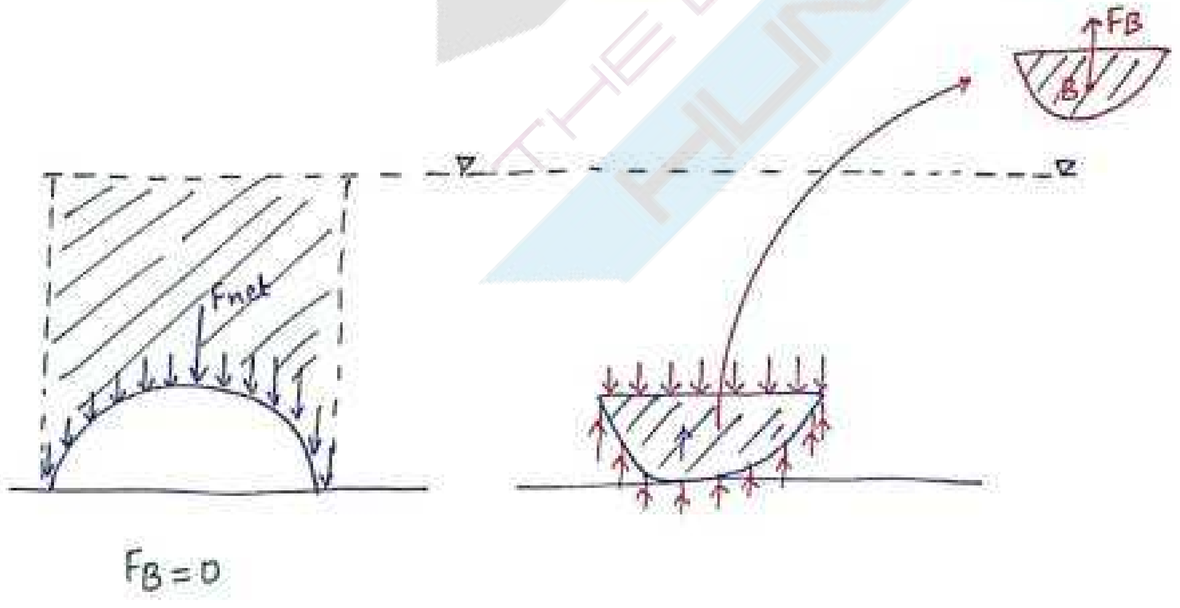
hence, $F_B = (\uparrow) - (\downarrow)$
$\downarrow$
F_{vert}

BUOYANCY AND FLOATING/FLOATATION:

when a Body is immersed partially or completely in the fluid, the net vertical upward Hydrostatic force exerted by the fluid on the body is known as Buoyant force.

Buoyant force is the weight of the fluid displaced.





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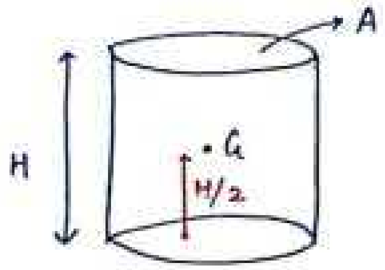
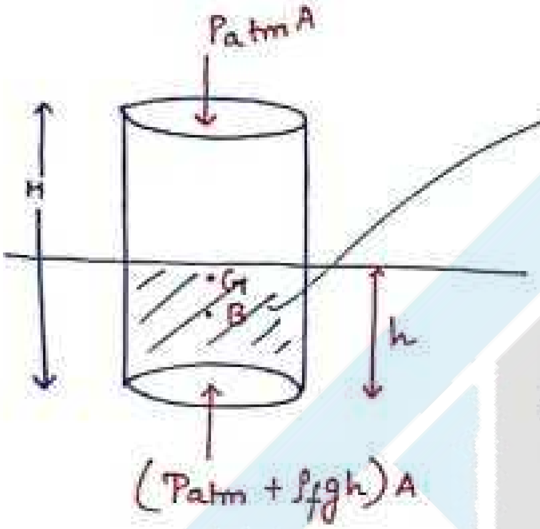
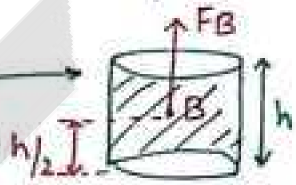


$$F_{net_V} = \rho_f g h A$$

$$F_B = F_{net_V} = \omega_f h A (\uparrow)$$

weight of the volume displaced of ~~Body~~ fluid





weight density

$$\left[\omega = \frac{W_t}{Vol}\right] = \rho g = \frac{mg}{V}$$

$$W_{fd} = \omega_f V_{fd} = (\rho_f g)(V_{fd})$$

weight of displaced fluid

$$W_{fd} = \omega_f A h$$

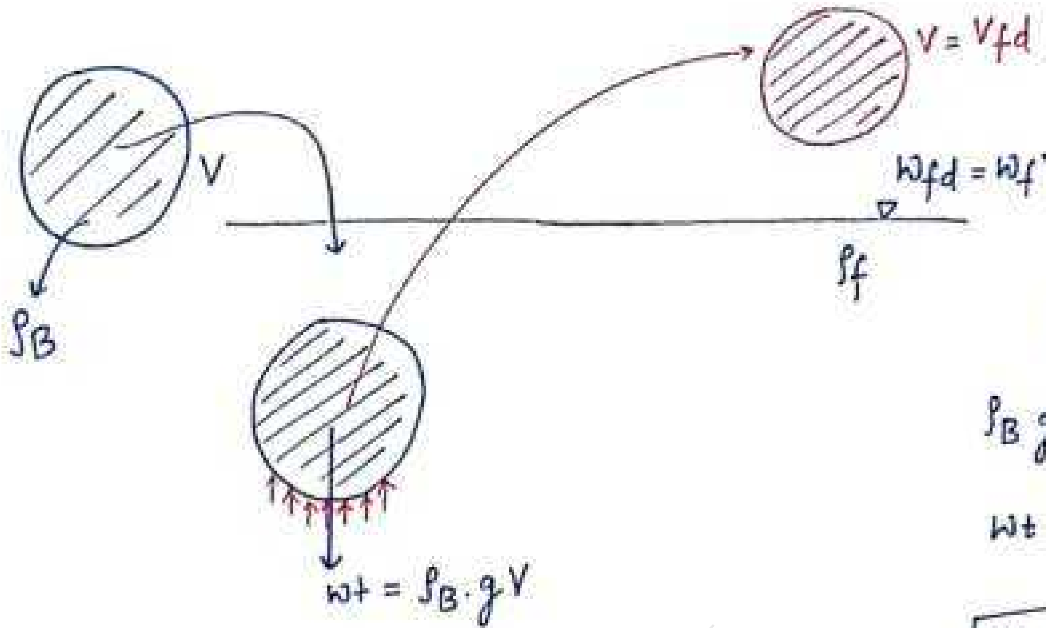
$$|W_{fd}| = |F_B|$$

Note :- Centre of Buoyancy (lies inside the Body) is the point of application of Buoyant force and it is the centre of gravity of displaced fluid.

PRINCIPLE OF Floatation :- (Archimedes Principle)

In equilibrium of submerged Body (partially / completely), the weight of the body is equals to the Buoyant force (weight of the fluid displaced).

→ The Buoyant force acting on the body immersed in a fluid is equal to the weight of the fluid displaced by body ($\rho_f g V_{fd}$) and it acts upward through the centroid of displaced volume.



$\rho_B > \rho_f$

$\rho_B g > \rho_f g$

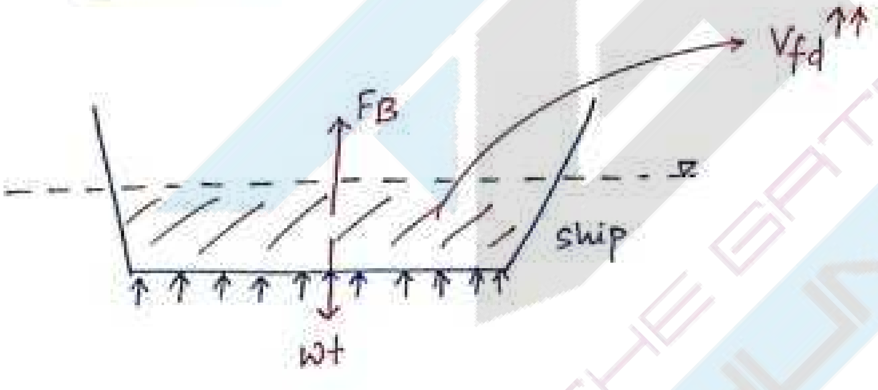
$\rho_B g V > \rho_f g V$

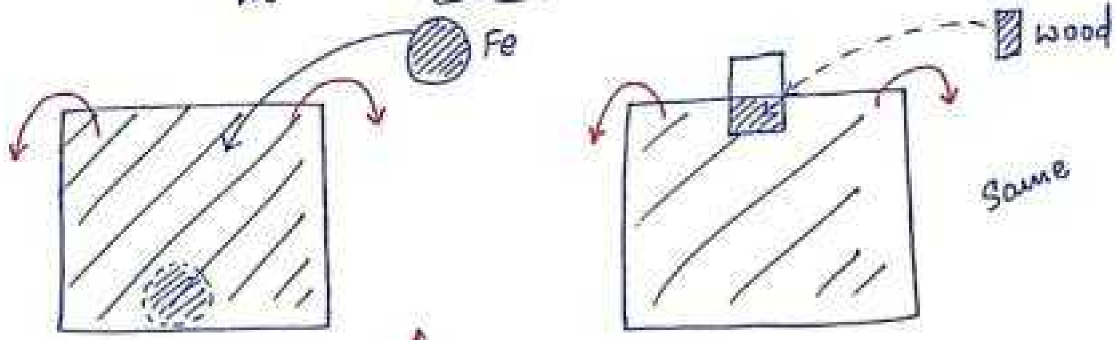
$wt > \rho_f\, g\, V_{fd}$

$\boxed{Wt > F_B}$ sink

Floating :-

$\boxed{Wt = F_B}$





* Equilibrium Condition :→

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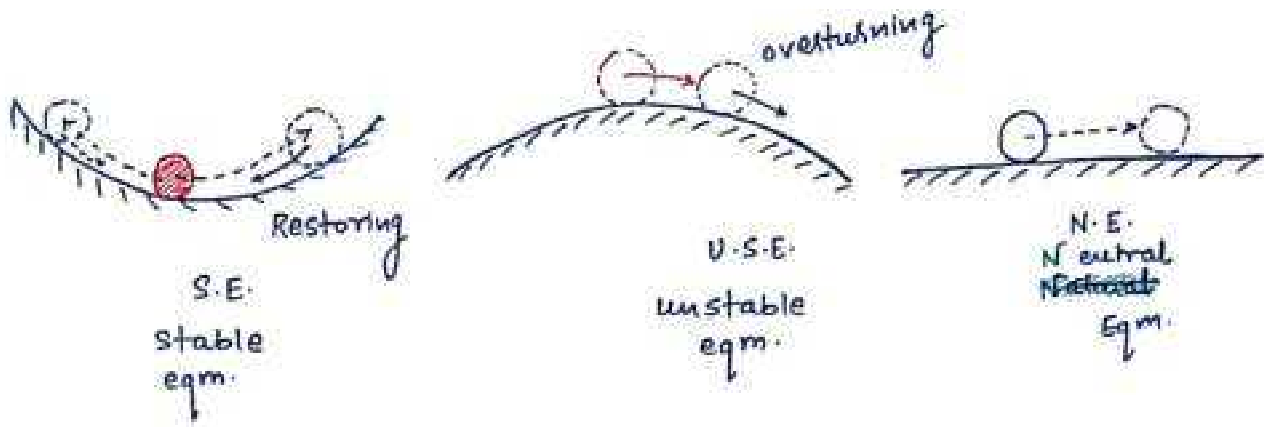
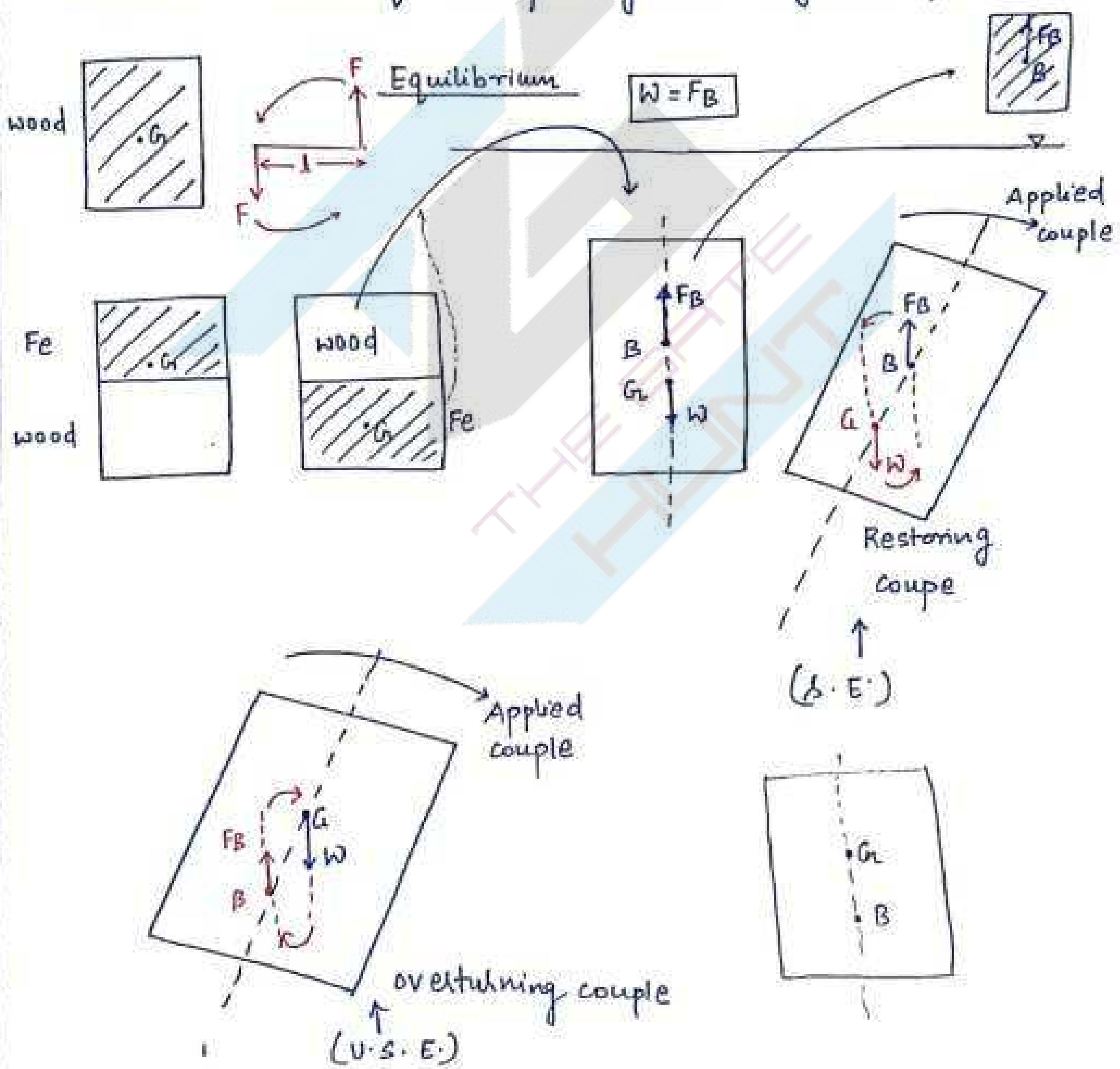
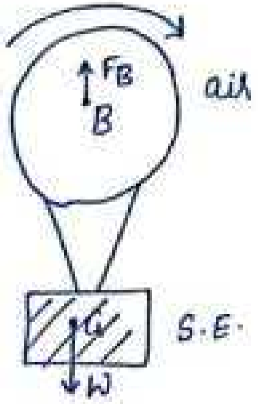
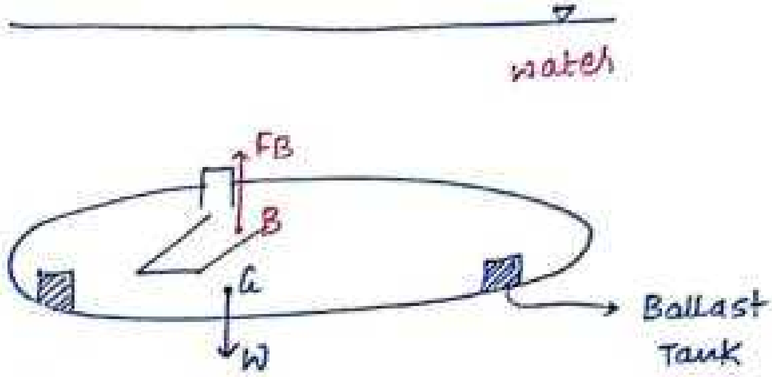


* Equilm. condition for Completely submerged Body :-





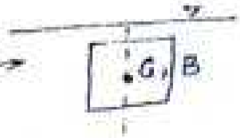


For completely submerged Bodies:-

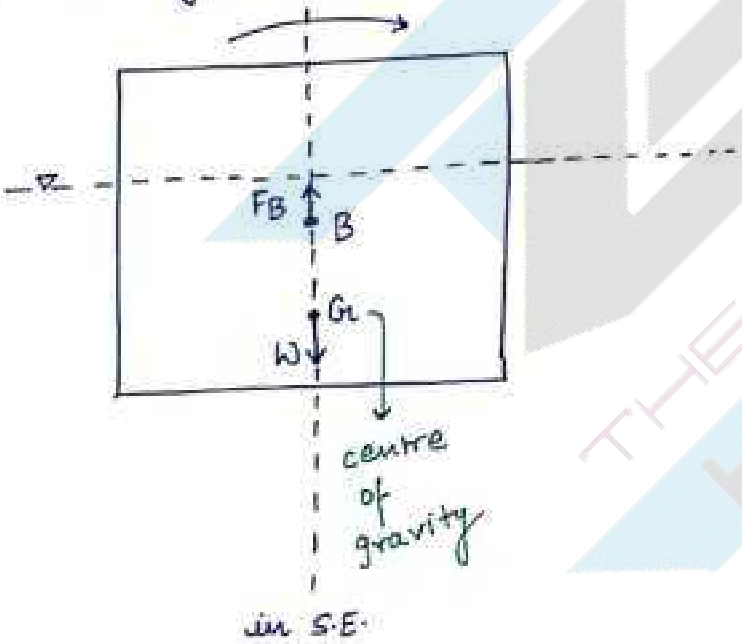
① A Body is said to be in stable equilibrium when B above G.

② In unstable equilibrium; when B below G.

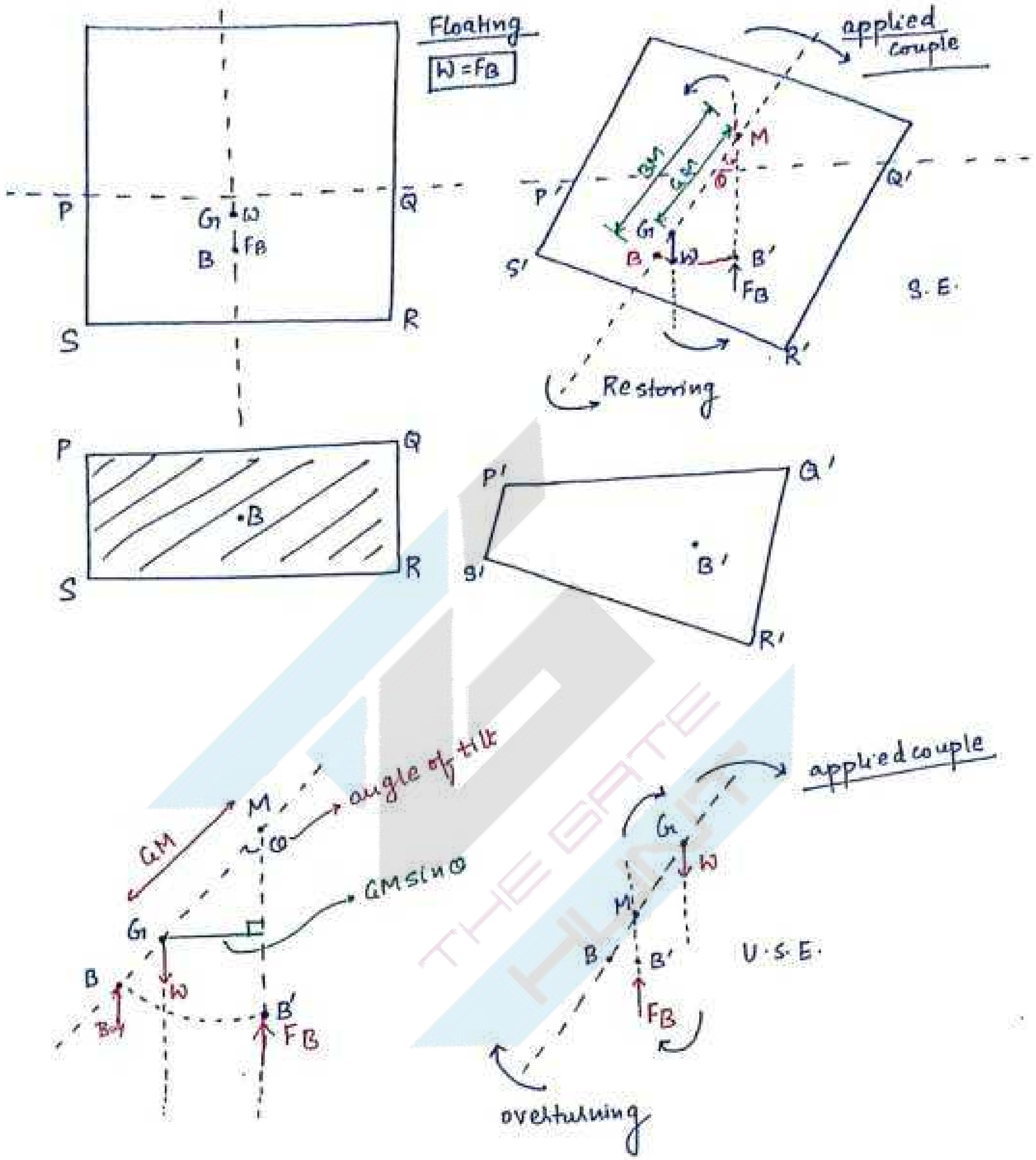
③ Neutral equilibrium ; when B coincides G. →



*** Stability Conditions for Partially Submerged Bodies :-**



MOHIT CHOUKSEY



or Because in the change of
When shape of a part which is emmersed in the fluid, we see that the line of action of the Buoyant force changes, the point where this line of action of this Buoyant force intersects this centroidal axis, that point of intersection is called Meta Centre.

GM → meta centric height

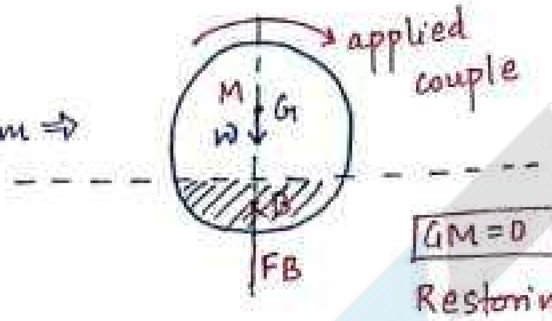
BM → ——"—— Radius

Restoring Couple = $W \cdot (GM) \sin\theta$

W ↓ +ve, $\sin\theta$ ↓ +ve

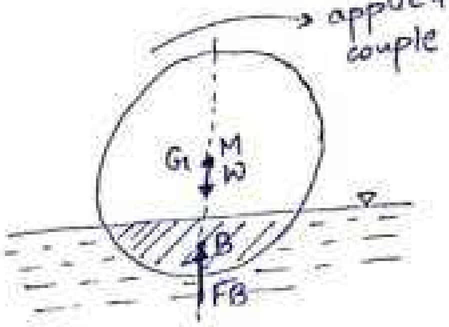
GM → +ve SE
GM → -ve USE
GM → 0 NE

Neutral Equilibrium ⇒



$\boxed{GM=0}$

Restoring couple = 0



* For partially submerged Bodies :-

① a body is said to be in stable eqm. when M above G.

② In unstable equilibrium, when M below G.

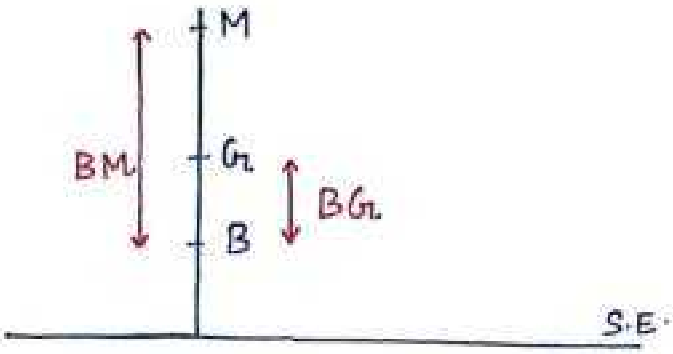
③ In Neutral equilibrium, when M coincides G.

Note :-
Metacentre may lie inside or outside the Body But centre of gravity & C.O.Buoyancy always lies inside the Bodies.

• In compl. sub. body, M.C. = C.O.B. (Meta centre = centre of Buoyancy)

Determination of Meta Centre height :-

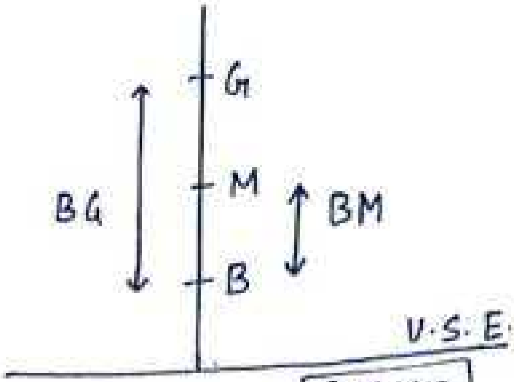
There are two methods :- ① Experimental Method
② Analytical Method



$\boxed{GM > 0}$

$\Rightarrow \boxed{GM = BM - BG}$

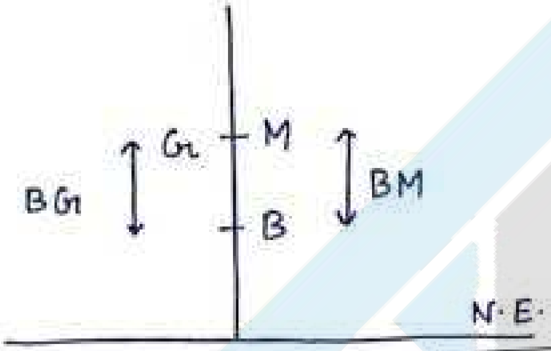
$\Rightarrow \boxed{BM > BG}$



$\boxed{GM < 0}$

$\Rightarrow \boxed{BM < BG}$

$\Rightarrow \boxed{GM = BM - BG}$



$\boxed{GM = 0}$

$\boxed{BM = BG}$

$$\boxed{BM = \frac{I}{V_{fd}}}$$

→ least MOI of water line area about tilting axis

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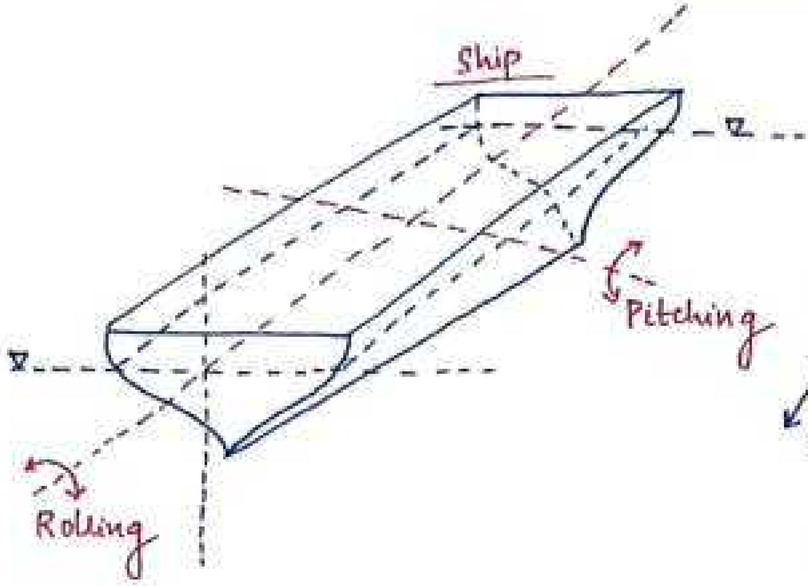
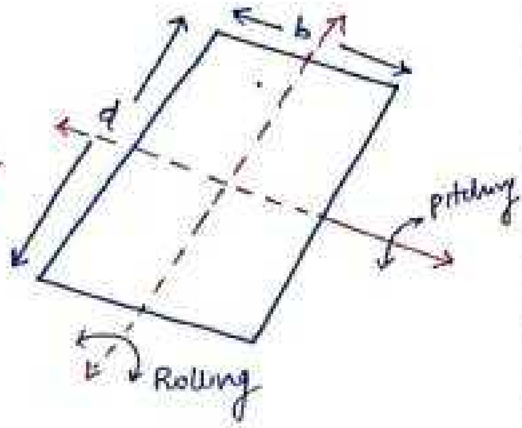




$I_{rolling} = \frac{db^3}{12}$ $\quad I_{pitching} = \frac{bd^3}{12}$

$b < d$

$I_{Roll} < I_{pitch}$

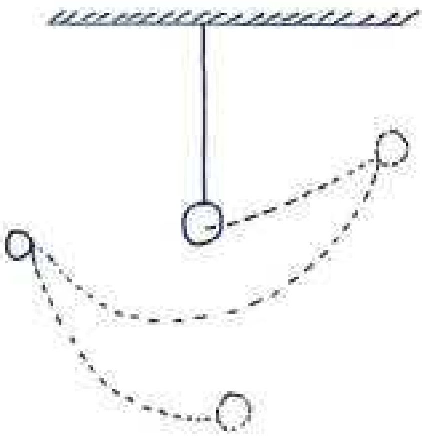
$BM_{Roll} < BM_{pitching}$

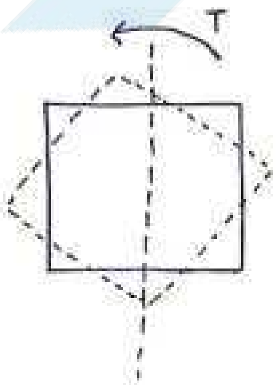
$$\boxed{GM_{Rolling} < GM_{pitching}}$$

$$\boxed{GM = \frac{I}{V_{fd}} - BG}$$

stable eqm (ship) M, G, B

Time Period of Oscillation :→





$$\boxed{T = 2\pi\sqrt{\frac{k^2}{gGM}}}$$

k = Radius of gyration

GM = Metacentric height

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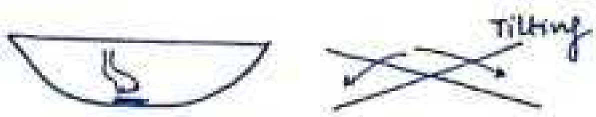
$\downarrow T \propto \frac{1}{\sqrt{GM}\uparrow}$ GM$\uparrow$ $\longrightarrow$ stability$\uparrow$ $\rightarrow$ T$\downarrow$ $\rightarrow$ comfort level ($\downarrow$) down

Warships :→ GM ⇒ 1.3m to 1.8m

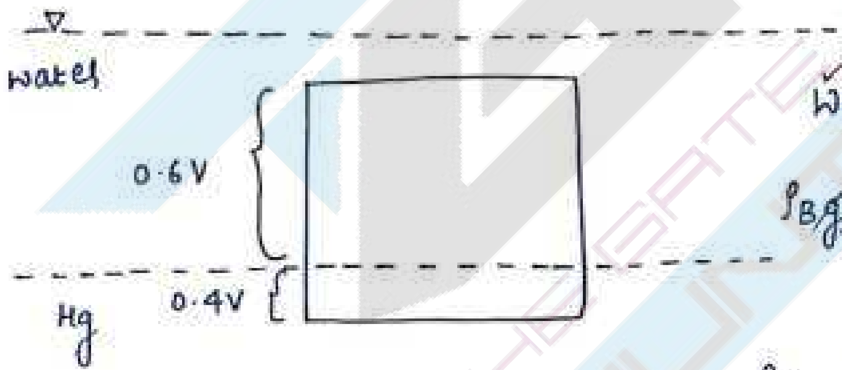
Cargoships :→ GM ⇒ 0.5m to 1m

River Boats



Q. A metallic Bodies/y floats at the inteface of Hg and water in such a way that 40% of its volume is submerged in mercury and 60% in water. find the ρ of the Body.

Sol. which case ⟶ mark floats



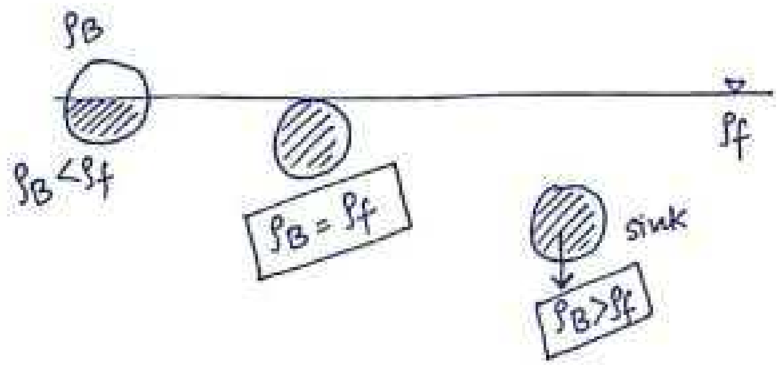
$\boxed{W_B = W_{fd}}$

$w_B \cdot V = w_w V_w + w_{Hg} V_{Hg}$

$\rho_B g V = \rho_w g \times 0.6V + \rho_{Hg} \times g \times 0.4V$

$\rho_B = 1000 \times 0.6 + 13.6 \times 1000 \times 0.4$

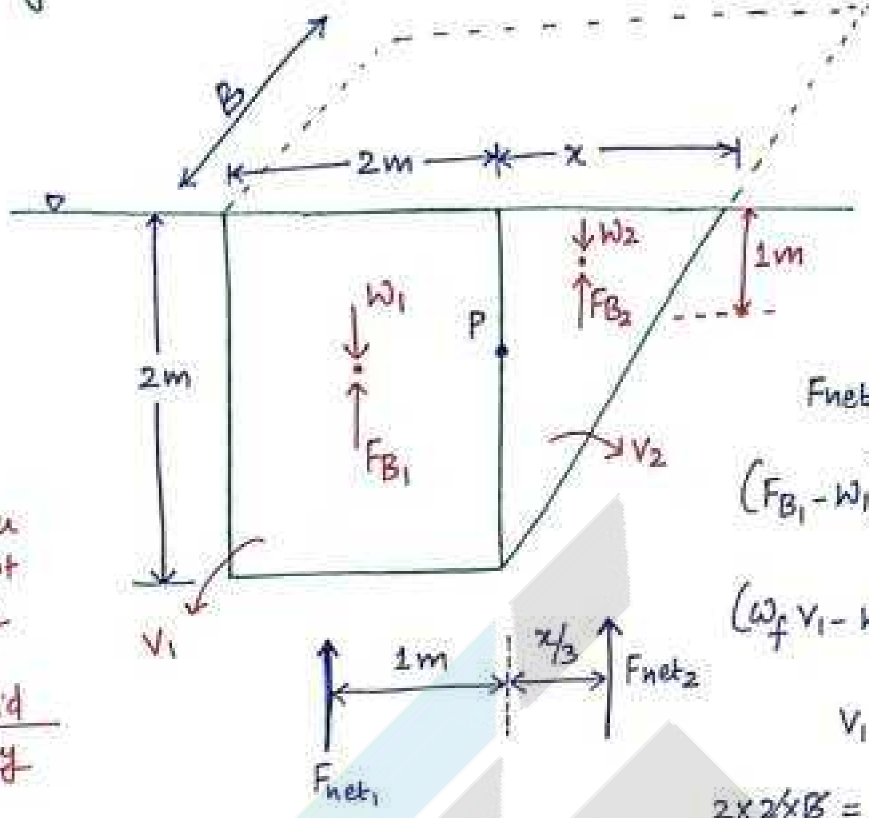
Ans $\boxed{\rho_B = 6040 \text{ Kg/m}^3}$



Note → Metacentric height does not depends on material of floating Body.

Pg 70 Q35 Submerged → balanced in horizontal condn.

Body → unbalanced → vertical forces



water is not given

fluid only

$$F_{net_1} \times 1 = F_{net_2} \times \frac{x}{3}$$

$$(F_{B_1} - W_1) = (F_{B_2} - W_2)\, x/3$$

$$(\omega_f V_1 - \omega_B V_1) = (\omega_f V_2 - \omega_B V_2)\frac{x}{3}$$

$$V_1 = V_2 \cdot x/3$$

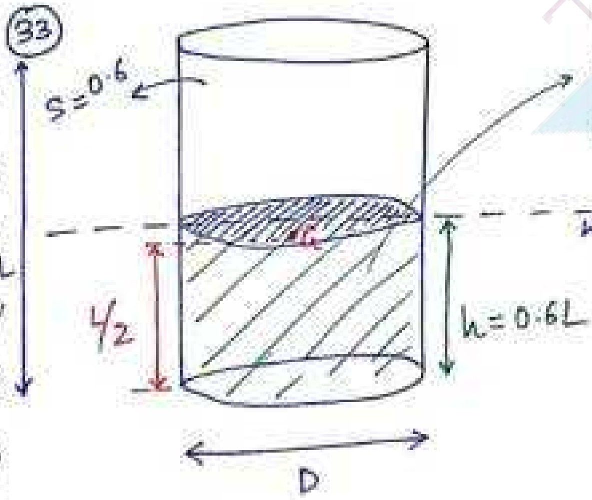
$$2 \times \cancel{2} \times \cancel{B} = \frac{1}{2} \times \cancel{2} \times x \cdot \frac{x}{3} \cdot \cancel{B}$$

$$x^2 = 4 \times 3$$

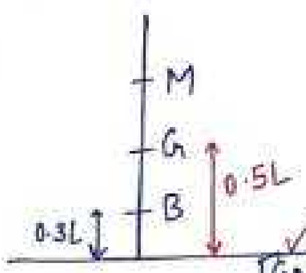
$$\boxed{x = \sqrt{4}\sqrt{3}}$$

$$\boxed{x = 2\sqrt{3}}$$

33



GM = BM − BG



$$\boxed{GM = BM - BG}$$

N.E. GM = 0

N.E. $\boxed{GM=0}$

$BM - BG = 0$

$BM - BG = 0$

$BG = 0.2L$

$W_t = W_{fd}$

$W_B \cdot V_B = W_w \times V_{fd}$

$\frac{W_B}{W_w} \times (\cancel{A} \times L) = \cancel{A} h$

$\boxed{S \cdot L = h}$

speci. gravity

$BM = \frac{I}{V_{fd}}$

$= \frac{\frac{\pi}{\cancel{64}_{16}} D^{\cancel{4}\,2}}{\frac{\cancel{\pi}}{\cancel{4}} D^2 \times h}$

$BM = \frac{D^2}{16 \times 0.6L}$

$BM - BG = 0$

$\frac{D^2}{9.6L} - 0.2L = 0$

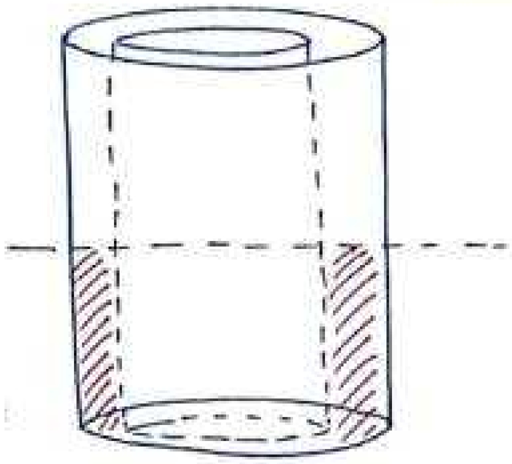
$\frac{L^2}{D^2} = \frac{1}{0.2 \times 9.6}$

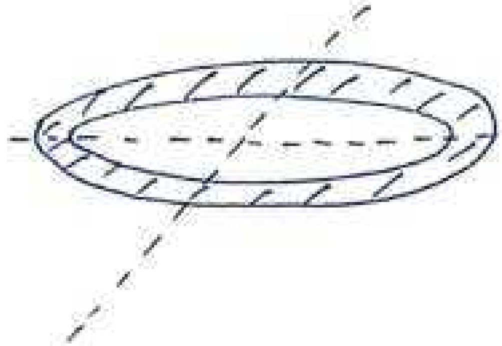
$\frac{L^2}{D^2} = \frac{\cancel{100}\ \cancel{50}\ 25}{\cancel{2} \times \cancel{96}\ 48} = \frac{25}{16 \times 3}$

$\boxed{\frac{L}{D} = \frac{5}{4\sqrt{3}}}$

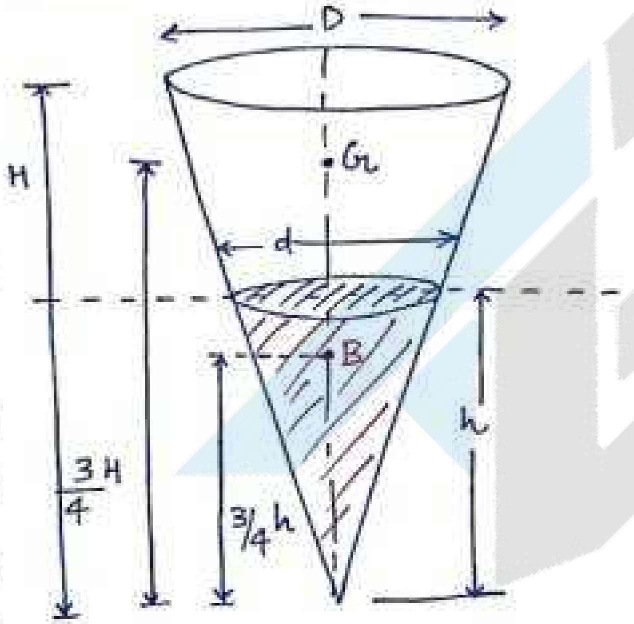
HW

Q A hollow cylinder of length 1m and have internal dia. 0.4m & external dia. 0.6m respectively and Both ends are opened assume the weight of cylinder as 700N. Analyse whether the cylinder is being stable while floating in H_2O with its axis vertical.





Solid Cone

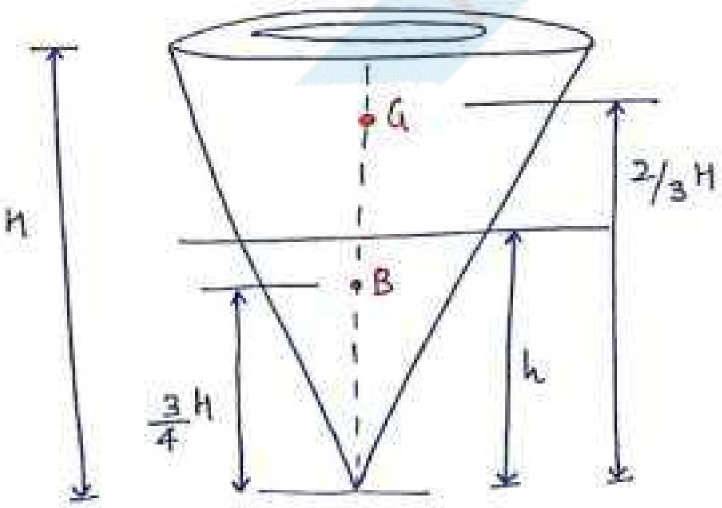


$BM = \frac{I}{V_{fd}}$

$V_{fd} = \frac{1}{3}\pi \left(\frac{d}{2}\right)^2 h$

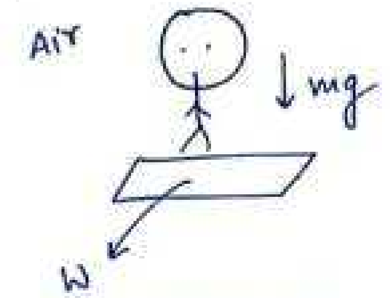
$I = \frac{\pi}{64} d^4$

T3 → HW

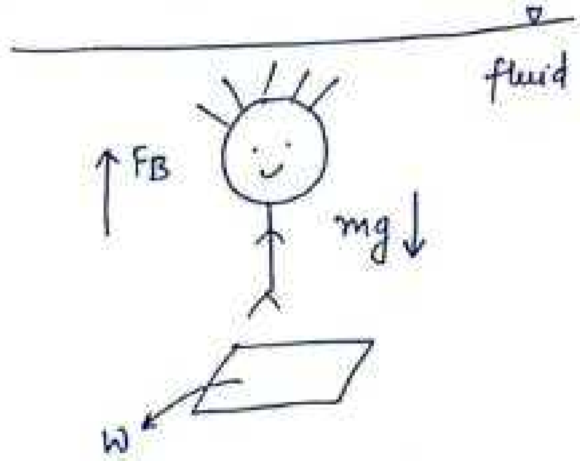


Real and Apparent Weight Concept :→

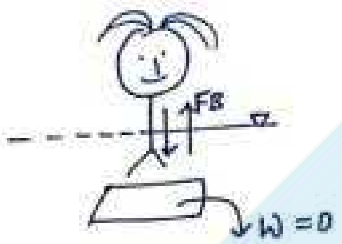
Ex :-



$\boxed{W = mg}$ → Real weight

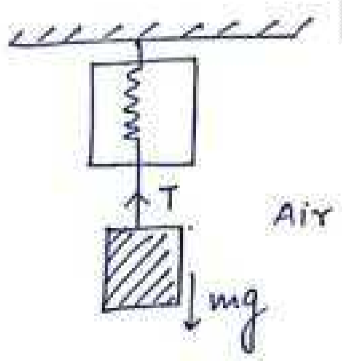


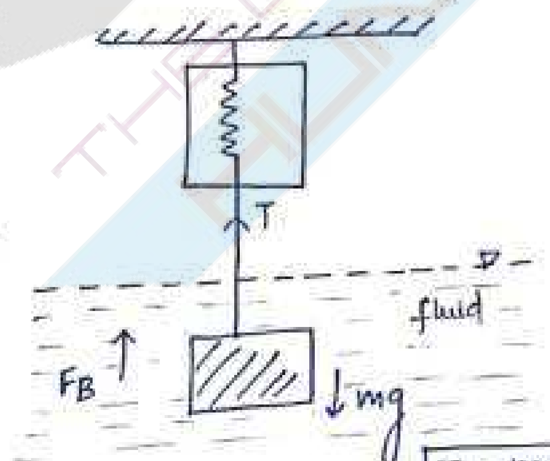
$\boxed{W = mg - F_B}$ → apparent



Ex :-

after dipping into another liquid



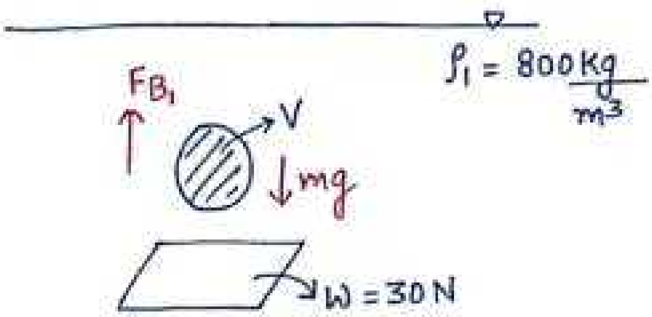


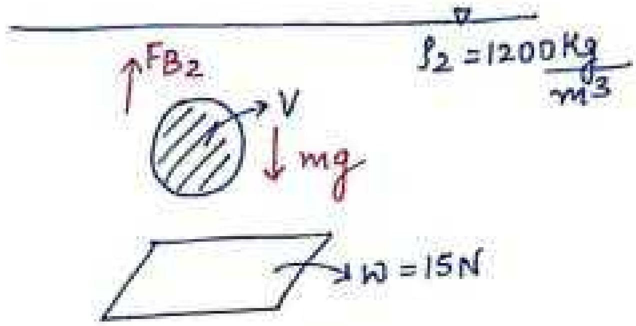
real weight $\boxed{T = mg}$

$\boxed{T = mg - F_B}$ → apparent weight

Now, because of the Bouyant force, the tension in the spring reduced and T new found → apparent weight

(: T only shows weight in previous figure)





$$30 = mg - F_{B_1}$$

$$15 = mg - F_{B_2}$$

$$30 = mg - \rho_1 g V \quad \text{---(i)}$$

$$15 = mg - \rho_2 g V \quad \text{---(ii)}$$

$$15 = (\rho_2 - \rho_1) \cdot g \cdot V$$

$$15 = (1200 - 800) \times 10 \times V$$

$$\boxed{V = 3.82 \text{ litres}}$$

$$\cancel{F_{B_1} = \rho_1 g h_1 A_1 = \rho_1 g V_1 = \rho_1 g V = \rho_1 g V}$$

$$\cancel{F_{B_2} = \rho g h A = \rho_2 g h_2 A_2 = \rho_2 g V}$$

F_{B_1} = Buoyancy force = weight of liquid displaced = net vertical upward hydrostatic force

$$F_{B_1} = w_1 V_1 = \rho_1 g V$$

$$F_{B_2} = w_2 V_2 = \rho_2 g V$$

(weight density)

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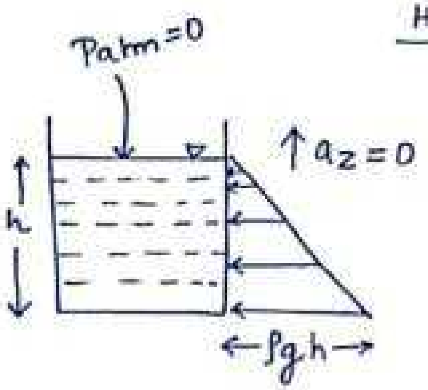
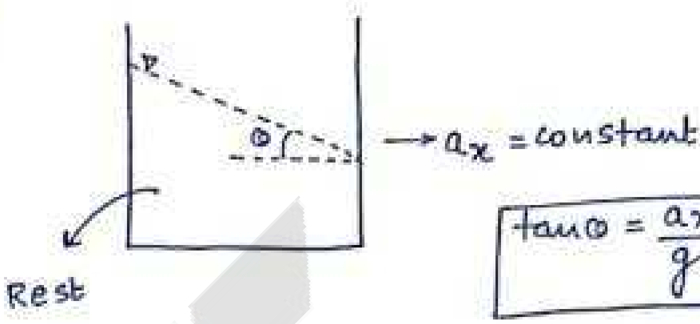
FLUID under Relative Motion :→

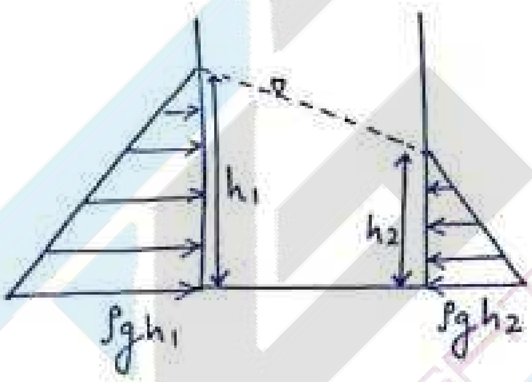
Ex :- ① Fuel tanks of cars, tankers, fuel tanks of aeroplanes etc.

Horizontal Motion

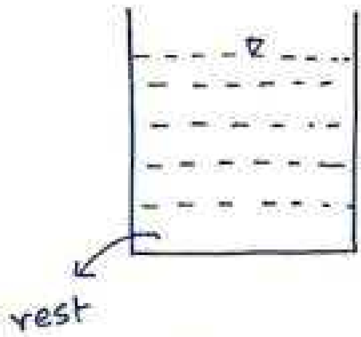




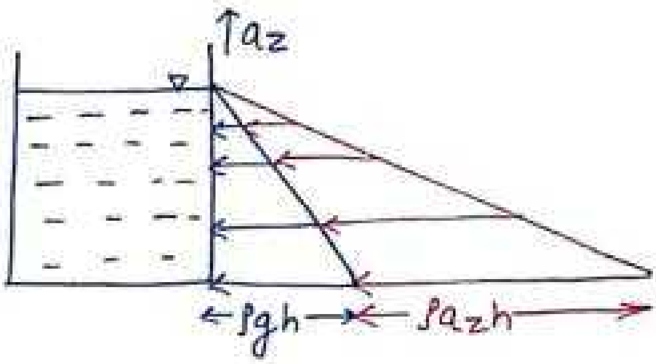
$$\boxed{\tan\theta = \frac{a_x}{g}}$$



Vertical Motion



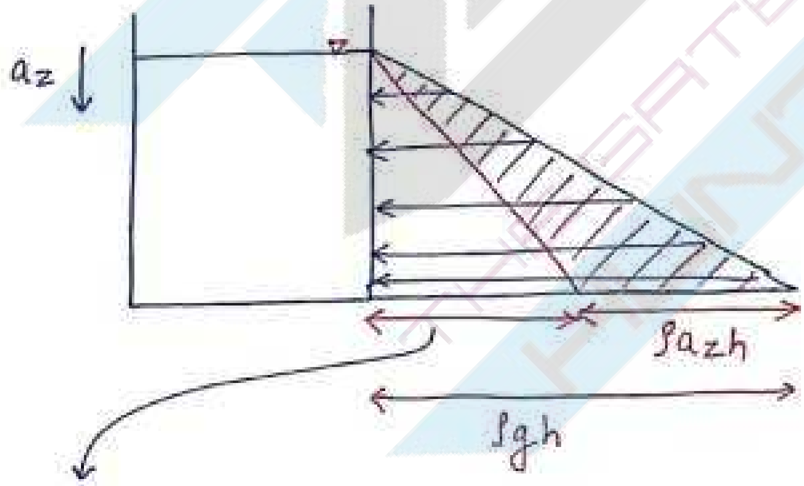
$\uparrow a_z = \text{const}$



$P = \rho g h + \rho a_z h$

$\boxed{P = \rho(g + a_z)h}$

$\boxed{P = \rho\, g_{eff}\, h}$



freefall

$a_z = g$

$\boxed{P = 0}$ Gauge

$\boxed{P_{abs} = P_{atm}}$

$P = \rho g h - \rho a_z h$

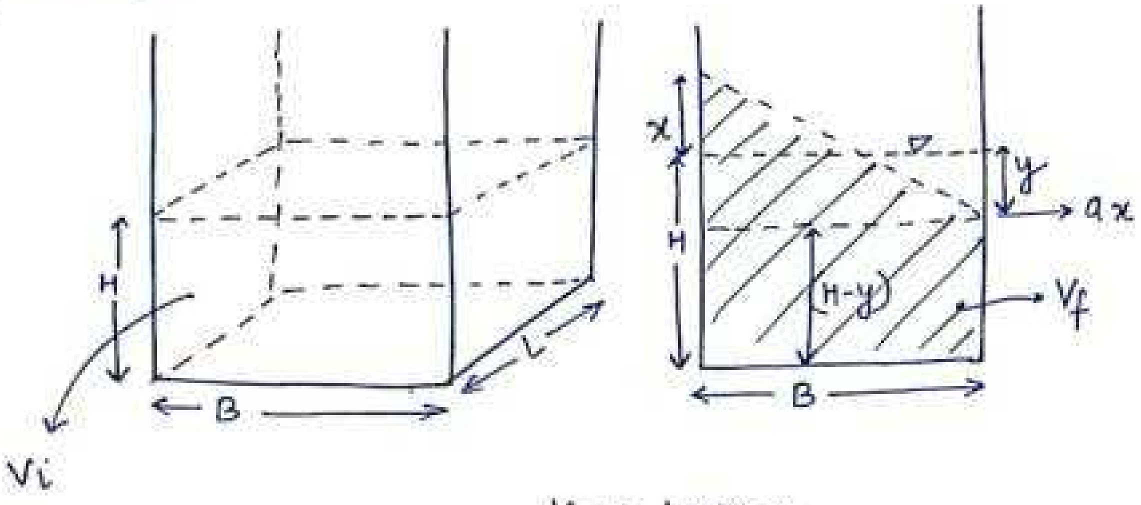
$P = \rho(g - a_z)h$

Q> Find the rise and fall of the liquid in the container with constant horizontal accn. if no liquid spilled out?

N.P.

Q 42 ← HW (WB) → completed

Sol Prev. Page



$$V_i = L \times B \times H$$

$$V_f = \left[\frac{1}{2}\left\{(x+y)\cdot B\right\} + \left\{H-y\right\}B\right]L$$

No spillage

$$V_i = V_f$$

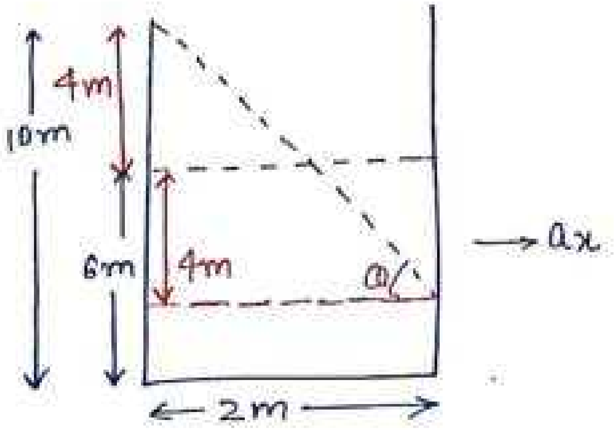
$$\cancel{LBH} = \frac{1}{2}xBL + \frac{1}{2}yBL + \cancel{HBL} - yBL$$

$$\boxed{x = y}$$

Pg 70
Q40 (d)

(41) (c)

(52)



Horizontal accⁿ·

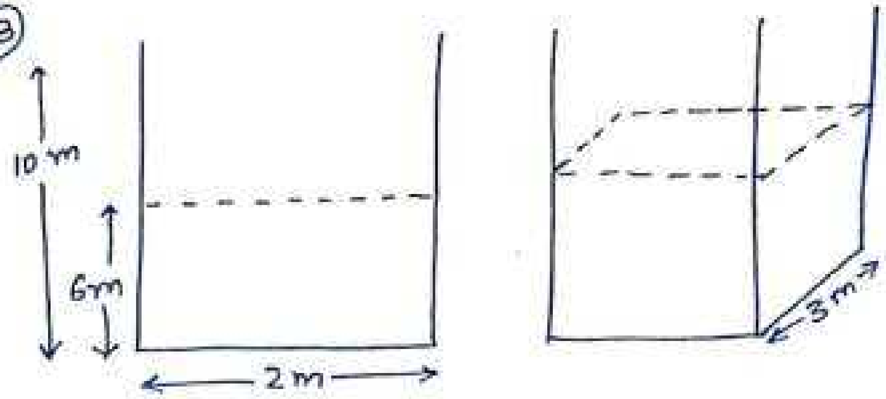
$\tan\theta = 8/2$

$\tan\theta = 4$

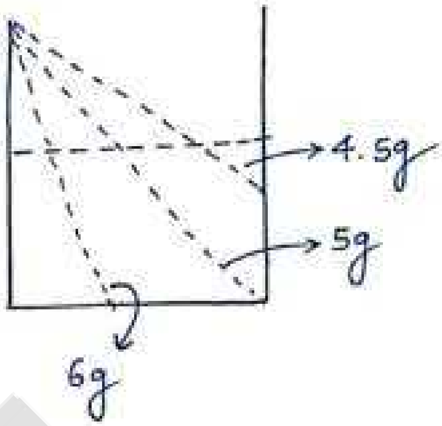
$\frac{a_x}{g} = \tan\theta$

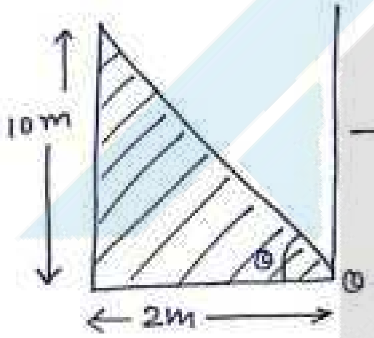
$$\boxed{a_x = 4 \cdot g}$$

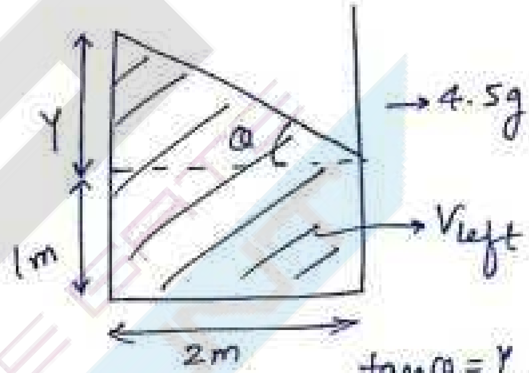
Q 53



$\vec{a}_x = 4g$







$\tan\theta = \frac{10}{2} = 5$

$\frac{a_x}{g} = 5$

$a_x = 5g$

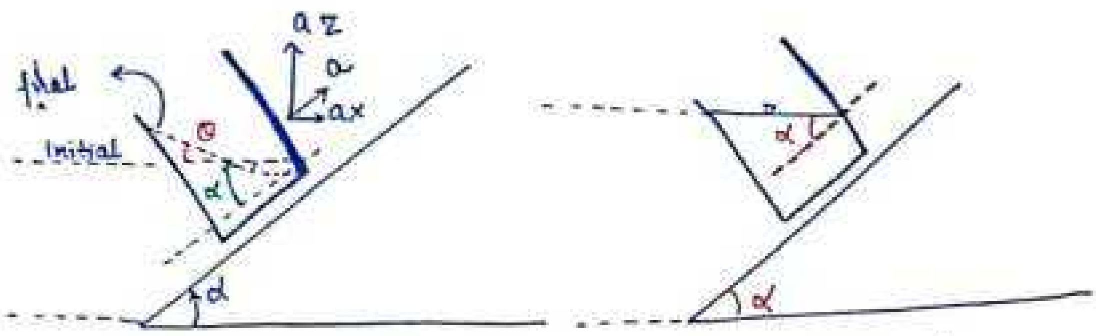
$\tan\theta = \frac{Y}{2}$

$\frac{a_x}{g} = \frac{Y}{2}$

$\frac{4.5g}{g} = \frac{Y}{2}$

$Y = 9\text{ m}$

10/11/16 Concept →



$$\tan\theta = \frac{a_x}{g_{eff}}$$

$$\boxed{\tan\theta = \frac{a_x}{g + a_z}}$$

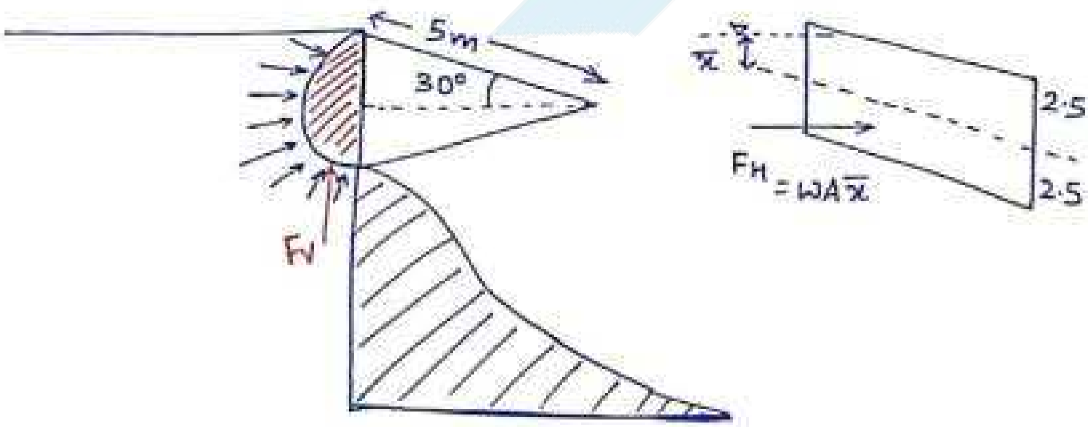
TS → $(s - 0.33)(s - 0.66) < 0$

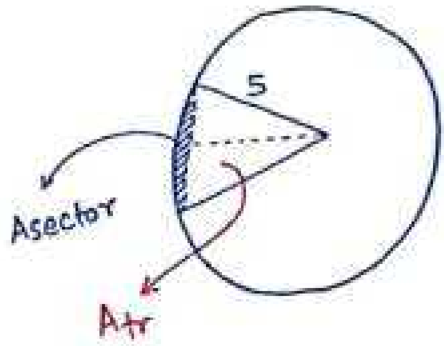
$-ve \times -ve > 0$

$\boxed{+ve \times -ve < 0}$

$+ve \times +ve > 0$

T.5 →





$$A_{arc} = \frac{\pi R^2}{360^\circ} \times 60^\circ = \frac{\pi R^2}{6}$$

$$A_{sector} = A_{arc} - A_{trg}$$

$$= \frac{\pi R^2}{6} -$$

HW → half cylinder Qn. → unstable ← Ans

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CHAPTER-5

FLUID KINEMATICS

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Langrangian Description

- Study is concentrated over the single fluid particle.
- Position vector are defined at different time intervals wrt given space
- Exact analysis
- More time consuming because of large no. of variables.

Eulerian description

- A finite volume called control volume is defined, through which fluid flows in and out
- we do not keep track of the position and velocity of a mass of fluid particle of fixed quantity
- Field variables which are function of space and time are defined within the control volume.

Position vector $= f(x,y,z,t)$

$\vec{V} = \vec{V}(x,y,z,t) = u\hat{i} + v\hat{j} + \omega\hat{k}$

$\Rightarrow \vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{\omega}\frac{\partial}{\partial z}$

$\Rightarrow \vec{a} = \frac{D\vec{V}}{dt} = (\vec{V}\cdot\vec{\nabla})\vec{V} + \frac{\partial \vec{V}}{\partial t}$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d\vec{V}}{dt}(x,y,z,t)$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t}\cdot\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial \vec{V}}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial \vec{V}}{\partial z}\cdot\frac{\partial z}{\partial t}$$

$$\vec{a} = \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local/temporal accl}^r} + \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + \omega\frac{\partial \vec{V}}{\partial z}}_{\text{convective accl}^r}$$

$$\frac{D\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + \omega\frac{\partial \vec{V}}{\partial z}$$

$$\frac{D}{dt} = \frac{\partial}{\partial t} + \underbrace{u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial z}}_{\vec{V}\cdot\vec{\nabla}}$$

CHAPTER - 5

FLUID KINEMATICS

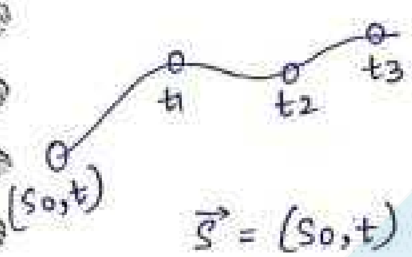
Study of motion of the fluid without references of forces and moments known as fluid kinematics. There are two description to define the flow behaviour :-

① Lagrangian description.

② Eulerian description.

Motion Study

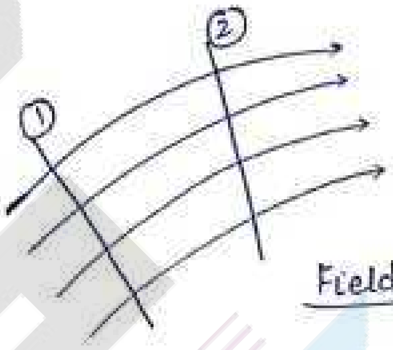
Lagrangian | Eulerian



$$\vec{S} = (S_o, t)$$

$$\vec{V} = \frac{d\vec{s}}{dt}$$

$$\vec{a} = \frac{d^2\vec{s}}{dt^2}$$



Field variables

$$\left[\begin{array}{l} \vec{V} = f(x, y, z, t) \\ \vec{a} = f(x, y, z, t) \end{array}\right.$$

$$\left[\begin{array}{l} P = f(x, y, z, t) \\ \rho = f(x, y, z, t) \\ T = f(x, y, z, t) \end{array}\right.$$

$$\vec{V} = f(x, y, z, t)$$

$$d\vec{V} = \frac{\partial \vec{V}}{\partial x}dx + \frac{\partial \vec{V}}{\partial y}dy + \frac{\partial \vec{V}}{\partial z}dz + \frac{\partial \vec{V}}{\partial t}dt$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{D\vec{V}}{Dt} \text{ (total derivative)}$$

$$\Rightarrow \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial x}\left(\frac{\partial x}{dt}\right) + \frac{\partial \vec{V}}{\partial y}\left(\frac{dy}{dt}\right) + \frac{\partial \vec{V}}{\partial z}\left(\frac{dz}{dt}\right) + \frac{\partial \vec{V}}{\partial t}\cancel{\left(\frac{dt}{dt}\right)}$$

$$\Rightarrow \frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

$$\left[u = \frac{\partial x}{\partial t}\right] \quad \left[v = \frac{\partial y}{\partial t}\right] \quad \left[w = \frac{dz}{dt}\right]$$

$$\boxed{\begin{aligned} \vec{\nabla} &= \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \\ \vec{V} &= u\hat{i} + v\hat{j} + w\hat{k} \end{aligned}}$$

- $(\vec{V}\cdot\vec{\nabla}) = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$

$\Downarrow$

Convective operator / time space derivative

- $\frac{\partial}{\partial t} \Rightarrow$ local operator / temporal operator

- $\frac{D}{Dt} = (\vec{V}\cdot\vec{\nabla}) + \frac{\partial}{\partial t} \longrightarrow$ (Total derivative operator)

$$\Rightarrow \vec{V}(x,y,z,t) = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$$

$$\Rightarrow \vec{a} = \frac{D\vec{V}}{Dt} \Rightarrow \boxed{\vec{a} = \underbrace{(\vec{V}\cdot\vec{\nabla})\vec{V}}_{\text{Convective accl}^n} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local accl}^r}}$$

$$\boxed{\frac{D\rho}{Dt} = (\vec{V}\cdot\vec{\nabla})\rho + \frac{\partial \rho}{\partial t}}$$

Incompressible flow

$$\rho = 2x^3y + 3y^2z$$

$$\boxed{\frac{D\rho}{Dt} = 0}$$

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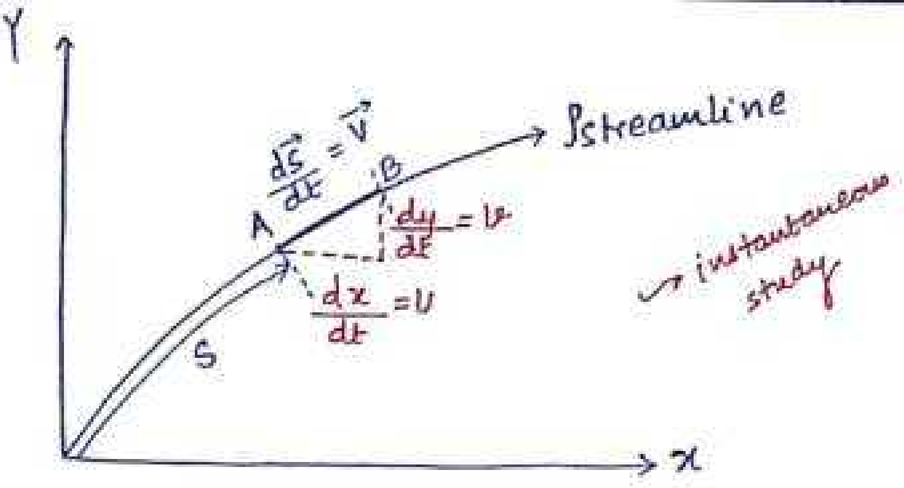
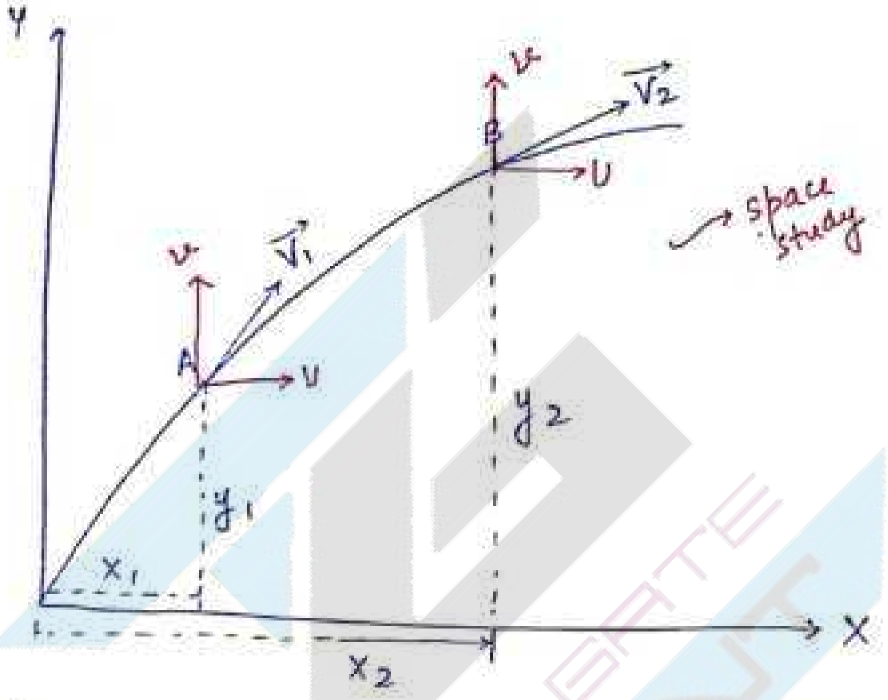




L.S.

The study is done on single fluid particle by defining the position vector with some reference point. It is accurate analysis due to large no. of variables. It has mathematical difficulty.

E.S.

The study is done on Bulk motion of the flow at the given section by defining the field variables. It is an average analysis.

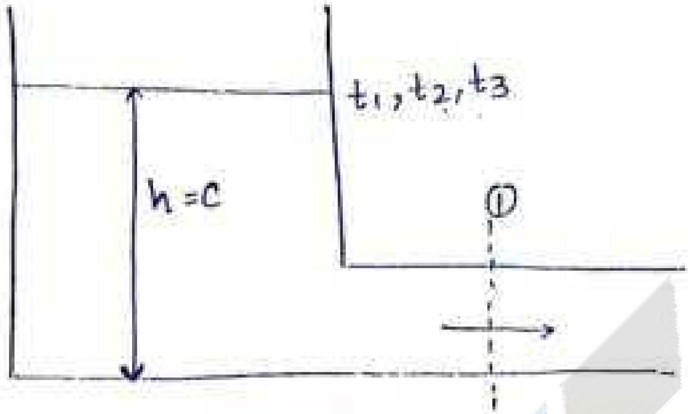
✓ we Prefer this

✓ Space study

Types of Fluid Flow :-

① STEADY AND UNSTEADY :- (Time is variable and space is fixed) (t_1, t_2, t_3)

A flow is said to be study when properties does not change with time in the given space. otherwise the flow is unsteady



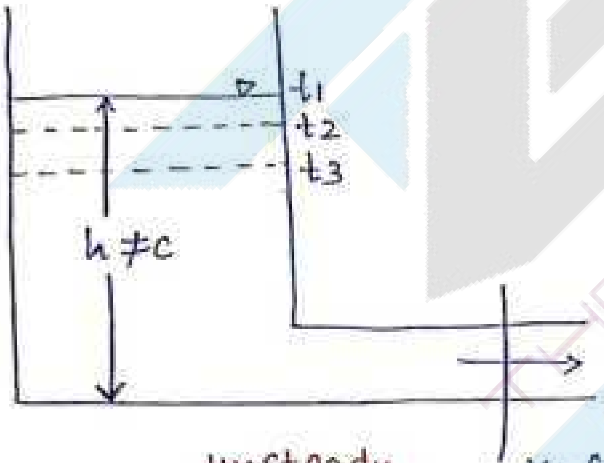
steady $V \neq f(\text{time})$

$$\frac{\partial V}{\partial t} = 0$$ → space study

Steady flow

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial P}{\partial t} = 0$$



unsteady $V = f(\text{time})$

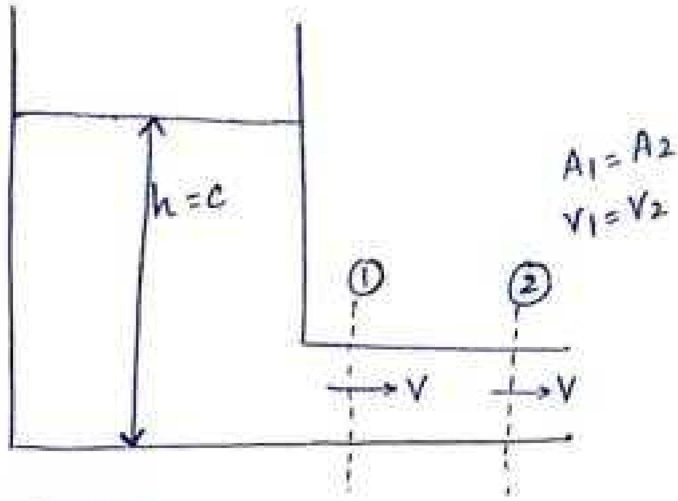
→ Time study

$$\frac{\partial V}{\partial t} \neq 0$$

unsteady

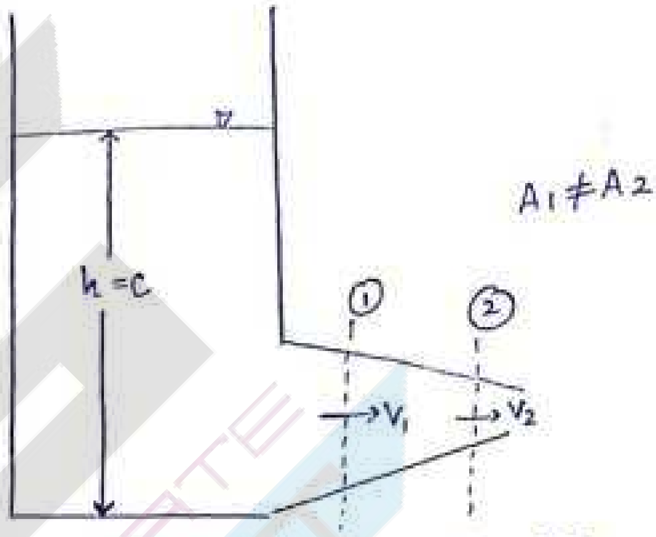
② UNIFORM & NON UNIFORM:- (time is fixed and space is variable)

A flow is said to be uniform when properties does not change with space in given time otherwise the flow is not uniform



Uniform

$$\frac{\partial V}{\partial s} = 0$$



Non-Uniform

$$\frac{\partial V}{\partial s} \neq 0$$

③ # LAMINAR AND TURBULENT FLOW:-

When fluid flows in the form of lamina, it is known as laminar flow. It generally occurs at low velocities and high viscosities that is at low Reynold's no. In laminar flow, there is negligible mixing b/w the layers. It is defined by viscous flow, so in laminar flow, viscous stresses are defined by Newton's law of viscosity.

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$$Re = \frac{F_i}{F_v}$$

Turbulent

It is highly disordered flow. There is rapid mixing b/w the layers. Due to rapid mixing, the fluctuating component velocity gives additional stresses in the flow known as Eddy shear stresses.

In turbulent flow, eddy stresses are very high than viscous stresses. Turbulent flow generally occurs at high velocities or low viscosities that is at high reynold's no

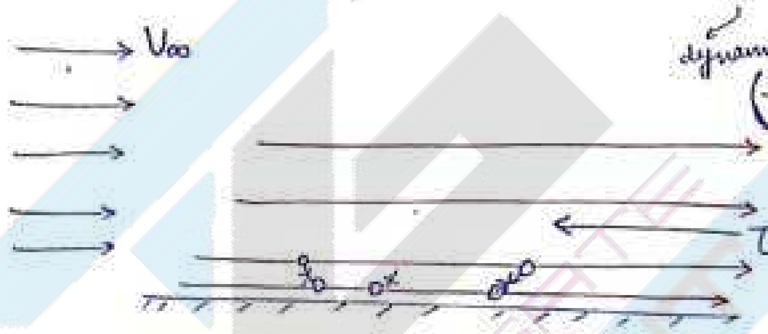
$$Re = \frac{\rho V L_c}{\mu}$$

$$Re = \frac{F_i}{F_v}$$

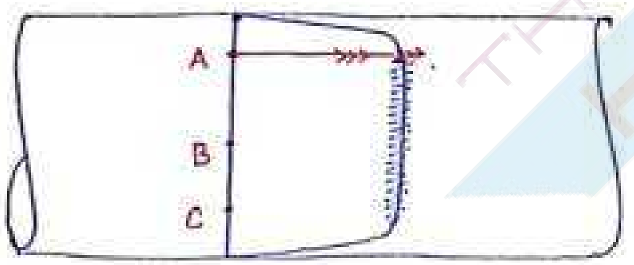
$$Re = \frac{\rho V D}{\mu} = \frac{VD}{(\mu/\rho)} = \frac{VD}{\nu}$$

kinematic $\left(\frac{m^2}{s}\right)$

dynamic $\left(\frac{N\cdot s}{m^2} = \frac{kg}{m\cdot s}\right)$ (Pa-sec)



$$\tau_v = \mu \frac{du}{dy}$$



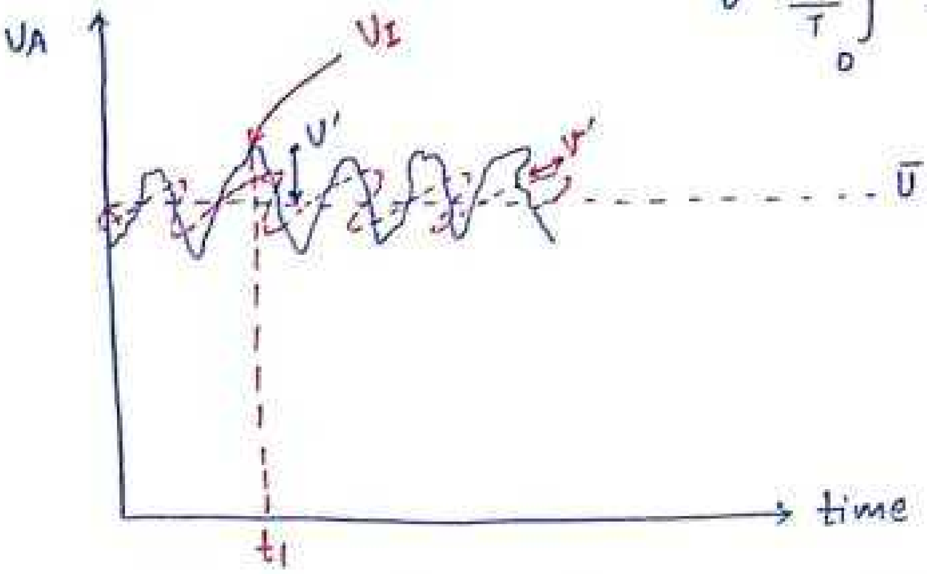
$$\vec{V} = u\hat{i} + v\hat{j} + \omega\hat{k}$$

$$u = \bar{u} + u'$$

$$v = \bar{v} + v'$$

$$\omega = \bar{\omega} + \omega'$$

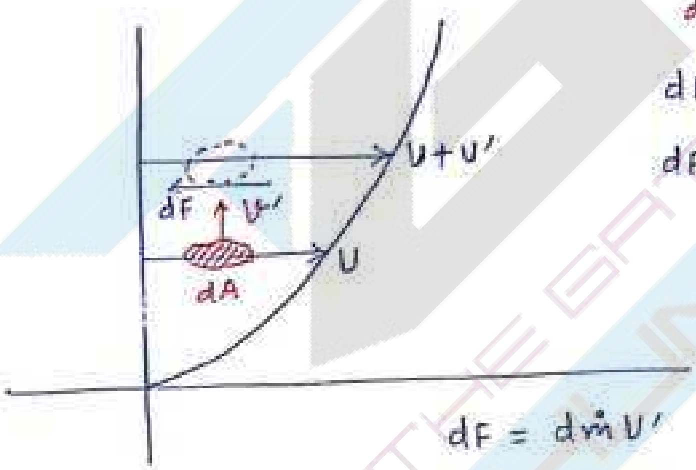
$\bar{U} = \frac{1}{T}\int_0^T U_I\,dt \qquad U_I = \bar{U} + \bar{U}$



$\bar{U} \to$ time average velocity

$\boxed{U' = V' = \omega'}$

$\frac{1}{T}\int_0^T U'\,dt = 0 \to \because \bar{U} = 0$



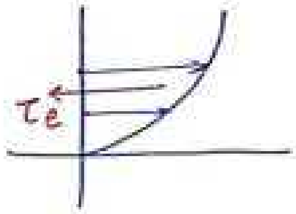
$d\dot{m} = \rho\, dA\, V'$

$dF = d\dot{m}\, V_2 - d\dot{m}\, V_1$

$dF = d\dot{m}\,(U + U') - d\dot{m}\, U$

$dF = d\dot{m}\, U'$

$dF = (\rho\, dA\, V') \cdot U'$



eddy shear stress

$\boxed{\tau_e = \rho U' V'}$

Reynolds

$\frac{dF}{dA} = \rho U' V'$

Rate mass flow $\dot{m} = \rho A V$ (density, area, velocity)

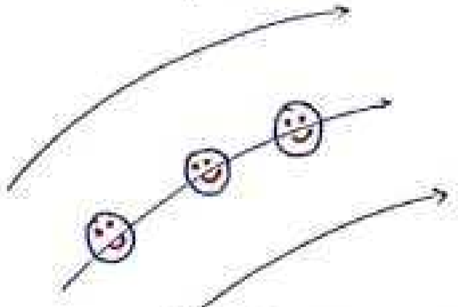
$= \frac{kg}{m^3} \times m^2 \times \frac{m}{sec}$

$= kg/sec$

④ ROTATIONAL & IRROTATIONAL FLOW :→

when fluid particle rotate about their own mass center along the flow is called as Rotational flow. It is associated with Viscous flow.

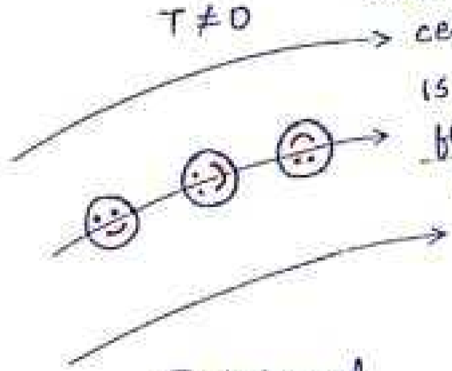
$T = 0$



IRRotational

$(\tau = 0)$

$T \neq 0$



Rotational

$(\tau \neq 0)$

$T \neq 0$

$F_s \times r \neq 0$

$r \neq 0$

$F_s \neq 0$

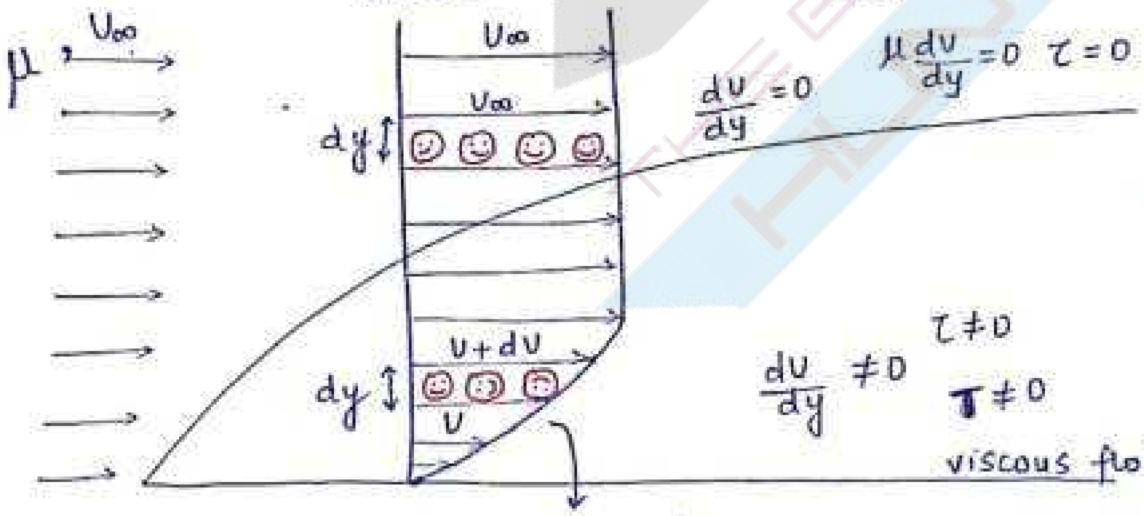
$\tau \times A = F_s$

$\tau \neq 0$

$\tau = \mu \frac{du}{dy}$

$\tau \neq 0 \quad \frac{du}{dy} \neq 0$

Non-viscous flow
Irrotational



$T = 0$

$\frac{du}{dy} = 0 \quad \mu \frac{du}{dy} = 0 \quad \tau = 0$

$v = f(y)$

$\frac{du}{dy} \neq 0 \quad \tau \neq 0 \quad T \neq 0$

viscous flow

rotational

$v = f(\cancel{x}, y, z)$ $\quad \omega = f(\cancel{x}, y, \cancel{z})$

$v = f(x, \cancel{y}, z)$

A flow is said to be irrotation if there is no external torque, No viscous shear force, no viscous shear stress that is the flow is non viscous flow.

(5) COMPRESSIBLE & INCOMPRESSIBLE FLOW :-

incompressible flow → $\boxed{\frac{D\rho}{Dt} = 0}$ → ρ does not changes wrt time and space

or

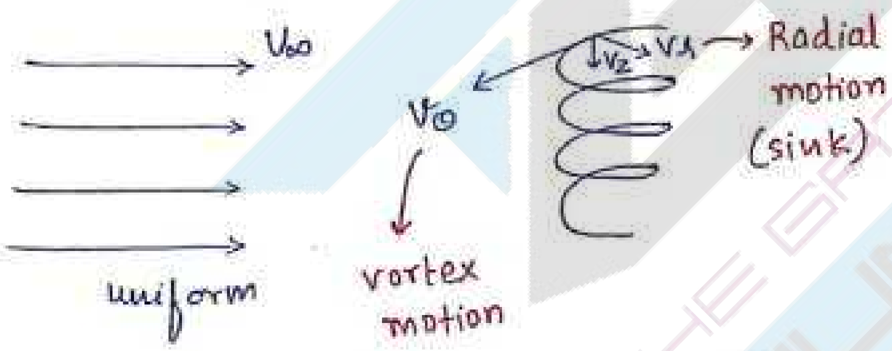
$\boxed{\frac{d\rho}{\rho} < 5\%}$

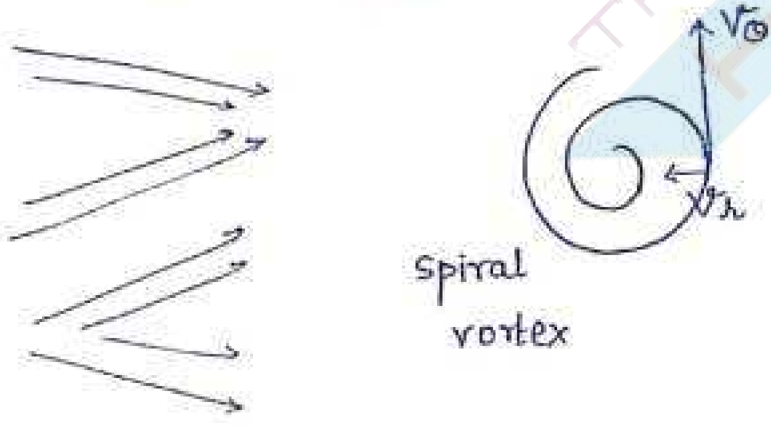
• density changes wrt space and time → compressible flow

(6) 1-D, 2-D, 3-D flow :- from theory Book.

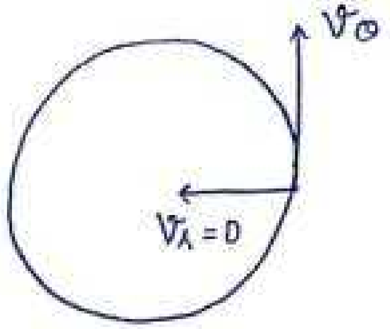
↳ Depending upon how the field variables varies.

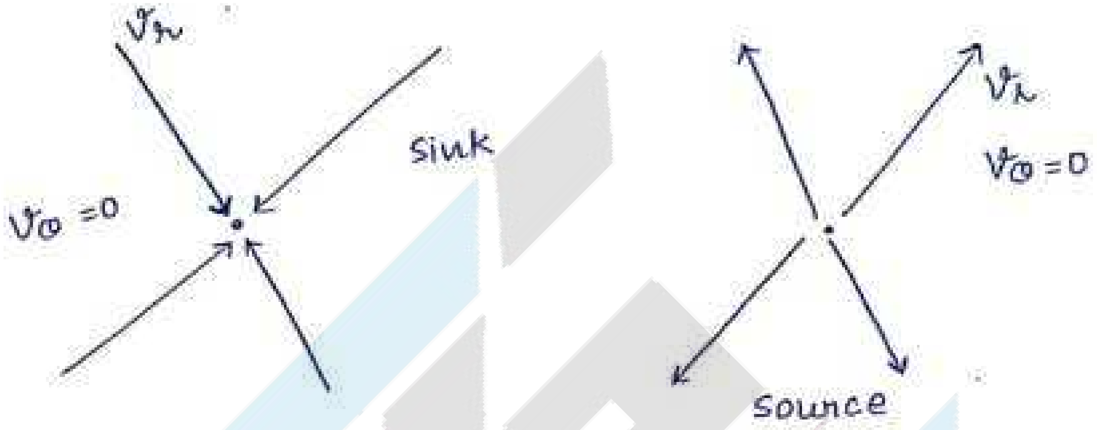
VORTEX MOTION :-

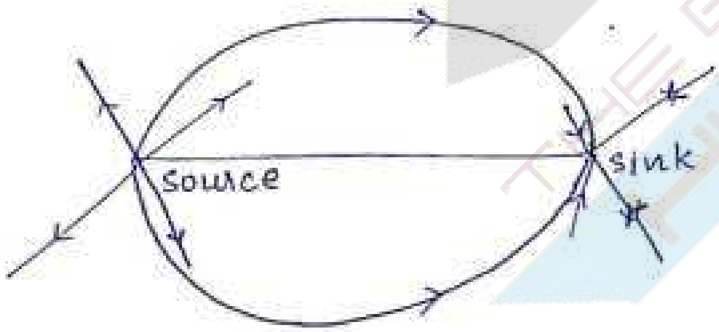




Vortex :-
motion of fluid mass along circular path.







Doublet

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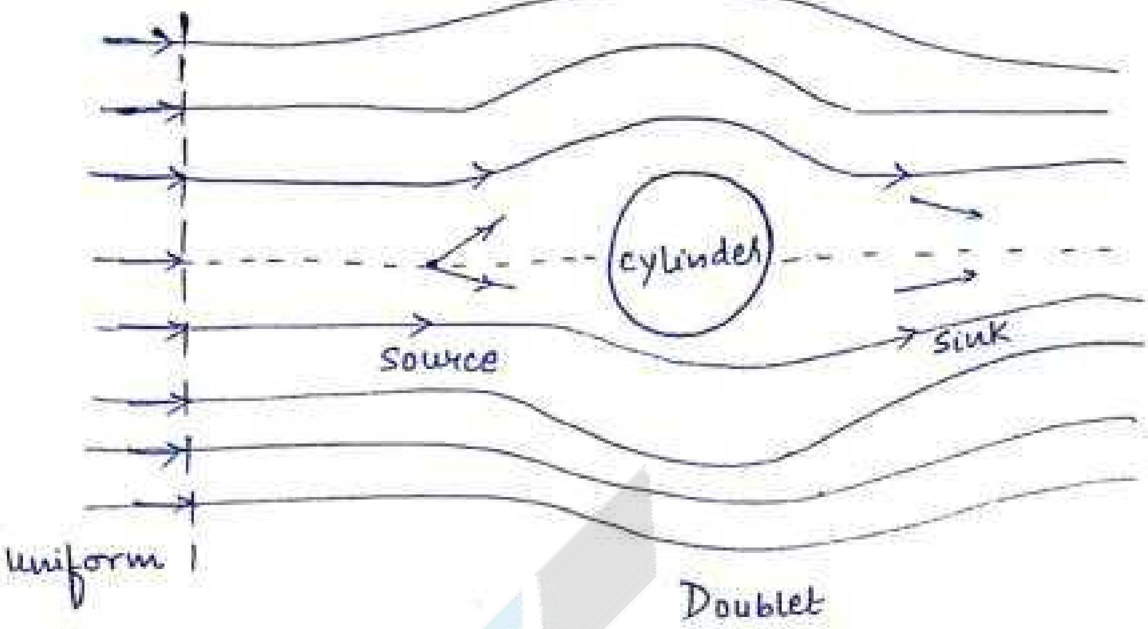
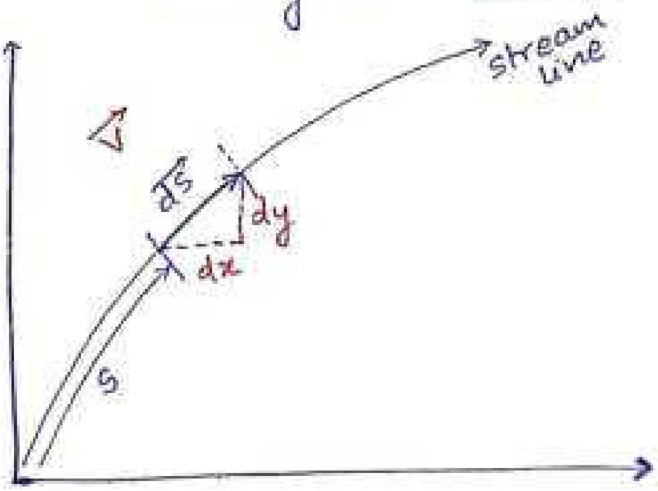


FLOW LINES:-

Fluid Motion can be described by three flow lines:-

(a) stream line
(b) Path line
(c) streak line

(a) Stream line → It is an imaginary line or curve drawn in space such that Tanget drawn gives velocity vector i.e. velocity vector and streamline vector coincides.



$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\vec{V} = u\hat{i} + v\hat{j} + \omega\hat{k}$$

$$\boxed{d\vec{s} \times \vec{V} = 0}$$ stream line eqn.

$[\vec{A} \times \vec{B} = |AB|\sin\theta\,\hat{n} = |AB|\sin 0^\circ\,\hat{n} = 0\hat{n}]$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

$$\Rightarrow \underbrace{(\omega dy - v dz)}_{\Downarrow\ 0}\hat{i} + \underbrace{(u dz - \omega dx)}_{\Downarrow\ 0}\hat{j} + \underbrace{(v dx - u dy)}_{\Downarrow\ 0}\hat{k} = 0$$

$\omega dy - v dz = 0$

$u dz - \omega dx = 0$

$v dx - u dy = 0$

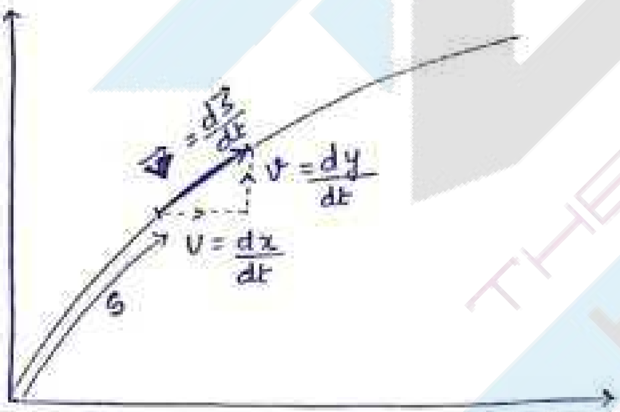
$$\boxed{\frac{dy}{v} = \frac{dz}{\omega}}$$

$$\boxed{\frac{dz}{\omega} = \frac{dx}{u}}$$

$$\boxed{\frac{dx}{u} = \frac{dy}{v}}$$

$$** \boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{\omega}} **$$

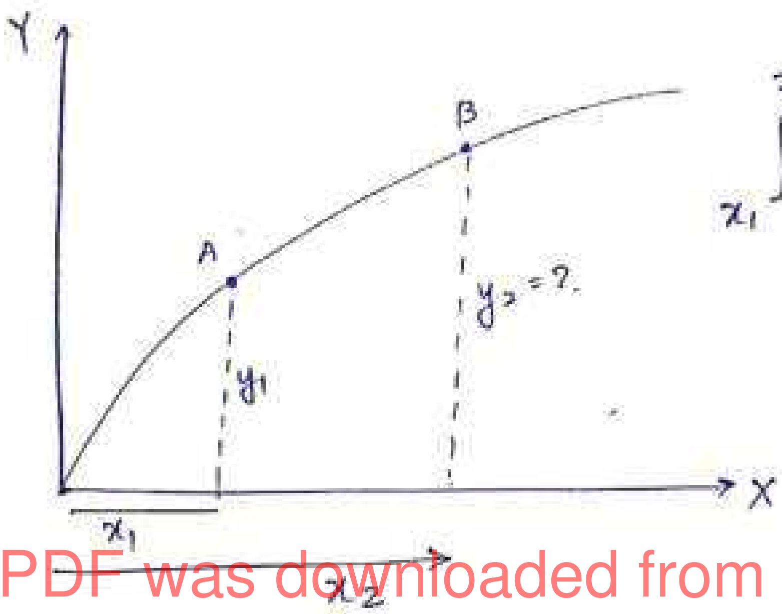
stream lines



$dt = \frac{dx}{u}$ $\quad dt = \frac{dy}{v}$

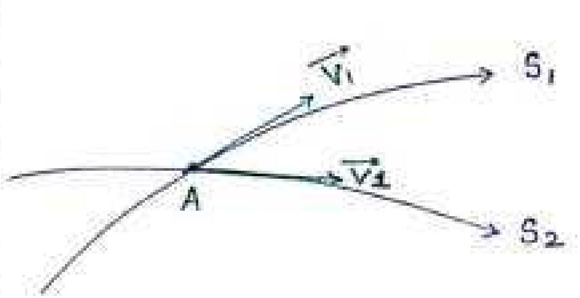
$$\boxed{\frac{dx}{u} = \frac{dy}{v}}$$ stream line eqn

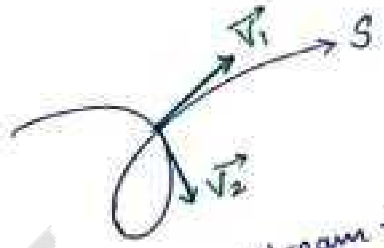
$u = 2x^2y\,, \quad v = 3xy$

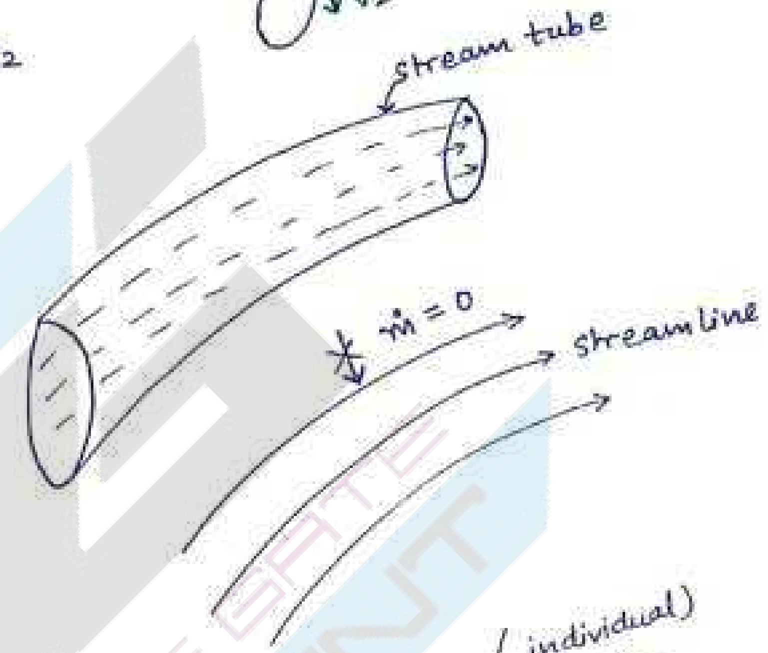


$$\int_{x_1}^{x_2} \frac{dx}{2x^2y} = \int_{y_1}^{?} \frac{dy}{3xy}$$

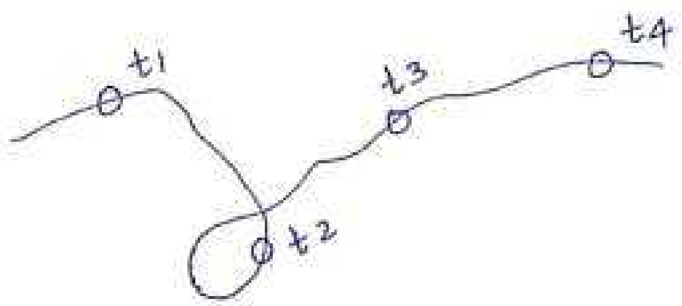
*** NOTE :-** Two stream lines never intersects each other as well as stream line also never intersects itself because at the point of intersection, there will be two velocity fields which is impossible. So there is no flow across the stream line.







(b) Path line → The line drawn By tracing the path of (individual) single fluid particle at different time intervals. It is defined on the basis of lagrangian description.



- actual path followed by an individual fluid particle over some time period.

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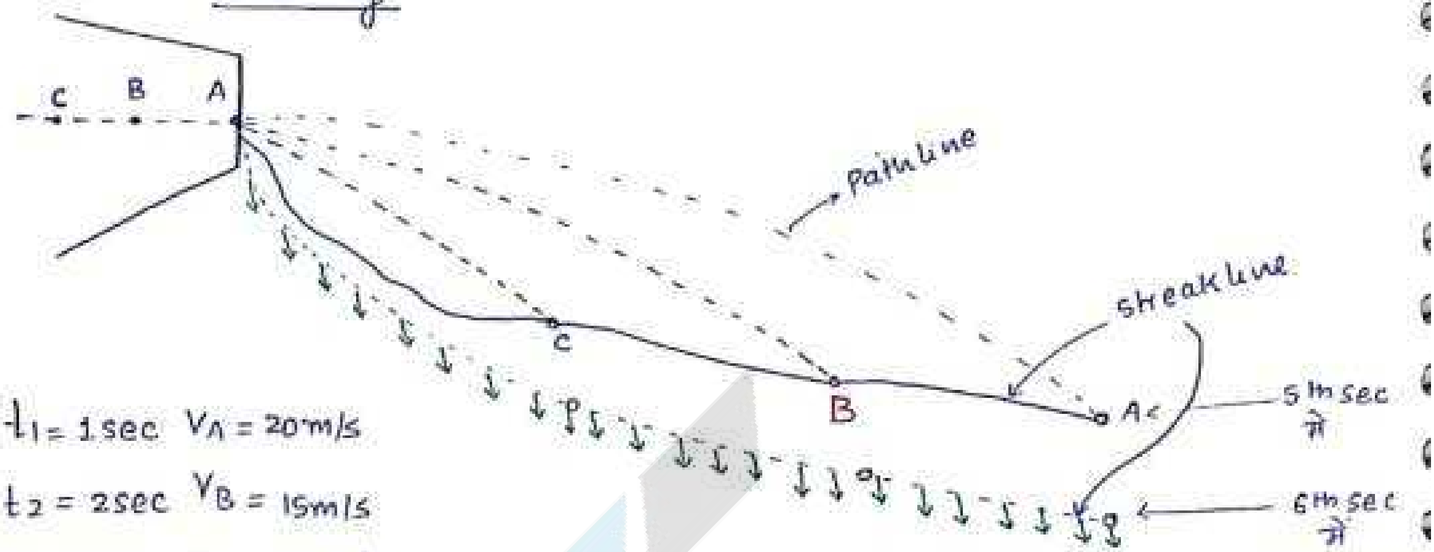
Ⓒ Streak line → It is an instantaneous picture of all fluid particles passing through single point.

Unsteady



$t_1 = 1\,sec$ $V_A = 20\,m/s$

$t_2 = 2\,sec$ $V_B = 15\,m/s$

$t_3 = 3\,sec$ $V_C = 10\,m/s$

- it is the locus of the fluid particles that have passed sequentially through a prescribed point in the flow.

Steady

$t_1 = 1\,sec$ $V_A = 20\,m/s$

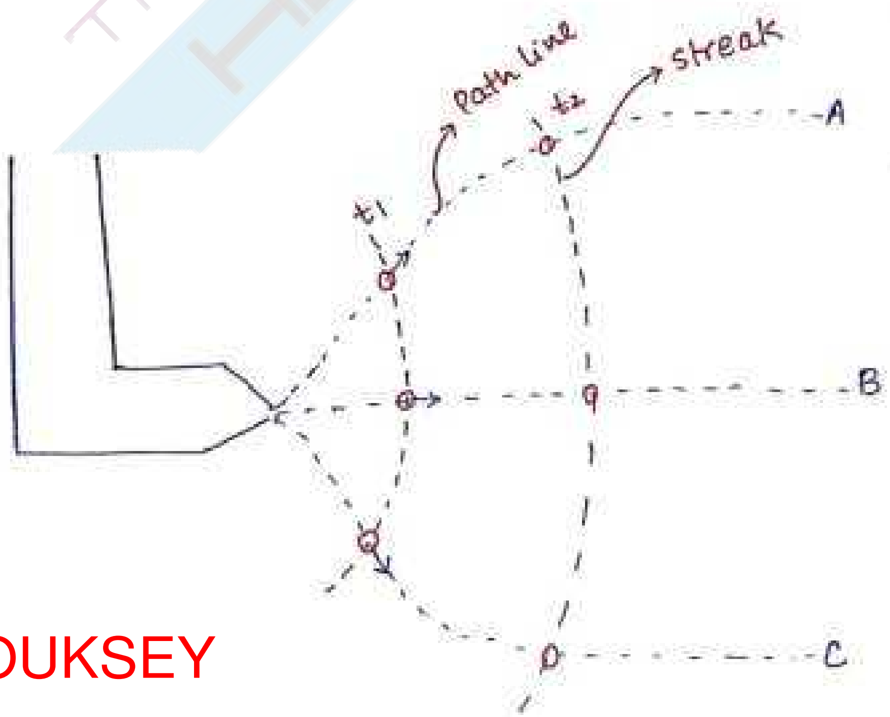
$t_2 = 2\,sec$ $V_B = 20\,m/s$

$t_3 = 3\,sec$ $V_C = 20\,m/s$

[nozzle connected to a tank of constant height]

↓

velocity will not change



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for steady flow, all three lines are identical that is all three lines coincides

03

Q In the flow field, if $\boxed{V = 5x^3\hat{i} - 15x^2y\hat{j}}$, obtain the equation for stream line.

Sol

$$V = 5x^3\hat{i} - 15x^2y\hat{j}$$

$$V = u\hat{i} + v\hat{j}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow \frac{dx}{\cancel{5}x^{\cancel{3}}} = \frac{dy}{-\cancel{15}_{3}x^{\cancel{2}}y}$$

$$\Rightarrow \int \frac{dx}{x} = -\frac{1}{3}\int \frac{dy}{y}$$

$$\ln x = -\frac{1}{3}\ln y + \ln c$$

$$\ln(xy^{1/3}) = \ln c$$

$$\boxed{xy^{1/3} = c}$$

OR

$$\boxed{x^3y = c}$$

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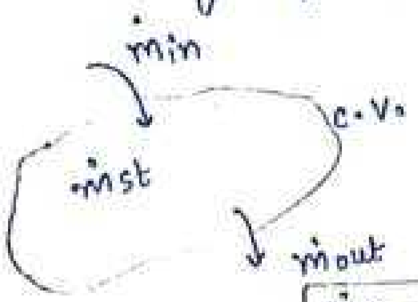
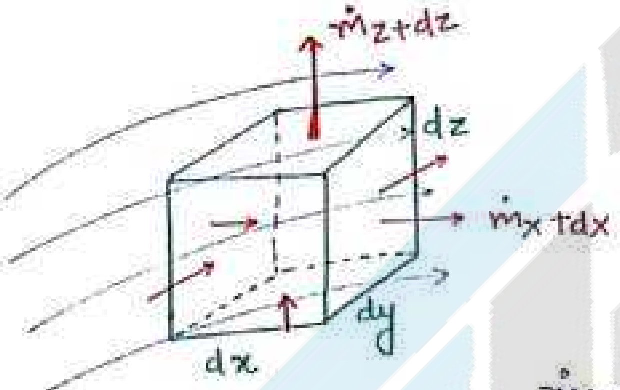
CONSERVATION OF mass equation / Continuity Equation :-

✓ Continuity equation in cartesian coordinate system :-



$$\boxed{\dot m_{in} - \dot m_{out} = \dot m_{st}} \rightarrow (1)$$

storage



$$\dot m_{in} - \dot m_{out} = \dot m_{st}$$

$$-\frac{\partial(\rho u)}{\partial x}\cancel{dV} - \frac{\partial(\rho v)}{\partial y}\cancel{dV} - \frac{\partial(\rho w)}{\partial z}\cancel{dV} = \cancel{dV}\frac{\partial \rho}{\partial t}$$

***

$$\boxed{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0}$$

→ Ch IC
→ V & NV
→ S & US
→ U & NU

General Continuity eqn.

valid for :

- compressible and incompressible
- steady and unsteady
- Viscous and Non Viscous flow

(1) $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$

Steady flow $\boxed{\frac{\partial \rho}{\partial t} = 0}$

***

$$\boxed{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0}$$ ***

(2) $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$

$\rho = f(x,y,z,t) \quad u = f(x,y,z,t)$

$\Rightarrow \rho\frac{\partial u}{\partial x} + \underbrace{u\frac{\partial \rho}{\partial x}} + \rho\frac{\partial v}{\partial y} + \underbrace{v\frac{\partial \rho}{\partial y}} + \rho\frac{\partial w}{\partial z} + \underbrace{w\frac{\partial \rho}{\partial z}} + \underbrace{\frac{\partial \rho}{\partial t}} = 0$

$\Rightarrow \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] + \left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}\right) + \frac{\partial \rho}{\partial t} = 0$

$\Rightarrow \rho(\vec{\nabla}\cdot\vec{V}) + (\vec{V}\cdot\vec{\nabla})\rho + \frac{\partial \rho}{\partial t} = 0$

*** $\Rightarrow \boxed{\rho(\vec{\nabla}\cdot\vec{V}) + \frac{D\rho}{Dt} = 0}$ *** General Continuity eqn

(ii) Incompressible flow, steady

$\frac{D\rho}{Dt} = 0 \qquad \rho(\vec{\nabla}\cdot\vec{V}) = 0$

$\boxed{\vec{\nabla}\cdot\vec{V} = 0}$

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$\Rightarrow \boxed{\frac{\partial u}{dx} + \frac{dv}{\partial y} + \frac{dw}{dz} = 0}$

Note → if the fluid flow given satisfies the continuity eqn., then only it is possible flow otherwise not possible.

(iii) Incompressible, unsteady

$\Rightarrow \frac{D\rho}{Dt} = 0$

$\Rightarrow \rho(\vec{\nabla}\cdot\vec{V}) = 0$

$\Rightarrow (\vec{\nabla}\cdot\vec{V}) = 0$

$\Rightarrow (\text{Div } \vec{V}) = 0$

Q Gate 2014 (a)
8 WB

Q The velocity components of x & y directions are given by

$u = \lambda x y^3 - x^2 y$

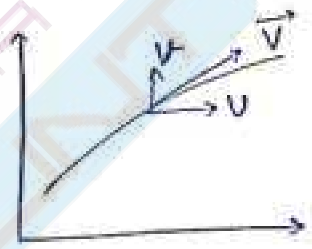
$v = xy^2 - \frac{3}{4}y^4$

then the value of λ for incompressible fluid flow

Sol

Incompressible flow

$\nabla \cdot V = 0$



$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$[\lambda y^3 - \cancel{2xy}] + [\cancel{2xy} - 3y^3] = 0$

$(\lambda - 3)y^3 = 0$

$\lambda = 3 \quad \underset{0}{\Downarrow} \quad = 0$

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⑥ WB

$$\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z}$$

$\rho = 1.2\,kg/m^3$

$$\rho(\nabla\cdot V) + \underbrace{\frac{D\rho}{Dt}}_{2.4} = 0$$

$$1.2[\nabla\cdot V] + 2.4 = 0$$

$$\nabla V = -2/s$$

WB ⑦

$$\vec{V} = (5x + 6y + 7z)\hat{i} + \underbrace{(6x+5y+9z)}_{v}\hat{j} + \underbrace{(3x+2y+\lambda z)}_{\omega}\hat{k}$$

scalar

$\rho = \rho_0 e^{-2t} \longrightarrow$ flow is unsteady (fn. of time)

$\lambda = ?$ (mass $\rightarrow$ conserved)

hence

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho\omega)}{\partial z} + \frac{\partial\rho}{\partial t} = 0$$

$$\Rightarrow \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] + \frac{\partial\rho}{\partial t} = 0$$

$$\Rightarrow \rho_0 e^{-2t}[(5+5+\lambda)] + (-2\underbrace{\rho_0 e^{-2t}}_{\text{cant be 0}}) = 0$$

$$\frac{\partial\rho}{\partial t} = -2\rho_0 e^{-2t} \neq 0$$

$$10 + \lambda - 2 = 0$$

$$\boxed{\lambda = -8}$$

NOTE :-> each velocity profile must satisfies its Continuity eqn.

⑤ HW $\quad u = x^2 \quad \rho = 1/x$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \rightarrow \frac{\partial\left[\frac{1}{x}\cdot x^2\right]}{\partial x} + \frac{1}{x}\frac{\partial v}{\partial y} = 0$$

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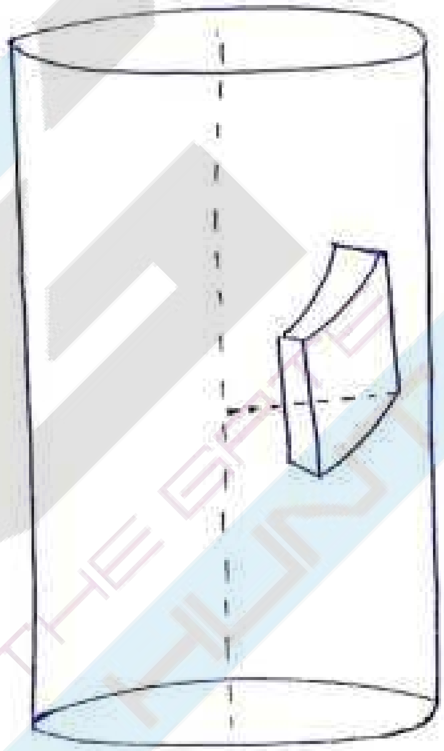
$$\frac{\partial v}{\partial y} = -x$$

$$v = -xy +$$

$$\boxed{v = -xy}$$

WB (4)

CONTINUITY EQ^N IN POLAR COORDINATE SYSTEM →



cylinder

In Polar co-ordinate system two velocities :-

1) Radial Velocity
2) Tangential Velocity

$$\left.\begin{aligned} -\frac{\partial \phi}{\partial r} &= v_r \\ -\frac{\partial \phi}{r\partial \theta} &= v_\theta \end{aligned}\right] \phi$$

$$\left.\begin{aligned} \frac{\partial \psi}{\partial r} &= v_\theta \\ \frac{\partial \psi}{r\partial \theta} &= -v_r \end{aligned}\right] \psi$$

" N.P "

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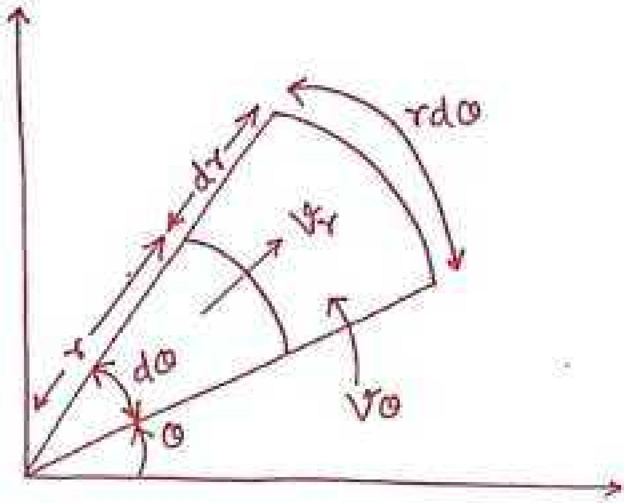
D

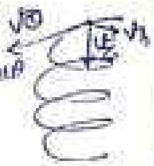


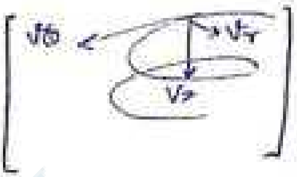
$$v_r = \frac{dr}{dt}$$
$$v_\theta = \frac{r\,d\theta}{dt}$$
$$v_z = \frac{dz}{dt}$$

$$\dot m_{in} - \dot m_{out} = \dot m_{st}$$

$x \to r$, $y \to r\,d\theta$, $z \to z$

$x \to u \to V_r$
$y \to v \to V_\theta$
$z \to w \to V_z$





$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{dt} = 0$$

$$\boxed{\frac{\partial(\rho r V_r)}{r\,\partial r} + \frac{\partial(\rho V_\theta)}{r\,d\theta} + \frac{\partial(\rho V_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0}$$

General Eqn.

2D, incompressible flow:

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

$$\boxed{\frac{\partial (r V_r)}{r\,\partial r} + \frac{\partial V_\theta}{r\,\partial \theta} = 0}$$

Stream line Eqn.

$$\boxed{\frac{dx}{u} = \frac{dy}{v}}$$

$$\boxed{\frac{dr}{v_r} = \frac{r\,d\theta}{v_\theta}}$$

$$\frac{dr}{dt} = v_r \qquad u = dx/dt$$

$$\frac{r\,d\theta}{dt} = v_\theta \qquad v = dy/dt$$

$$\frac{dz}{dt} = v_z \qquad \omega = dz/dt$$

Q A leaf caught in a whirlpool at a given instant, the leaf at a distance of 120m from the centre of whirlpool. whirlpool can be described by the velocity distribution

$$v_r = \frac{-60\times10^3}{2\pi r}\ (m/s)$$

$$v_\theta = \frac{300\times10^3}{2\pi r}\ (m/s)$$

where r is the distance in metre what will be the distance of the leaf from the centre when it has move through half a revolution.

(a) 48m

(b) 64 m

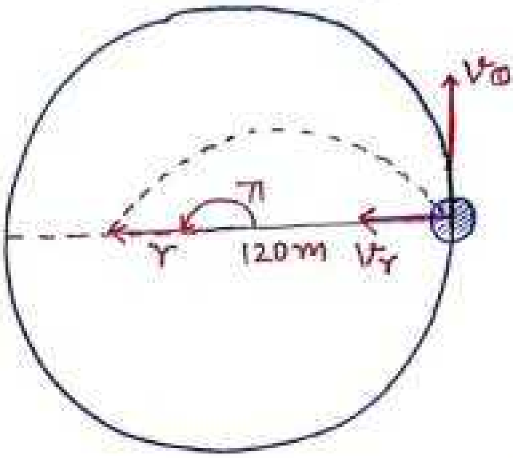
(c) 120m

(d) 142m

Sol → Continuity eqn ⟶ used ⟶ when to find one of the scalar (v_r or v_θ)

dt

(111)



$$v_\theta = \frac{r\,d\theta}{dt} \qquad dt = \frac{r\,d\theta}{v_\theta}$$

$$v_r = \frac{dr}{dt} \qquad dt = \frac{dr}{v_r}$$

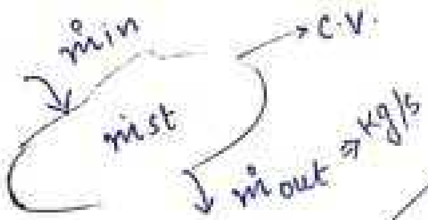
$$\boxed{\frac{dr}{v_r} = \frac{r\,d\theta}{v_\theta}}$$

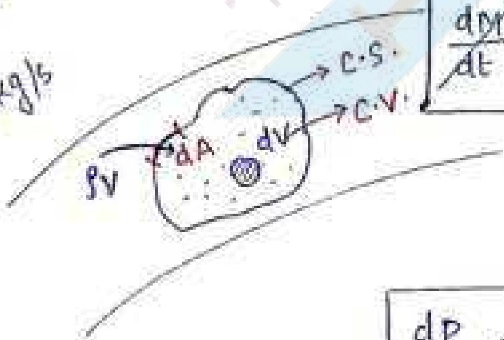
$$\frac{dr}{\frac{-60\times10^3}{2\pi r}} = \frac{r\,d\theta}{5\,\frac{300\times10^3}{2\pi r}}$$

$$-\int_{120}^{r} \frac{dr}{r} = \frac{1}{5}\int_0^{\pi} d\theta$$

$$\ln\left(\frac{r}{120}\right) = -\pi/5$$

$$\frac{r}{120} = e^{-\pi/5}$$





$$\boxed{\frac{d\dot{m}}{dt}\xrightarrow{}0 = \int_{C.S} \rho\, dA \cdot V + \frac{d}{dt}\int_{C.V} \rho \cdot dV}$$

Rate of flow

Rate of accumulation

$$\boxed{\frac{dP}{dt} = \int_{C.S.} \rho V^2 dA + \frac{\partial}{\partial t}\int_{C.V.} \rho \cdot V\, dV}$$

$$\dot{m} = \rho A V$$

$$\dot{P} = \dot{m} V$$

ACCELERATION IN FLUID MOTION :-

In fluid flow, velocity is the function of space and time. so the acceleration is the function of space and time.

Space component is known as convective acceleration and time component is known as local acceleration.

$$\vec{V} = f(x, y, z, t)$$

$$\vec{a} = \frac{D\vec{V}}{Dt}$$

$$\boxed{\vec{a} = (\vec{V}\cdot\vec{\nabla})\vec{V} + \frac{\partial \vec{V}}{\partial t}}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$a_x = \frac{du}{dt}, \quad a_y = \frac{dv}{dt}, \quad a_z = \frac{dw}{dt}$$

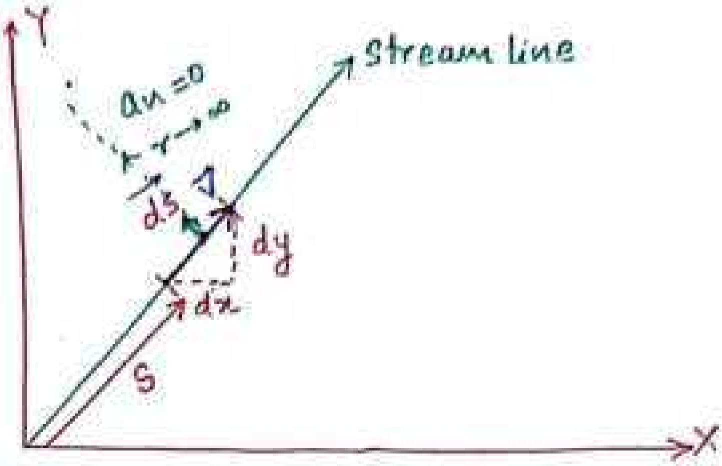
$$u = f(x, y, z, t)$$

$$\frac{du}{dt} = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z}}_{\nabla} + \frac{\partial u}{\partial t}$$

$$a_x = (\vec{V}\cdot\vec{\nabla})u + \frac{\partial u}{\partial t} = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = (\vec{V}\cdot\vec{\nabla})v + \frac{\partial v}{\partial t} = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = (\vec{V}\cdot\vec{\nabla})\omega + \frac{\partial \omega}{\partial t} = \frac{d\omega}{dt} = u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + \omega\frac{\partial \omega}{\partial z} + \frac{\partial \omega}{\partial t}$$



$$\vec{a} = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)\hat{s}$$

$$\vec{a} = a_x\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\boxed{a_x = u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}}$$

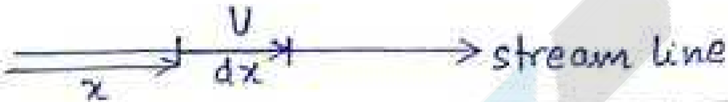
$\downarrow$

$$\cancel{v\frac{\partial u}{\partial x}} \to 0 + \cancel{\omega\frac{\partial u}{\partial x}} \to 0$$

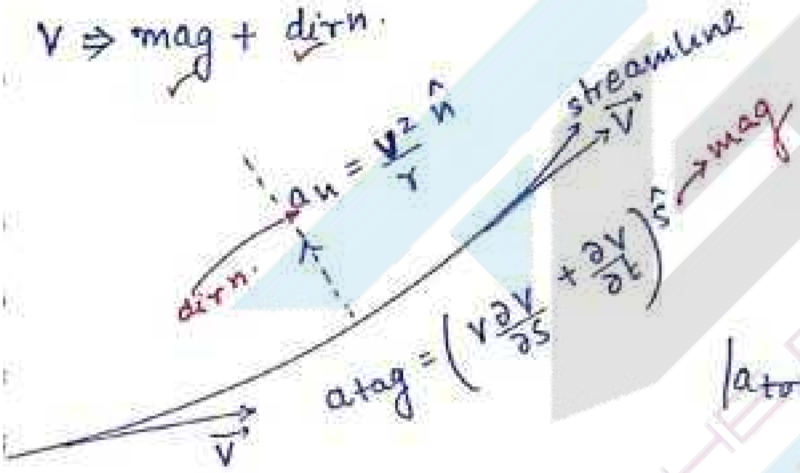
$(\because v = 0)$ $(\because \omega = 0)$

1D flow:- $\vec{V} = u\hat{i} + \overset{\vec{v}}{0\hat{j}} + \overset{\vec{\omega}}{0\hat{k}}$

$d\vec{s} = dx\hat{i} + 0\hat{j} + 0\hat{k}$



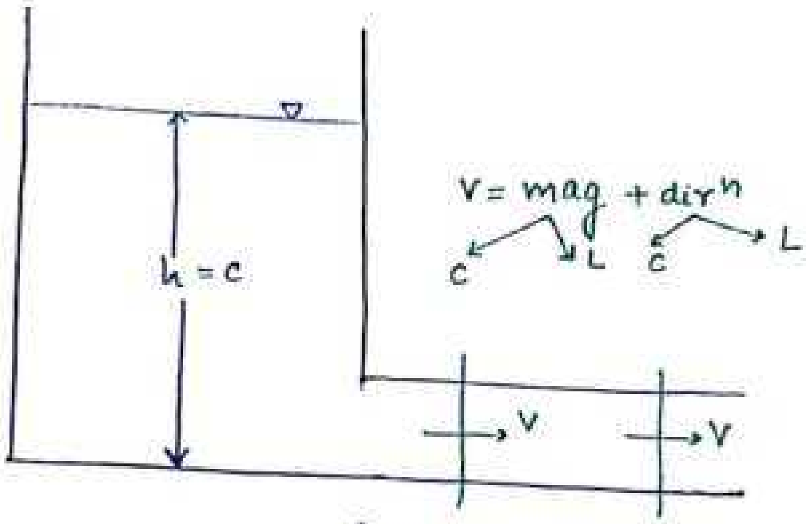
V ⇒ mag + dirn.



$$\vec{a}_{total} = a_{tang}\hat{s} + a_{normal}\hat{n}$$

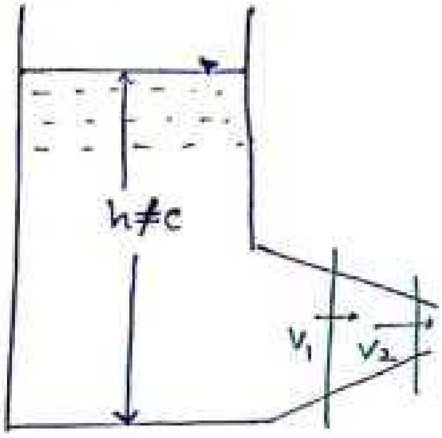
$$\vec{a}_{total} = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)\hat{s} + \left(\frac{V^2}{r}\right)\hat{n}$$

$$|a_{total}| = \sqrt{(a_{tag})^2 + a_n^2}$$



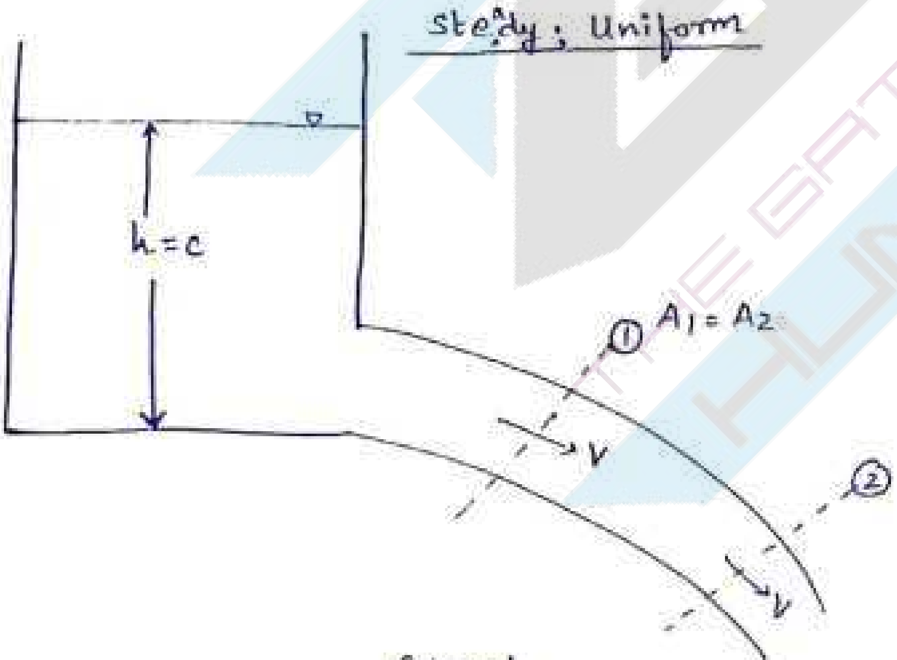
- a_{total}
  - a_{tang}
    - $a_{t_c} = 0$
    - $a_{t_L} = 0$
  - $a_n = 0$
    - a_{n_c}
    - a_{n_L}

Unsteady; Non-Uniform



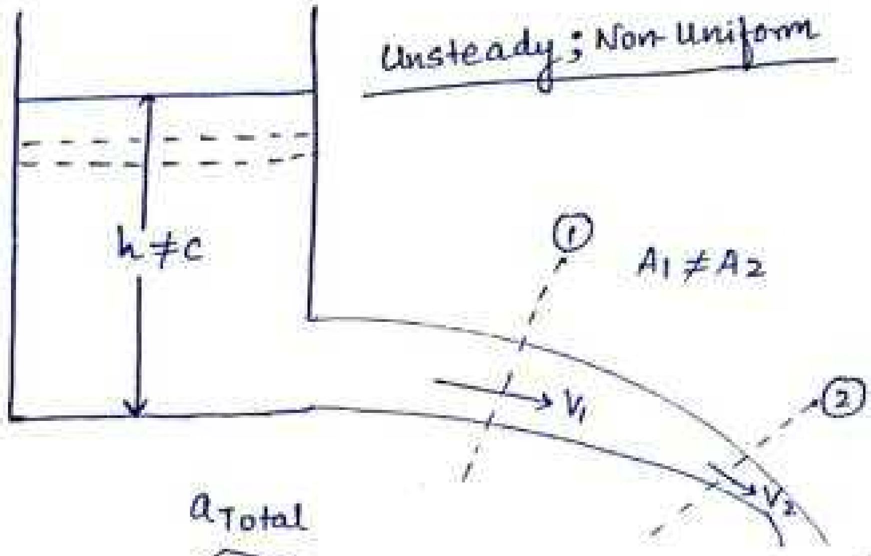
- a_{total}
  - a_{tag}
    - $a_{tc} \neq 0$
    - $a_{tL} \neq 0$
  - $a_n = 0$
    - a_{nc}
    - a_{nL}

Steady; Uniform



- a_{total}
  - a_{tag}
    - $a_{tc} = 0$
    - $a_{tL} = 0$
  - a_n
    - $a_{nc} \neq 0$
    - $a_{nL} = 0$

Unsteady ; Non Uniform



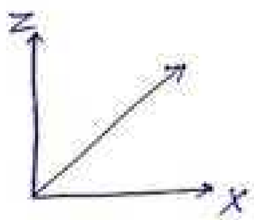
a_{Total}
- a_{tag}
  - $a_{tag_c} \neq 0$
  - $a_{tag_L} \neq 0$
- a_n
  - $a_{n_L} \neq 0$
  - $a_{n_L} = 0$

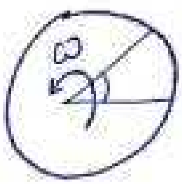
WB

G tb → (c) ✓

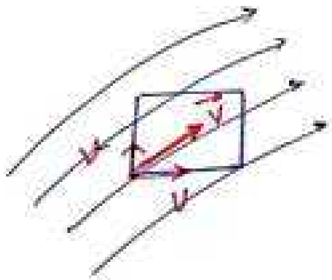
ROTATION IN FLUID MOTION :-

$u = f(x, y)$ $v = f(x, y)$

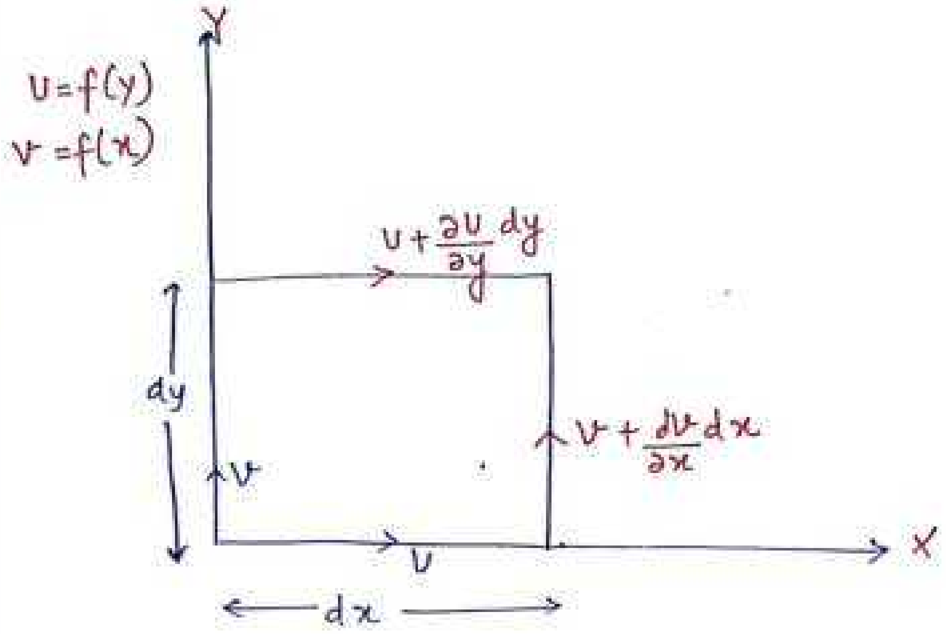


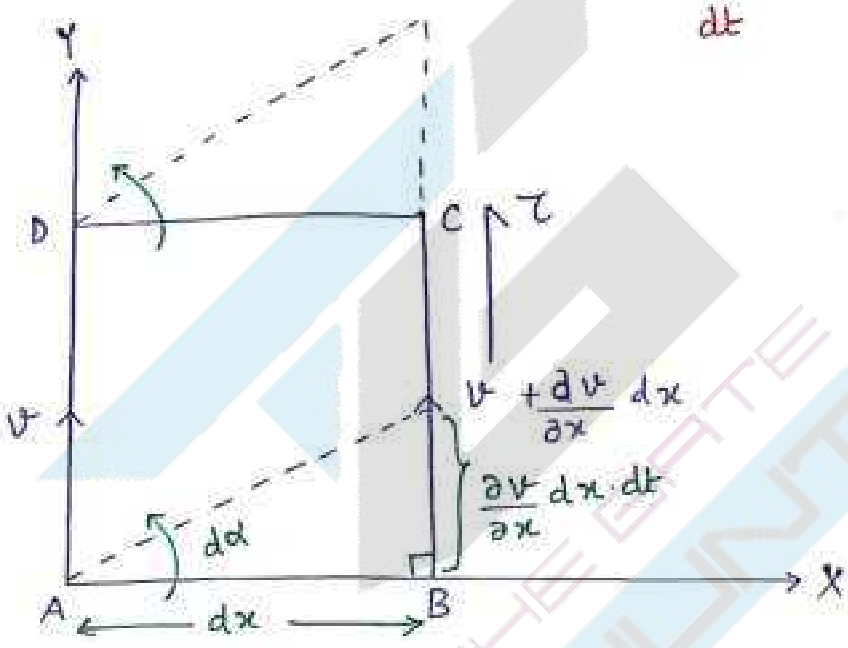


$\frac{d\theta}{dt} = \omega$



$$\omega_m = \frac{\omega_1 - \omega_2}{2}$$





$$\tan(d\alpha) = \frac{\frac{\partial v}{\partial x} dx \cdot dt}{dx}$$

MOHIT CHOUKSEY

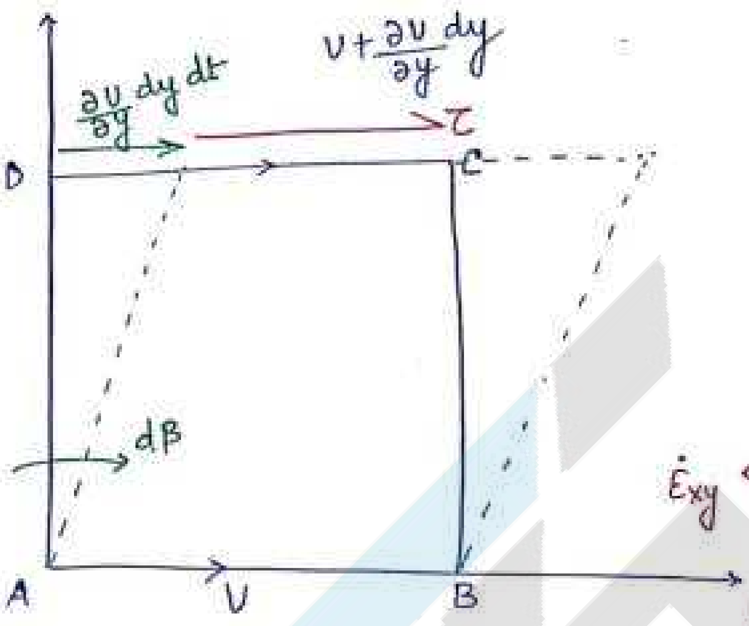
$$\frac{d\alpha}{dt} = \frac{\partial v}{\partial x}$$

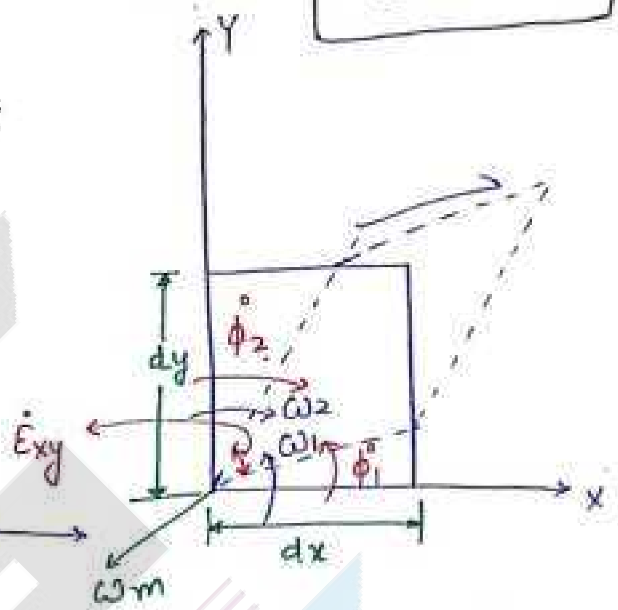
$$\dot{\epsilon}_{xy} = \frac{|\dot{\phi}_1| + |\dot{\phi}_2|}{2}$$

$$\boxed{\omega_1 = \frac{\partial v}{\partial x}}$$

Rate of shear strain $\leftarrow$ $\boxed{\dot{\epsilon}_{xy} = \frac{1}{2}\left\{\left|\frac{dv}{dx}\right| + \left|\frac{du}{dy}\right|\right\}}$

dt





$$\frac{d\beta}{dt} = \frac{\partial u}{\partial y}$$

$$\boxed{\omega_2 = \frac{\partial u}{\partial y}}$$

$$\boxed{\omega_m = \frac{\omega_1 - \omega_2}{2}}$$

$$\boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$

22) $u = 6y \qquad v = 0$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

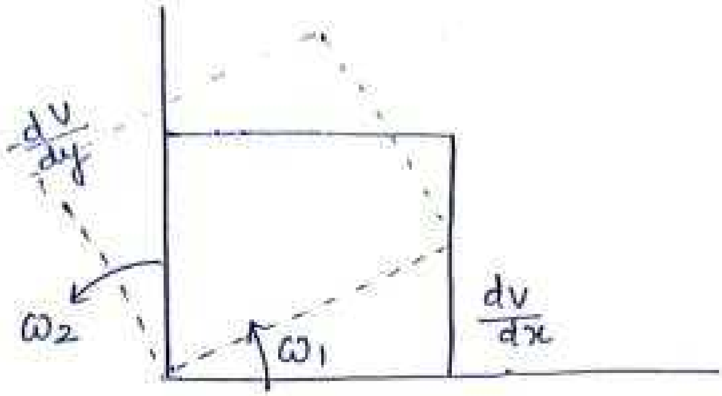
$$= \frac{1}{2}\left[0 - 6\right] = -3$$

(a) ✓

$$\dot{\epsilon}_{xy} = \frac{1}{2}\left[\left|\frac{\partial v}{\partial x}\right| + \left|\frac{\partial u}{\partial y}\right|\right]$$

$$= \frac{1}{2}\left[0 + 6\right] = 3$$

MOHIT CHOUKSEY



[Divergence $(\vec{\nabla} \cdot \vec{V})$]

[curl $(\vec{\nabla} \times \vec{V})$]

$$\omega_m = \frac{\omega_1 + \omega_2}{2}$$

$$\boxed{\omega_m = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]}$$

$\omega = \frac{1}{2}$ curl $\vec{V}$

$$\omega = \frac{1}{2}\left[\vec{\nabla} \times \vec{V}\right]$$

$$\omega = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & \omega \end{vmatrix}$$

$$\omega = \frac{1}{2}\left[\frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial z}\right]\hat{i} + \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial \omega}{\partial x}\right]\hat{j} + \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]\hat{k}$$

$$\boxed{\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}}$$

Q The laminar flow takes place b/w closely spaced ∥ plates as shown in fig. below, the velocity profile is given by

$$u = \frac{V}{h} y$$

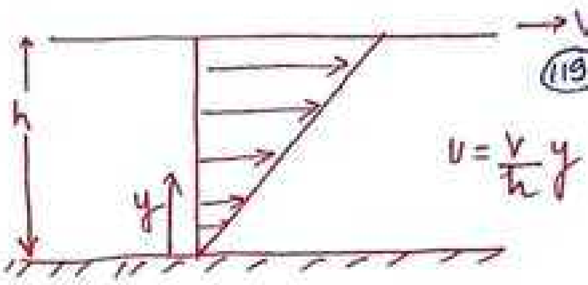
The Rate of Rotation of a fluid particle is given by

(a) $\omega_y = 0$ $\omega_z = \frac{-V}{2h}$

(b) $\omega_y = 0$ $\omega_z = -V/h$

(c) $\omega_y = \frac{V}{h}$ $\omega_z = \frac{-V}{2h}$

(d) $\omega_y = \frac{V}{h}$ $\omega_z = 0$



(119)

Sol
$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$
$$= \frac{V}{h} y\hat{i} + 0\hat{j} + 0\hat{k}$$

MOHIT CHOUKSEY

$$\omega = \frac{1}{2} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{V}{h} y & 0 & 0 \end{vmatrix}$$

$\omega_y = 0$ $\omega_z = \frac{1}{2}\left[0 - \frac{V}{h}\right]$

$$\omega_z = -\frac{V}{2h}$$

CIRCULATION :-

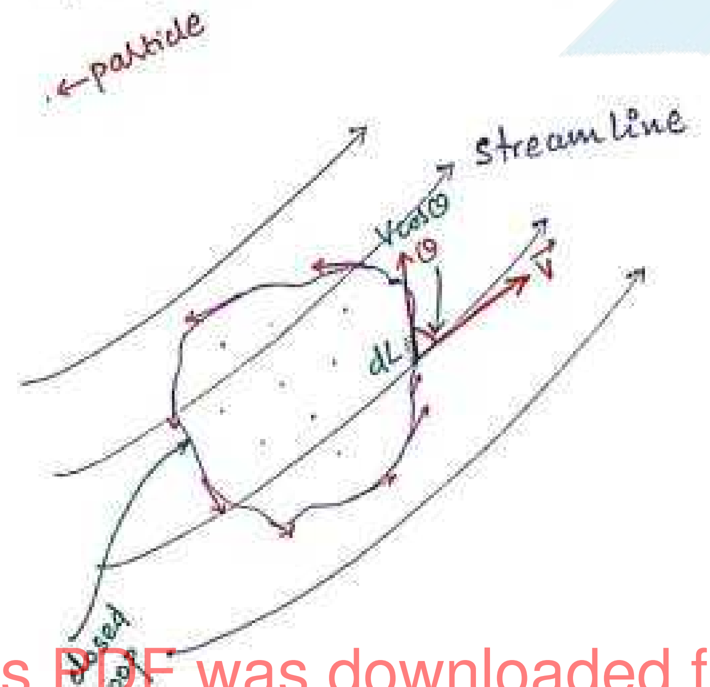
It is the line integration of tangential component of velocity along the closed loop (any shape). It is a scalar quantity anticlockw. movement should be considered '+ve'.

$$\boxed{\Gamma = \oint V\cos\theta \, dL}$$

$\vec{V} \cdot \vec{dL} = V dL \cos\theta$

$$\boxed{\Gamma = \oint \vec{V} \cdot \vec{dL}}$$

$\omega = \frac{1}{2}(\text{curl } \vec{V})$



$$\oint_L \vec{V}\cdot dL = \iint_S (\vec{\nabla}\times\vec{V})\, d\vec{s}\, \hat{n}$$

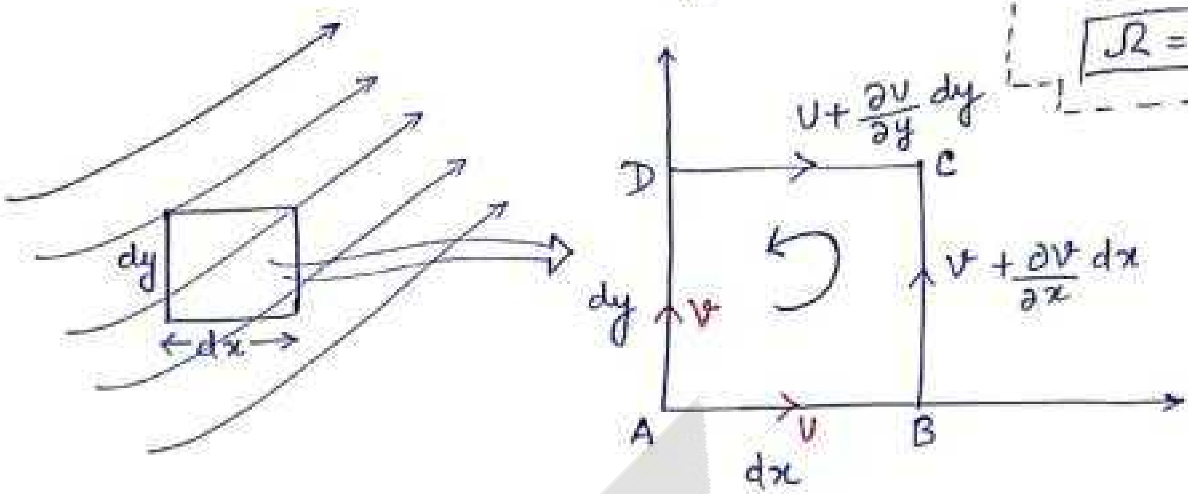
$$\Rightarrow \oint_L \vec{V}\cdot dL = \iint_S \vec{\Omega}\cdot d\vec{s}\, \hat{n}$$

$$2\omega = (\vec{\nabla}\times\vec{V})$$

$$\omega = \frac{1}{2}(\vec{\nabla}\times\vec{V})$$

$$\boxed{\Omega = \vec{\nabla}\times\vec{V}}$$

$$\boxed{\Omega = \text{curl}\, \vec{V}}$$



$$\Gamma_{ABCD} = \Gamma_{AB} + \Gamma_{BC} + \Gamma_{CD} + \Gamma_{DA}$$

$$= u\,dx\cos 0^\circ + \left(v + \frac{\partial v}{\partial x}dx\right)dy\cos 0^\circ + \left(u + \frac{\partial u}{\partial y}dy\right)dx\cos 180^\circ + v\,dy\cos 180^\circ$$

$$\Gamma_{ABCD} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)dx\,dy$$

$$\Gamma_{ABCD} = \left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\right)dA$$

$$\frac{\Gamma}{dA} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\boxed{\frac{\Gamma}{dA} = \Omega = \Omega}$$ vorticity

$$\boxed{\omega = \frac{1}{2}\Omega}$$

Vorticity → It is the mathematical measure of Rotationality. It is circulation per unit area. It is twice of Rotation the dirn of vorticity is as of Rotation. (121)

$\boxed{\Omega = \text{curl } \vec{V}}$

$$\Omega = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & \omega \end{vmatrix}$$

$$\boxed{\Omega = \Omega_x \hat{i} + \Omega_y \hat{j} + \Omega_z \hat{k}}$$

irrotational → curl $\vec{V} = 0$

$\boxed{\omega = 0}$ $[\omega_x = \omega_y = \omega_z = 0]$

$2\omega = 0$

$\boxed{\Omega = 0}$

$\Omega \times A = 0$

$\boxed{\Gamma = 0}$

WB (24) radius = 2 units

velocity field = $u = 2x + 3y$

$v = -2y$

SIR $u = 2x + 3y$ $v = -2y$

$\Gamma = \Omega \times A$

$$\Gamma = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \times A$$

$\Gamma = [0 - 3]\pi 2^2$

$\boxed{\Gamma = -12\pi}$

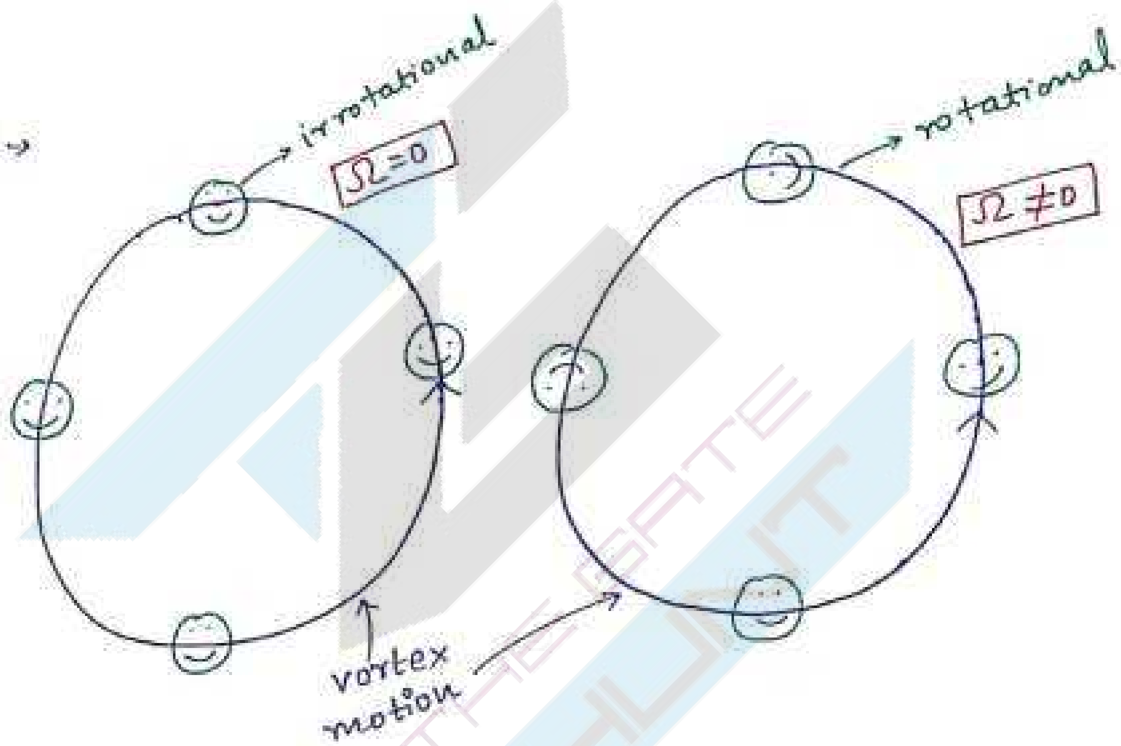
WB (25) $\vec{V} = k(y\hat{i} + x\hat{k})$ $\Omega_z = ?$

$$\vec{V} = ky\hat{i} + 0\hat{j} + kx\hat{k}$$

$$\Omega' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ky & 0 & kx \end{vmatrix}$$

$$\Omega_z = [0 - k]$$
$$= -k$$

* Concept →



WB (50)

$$V = 2t\left[1 - \frac{x}{2L}\right]^2$$

$$a_c = V\frac{\partial V}{\partial x}$$

$$a_c = 2t\left[1 - \frac{x}{2L}\right]^2 \cdot 2t \times 2 \times \left(\frac{-1}{2L}\right)\left(\frac{1-x}{2L}\right)$$

$$a_c = -14.623\ m/s^2$$

$$a_L = \frac{\partial V}{\partial t}$$

$$a_L = 2\left[1 - x/2L\right]^2$$

$$a_L = 0.945\ m/s^2$$

$$a_{total} = V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t}$$

$$a_t = -13.68\ m/s^2$$

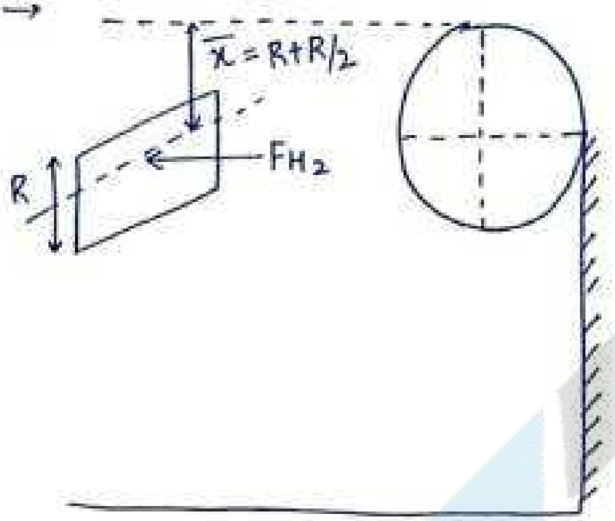
52 → Theory Book में solved है।

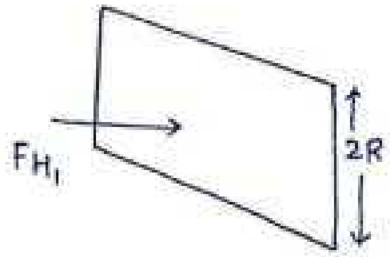
38

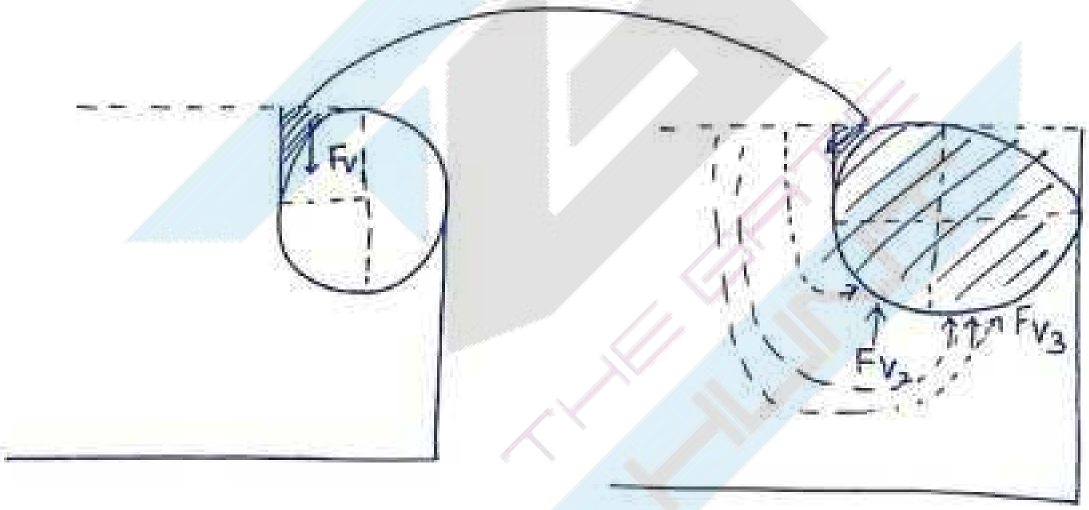
50 4.44 KN ① ③ 0
0.66 KN ②

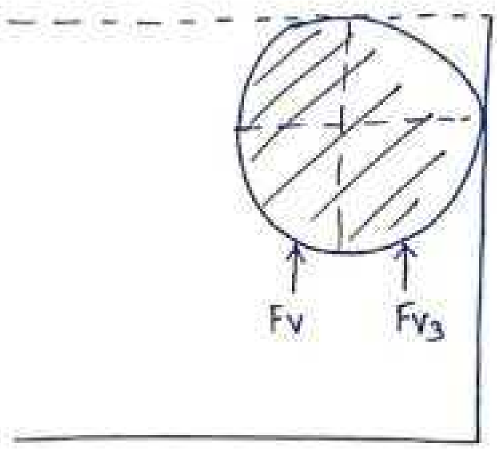
11/11/2016

HW →









MOHIT CHOUKSEY

(PPE)

Relationship b/w Equipotential line and constant stream f^n line

$$\psi = f(x,y)$$

$$\Rightarrow d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy \qquad \left(\because \frac{\partial \psi}{\partial x} = v \text{ and } \frac{\partial \psi}{\partial y} = -u \right)$$

$$\Rightarrow d\psi = v\,dx - u\,dy$$

$$\Rightarrow 0 = v\,dx - u\,dy \quad (\because \text{ constant stream } f^n \text{ so } d\psi = 0)$$

$$\Rightarrow u\,dy = v\,dx \Rightarrow \frac{dy}{dx} = \frac{v}{u}$$

$$\Rightarrow \boxed{m_1 = \frac{dy}{dx} = \frac{v}{u}} \quad \text{--- (1)}$$

and

$$\phi = f(x,y)$$

$$\Rightarrow d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$\Rightarrow d\phi = -u\,dx - v\,dy$$

$$\Rightarrow d\phi = -u\,dx - v\,dy = 0 \quad (\because \text{ constant potential } f^n \rightarrow \text{equipotential lines})$$

$$\Rightarrow 0 = -u\,dx - v\,dy \Rightarrow +u\,dx = -v\,dy \Rightarrow \frac{dy}{dx} = -\frac{u}{v}$$

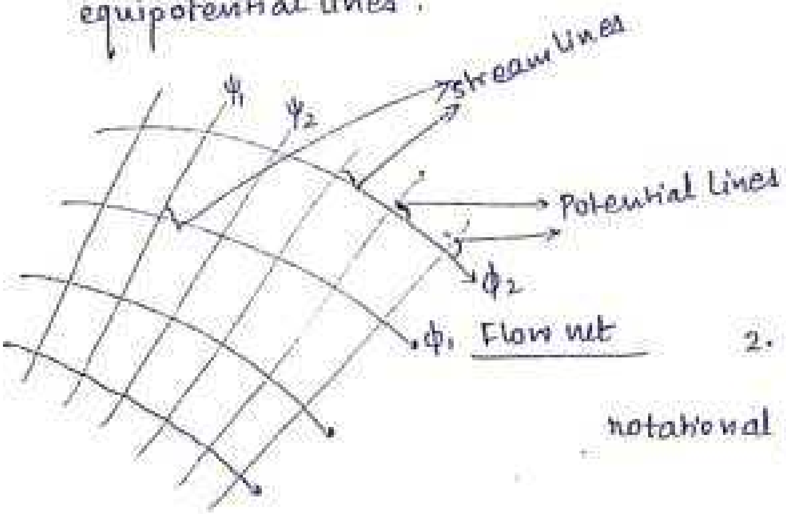
$$\Rightarrow \boxed{m_2 = \frac{dy}{dx} = \frac{-u}{v}} \quad \text{--- (2)}$$

eqn (1) × eqn (2)

$$\# \quad \boxed{m_1 \times m_2 = -1} \rightarrow \text{both lines are orthogonal to each other}$$

Flow net

- for a real and irrotational, 2D flow
- Graphical representation of constant stream f^n ($d\psi = 0$) and equipotential lines.



Features of flow net:

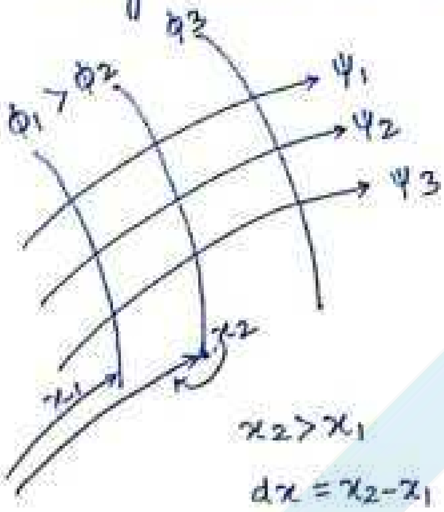
1. it is laplace representation of equipotential line and constant stream line.
2. Flow net cannot be drawn for rotational flow, within the Boundary layer etc.

Velocity Potential Function

$$\phi = f(x, y, z, t)$$

→ It is the scalar function of space and time. It is defined for irrotational flow.

It is define in such a way that -ve derivative of potential function in particular direction gives velocity scalar in that direction.



$$-\frac{\partial \phi}{\partial x} = u, \quad -\frac{\partial \phi}{\partial y} = v, \quad -\frac{\partial \phi}{\partial z} = \omega$$

$x_2 > x_1$

$dx = x_2 - x_1$

$dx > 0$

$d\phi < 0$

$\frac{d\phi}{dx} < 0$

$\left(-\frac{d\phi}{dx}\right) > 0$

Continuity eqn :

Steady flow, incompressible flow :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{-\partial \phi}{\partial z}\right) = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$\nabla^2 f(x)$

Laplace eqn. → $\nabla^2 f(x) = 0$

Poisson's eqn. → $\nabla^2 f(x) \neq 0$

$\nabla^2 \phi = 0$

Laplace eqn. → $\nabla^2 \phi$ ⇓ $\boxed{0 = 0}$

Poisson's eqn. → $\nabla^2 \phi \neq 0$, $(\,) \neq 0$

Rotational

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\omega_z = \frac{1}{2} \ldots \ldots = 0$$

- From this we conclude that if flow is irrotational then only velocity potential function exists or vice versa.

Stream Function (ψ) $\rightarrow$ 2D flow only

$$\psi = f(x,y,z,t)$$

It is the point fn. It is the scalar function of space and time. It is defined for conservation of mass i.e. for continuity eqn.

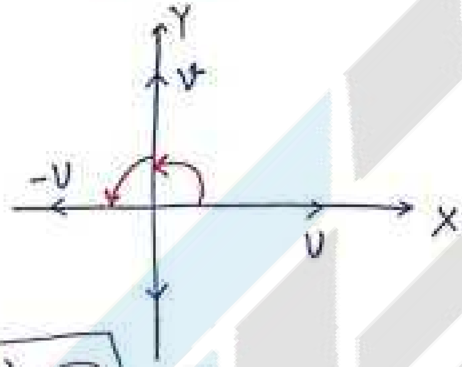
It is defined in such a way that derivative in particular direction gives velocity scalar in ⊥r direction (CCW and ~~CW~~)

(CCW)

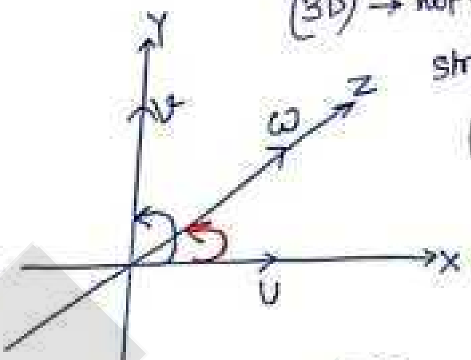
$\frac{\partial \psi}{\partial x} = v$

$\frac{\partial \psi}{\partial y} = -u$

(2D flow)



(3D) $\rightarrow$ not valid stream fn. (ψ)



(CW)

$\frac{\partial \psi}{\partial x} = -v$

$\frac{\partial \psi}{\partial y} = u$

~~$\frac{\partial \psi}{\partial x} = v$ or ω~~

Rotationality

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

** $\boxed{\omega_z = \frac{1}{2}(\nabla^2 \psi)}$ **

$$\omega_z = \frac{1}{2}(\nabla^2 \psi)$$

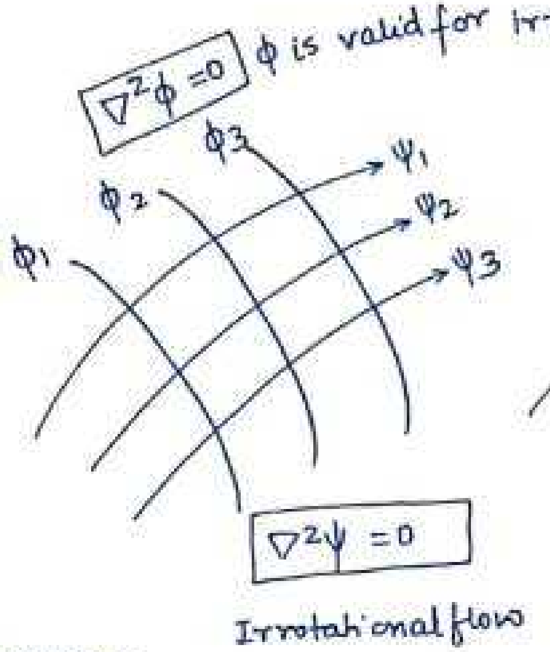
Laplace eqn.	Poisson's eqn.
$\nabla^2 \psi = 0$	$\nabla^2 \phi \neq 0$
$\boxed{\omega_z = 0}$	$\boxed{\omega_z \neq 0}$
irrotational	Rotational

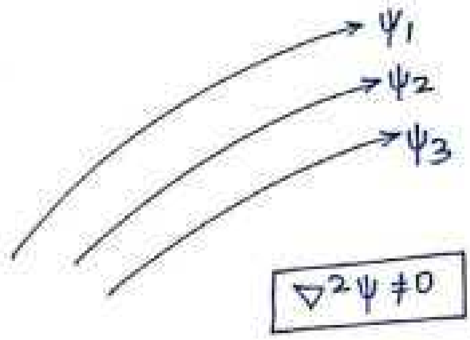
$\rightarrow$ Continuity eqn
- steady, 2D, incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{-\partial^2 \psi}{\partial x \, \partial y} + \frac{\partial^2 \psi}{\partial x \, \partial y} = 0$$

It always satisfy continuity eqn.





NOTE :-

Velocity potential fn. is valid for irrotational flow, so irrotational flow is also known as potential flow.

velocity poten. fn. is defined for 3D flow but stream function for 2D flow

(35) $\psi = px^2 + qy^2$ → irrotational

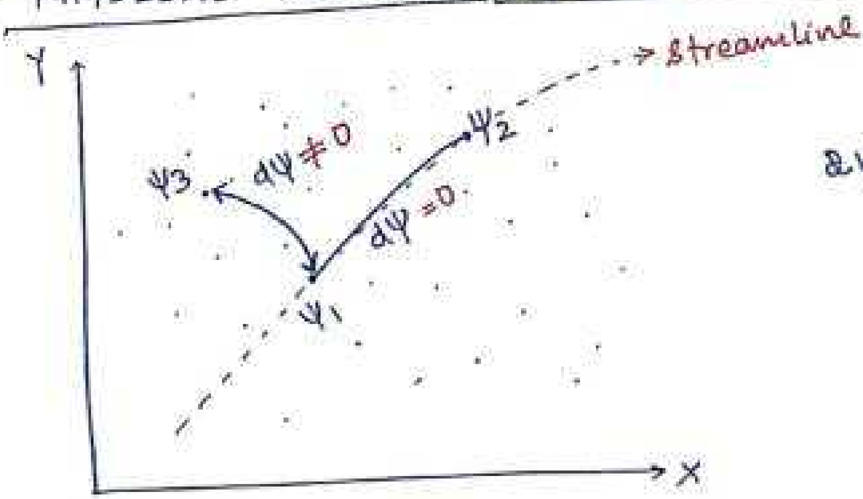
$\Rightarrow$ $\nabla^2\psi = 0$

$2p + 2q = 0$

$\boxed{p = -q}$

e.g. $\psi = 2x^2 - 2y^2$
$= 120x^2 - 120y^2$
$= 10000x^2 - 10000y^2$ } irrot.

PHYSICAL SIGNIFICANCE OF STREAM FUNCTION :-



$\psi = f(x,y)$ $\quad S = f(P,T)$

$$\partial\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy$$

$$\boxed{d\psi = v\,dx - u\,dy} \quad (1)$$

<u>Stream line eqn.</u>

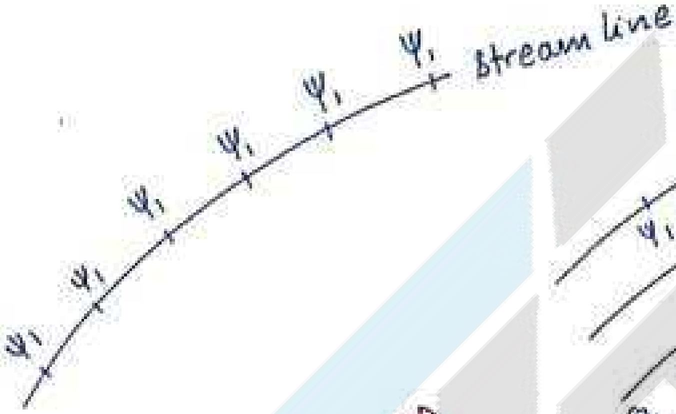
$$\frac{dx}{u} = \frac{dy}{v}$$

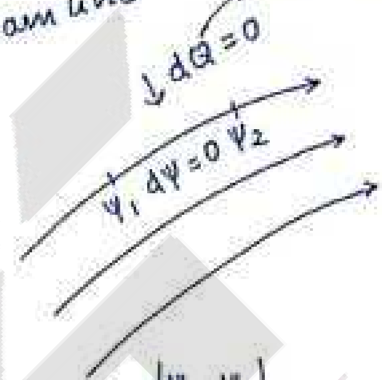
$$\boxed{v\,dx - u\,dy = 0}$$

$$\boxed{d\psi = 0}$$

$$\int d\psi = \int 0$$

$$\boxed{\psi = c}$$



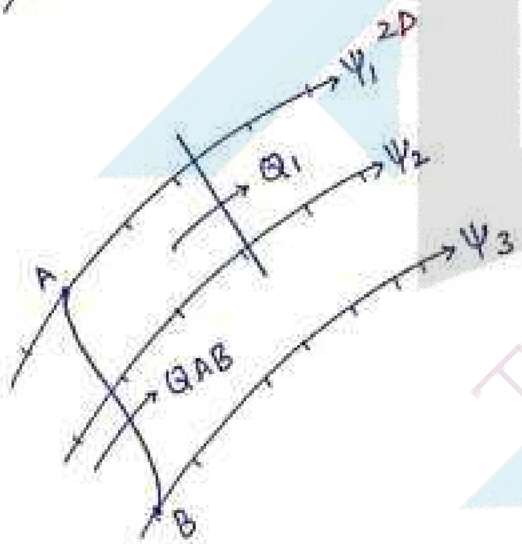


discharge

$$\boxed{d\psi = dQ}$$

$$\int d\psi = \int dQ$$

$$|\psi_1 - \psi_2| = Q$$



$$Q_1 = |\psi_1 - \psi_2|$$

$$Q_{AB} = |\psi_1 - \psi_3|$$

Discharge b/w the two stream lines

Q → discharge per unit depth

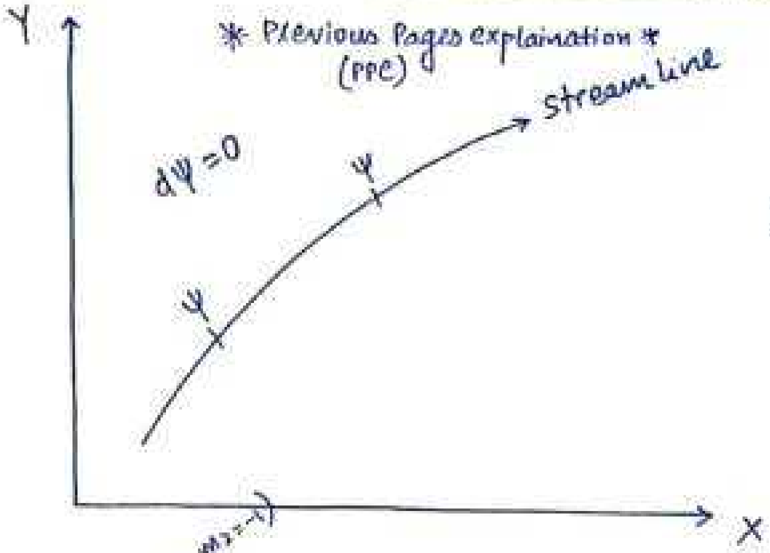
$$Q = |\psi_2 - \psi_1|$$

Discharge per unit width in the flow field is obtained by difference b/w 2 stream functions.

* Constant stream function (ψ) — ψ = constant
  ↳ along stream line ψ does not change

* stream line is the locus of constant stream function.

* Previous Pages explaination * (PPC)

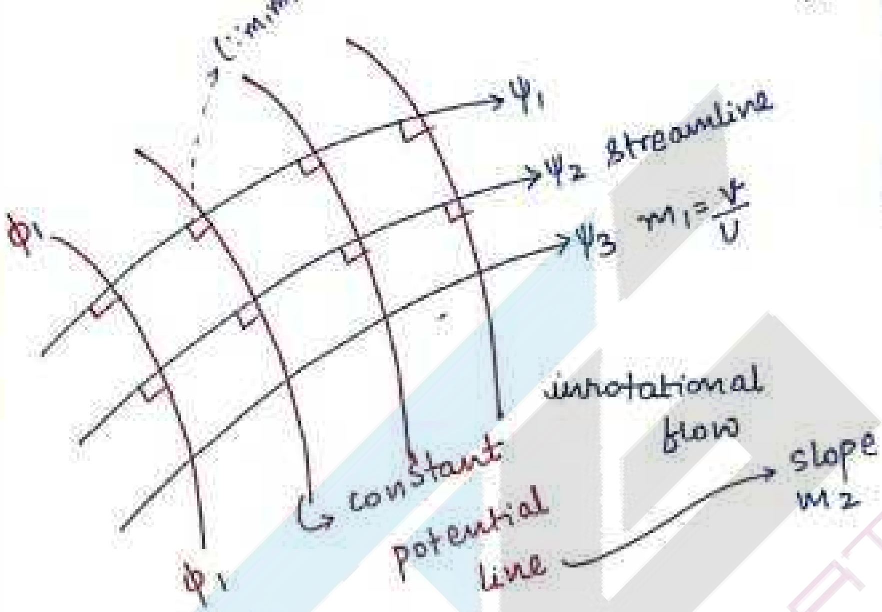


$$\frac{dx}{u} = \frac{dy}{v}$$

slope of stream line / slope of constant stream function line

$$\boxed{\frac{dy}{dx} = \frac{v}{u}}$$

$$\boxed{\frac{dy}{dx} = \frac{\left(\frac{\partial \psi}{\partial x}\right)}{-\left(\frac{\partial \psi}{\partial y}\right)}}$$

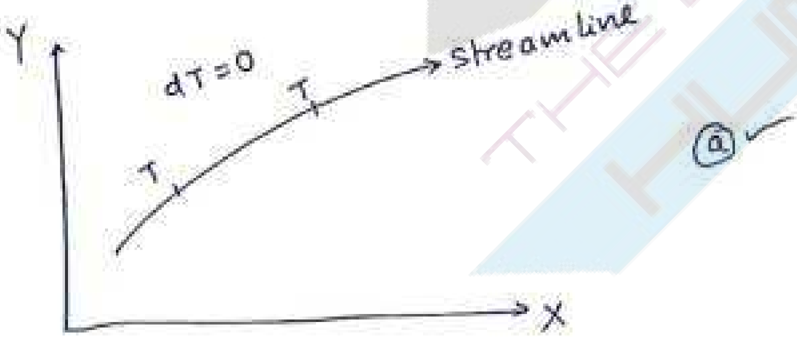


$m_1 \times m_2 = -1$

$m_2 = \frac{-1}{m_1}$

$$\boxed{m_2 = -\frac{u}{v}}$$

WB (20)



(a) ✓

Stream fn. is a point fn. so we make analogy from T-s fn

(37) 2D, steady flow (NOT given) (b) ✓

Note :- 1) Equipotential lines and constant stream fn. lines are orthogonal to each other

2) irrotational flow is also known as Potential flow.

CAUCHY - RIEMANN EQUATION :-

$$-\frac{\partial \phi}{\partial x} = u \qquad \frac{\partial \psi}{\partial y} = -u$$

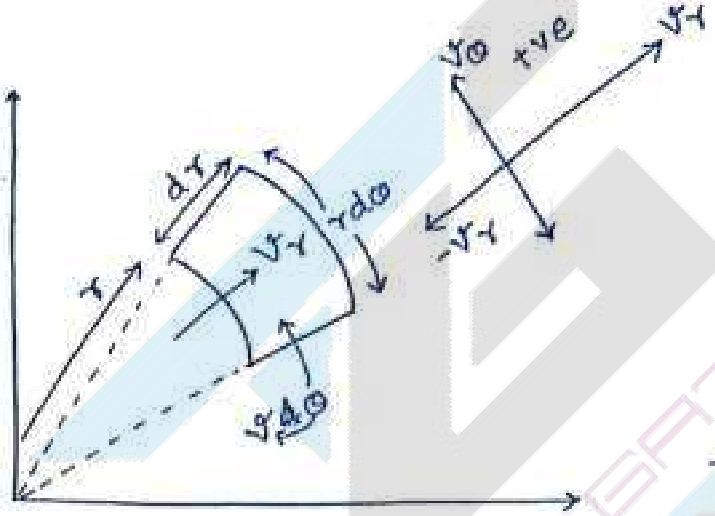
$$-\frac{\partial \phi}{\partial y} = v \qquad \frac{\partial \psi}{\partial x} = v$$

these are complimentary

** $\boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}}$ **

** $\boxed{\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}}$ ** → C.R. eqn.

Polar :-



$$-\frac{\partial \phi}{\partial r} = v_r$$

$$-\frac{\partial \phi}{r\,\partial \theta} = v_\theta$$

$$\frac{\partial \psi}{r\,\partial \theta} = -v_r$$

$$\frac{\partial \psi}{\partial r} = v_\theta$$

** $\boxed{\frac{\partial \phi}{\partial r} = \frac{\partial \psi}{r\,\partial \theta}}$ **

** $\boxed{\frac{\partial \phi}{r\,\partial \theta} = -\frac{\partial \psi}{\partial r}}$ ** → C.R.

WB (T3) $\psi = m \ln r$

$\phi = ?$

$$\frac{\partial \psi}{\partial r} = v_\theta$$

$$\frac{\partial \psi}{\partial r} = \frac{m}{r}$$

$$v_\theta = \frac{m}{r}$$

$$-\frac{\partial \phi}{\cancel{r}\,\partial \theta} = \frac{m}{\cancel{r}}$$

$$\int \partial \phi = -\int m\,\partial \theta$$

$$\boxed{\phi = -m\theta + C}$$

NB 84

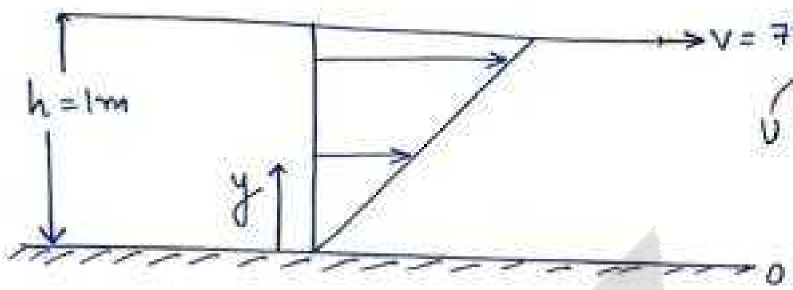
|| 2D flow $\longrightarrow$ +ve dir^n

$V \rightarrow$ linear

$y = 0$ to $75\, m/sec$ $y = 1$

$\psi = ?$

SIR



$u = \frac{V}{h} y$ rotational $\qquad u = 75y$

$v = 0$

$$\frac{\partial \psi}{\partial y} = -u$$

$$\Rightarrow \int \frac{\partial \psi}{\partial y} = -\int 75y$$

$$\Rightarrow \psi = -\frac{\overset{37.5}{\cancel{75}}}{\cancel{2}} y^2 + f(x)$$

for $f(x) \rightarrow$ Boundary condition needed

$$\frac{\partial \psi}{\partial x} = 0 + f'(x)$$

$$v = f'(x)$$

$$0 = f'(x)$$

$$\boxed{f(x) = c}$$

$$\boxed{\psi = -37.5y^2 + c}$$

(c) ✓

$\downarrow$

Ans is clockwise sense में मान के निकाला]

Qn. & Ans^r both Right.

$\cancel{\frac{\partial \psi}{\partial y} = -75y}$

$\cancel{+u = +75y}$

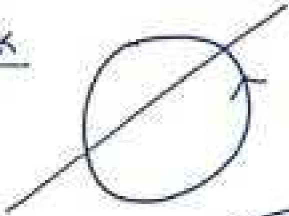
$\cancel{u = 75y}$

$$-\frac{\partial\phi}{\partial x} = v$$

$$-\frac{\partial\phi}{\partial x} = 75y$$

$$\boxed{\phi = -75yx + f(y)}$$

TRICK



$\psi \rightarrow$ valid on both Rot. / N.R.

NOTE :-

- $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

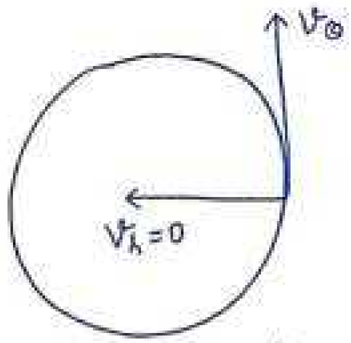
$$\boxed{\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]}$$

- $$\frac{\partial (rv_r)}{r\,\partial r} + \frac{\partial v_\theta}{r\,\partial\theta} = 0$$

$$\boxed{\omega_z = \frac{1}{2}\left[\frac{\partial (r v_\theta)}{r\,\partial r} - \frac{\partial v_r}{r\,\partial\theta}\right]}$$

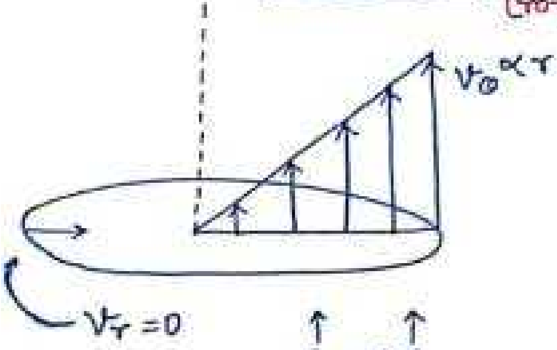
VORTEX MOTION :-

$V_r = 0$



Forced Vortex

$\boxed{V_\theta = r\omega}$ $T \neq 0$ (rotational)



$\uparrow V_\theta \propto \uparrow r$

$$\omega_z = \frac{1}{2}\left[\frac{\partial(rV_\theta)}{r\partial r} - \cancel{\frac{\partial V_r}{r\partial\theta}}^{\,0}\right]$$

$$\omega_z = \frac{1}{2\times r} \cdot \frac{\partial(r^2\omega)}{\partial r}$$

$$\omega_z = \frac{1}{\cancel{2r}} \times \cancel{2r}\,\omega$$

$$\boxed{\omega_z = \omega}$$

hence, Rotational

Example :-

- Flow over impeller of pump or runner of turbine.
- Fluid in a container with constant angular velocity.

Free vortex

$F \times r = 0$ $T = 0$ external torque (irrotational)

$xy = c$

$\uparrow y \propto \frac{1}{x} \downarrow$

$$\frac{d\,mv}{dt} \cdot r = 0$$

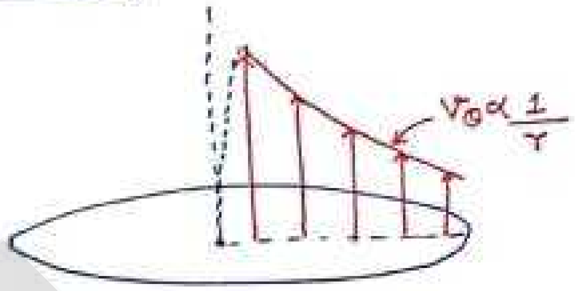
$$\frac{d(mv \cdot r)}{dt} = 0$$

$mv \cdot r = c$

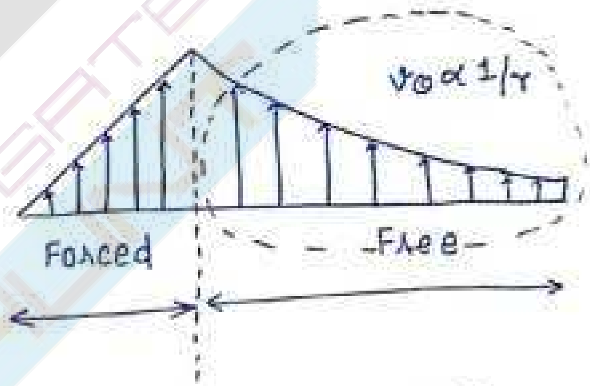
mass = constant

$\boxed{V_\theta = \frac{c}{r}}$

$\boxed{V_\theta \cdot r = c}$



$r \to 0$ $V_\theta \to \infty$



$$\omega_z = \frac{1}{2}\left[\frac{\partial(rV_\theta)}{r\partial r} - \cancel{\frac{\partial V_r}{r\partial\theta}}^{\,0}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial c}{r\partial r} - 0\right]$$

$\boxed{\omega_z = 0}$ sys, irrotational.

Ex → • cyclones • flow in whirlpool.
• flow in washbasins, pipelines, diffusion of pump etc.

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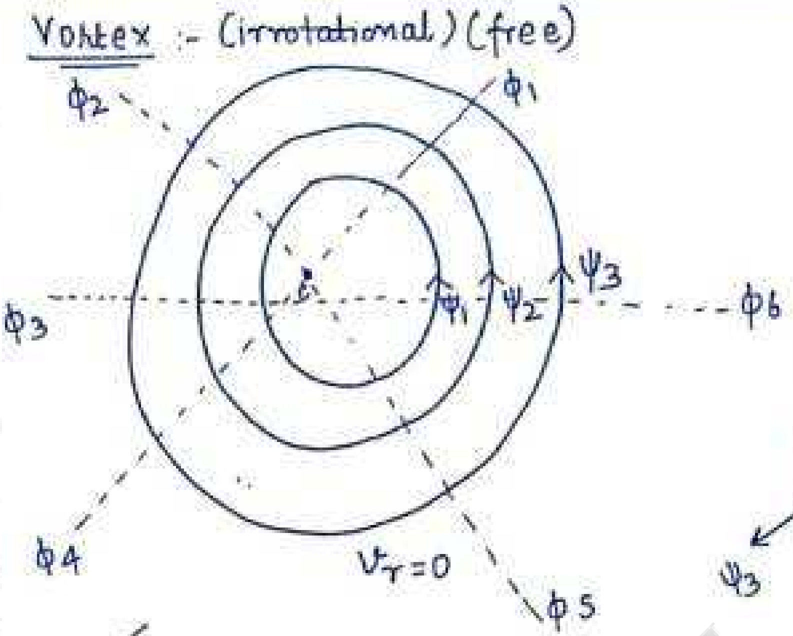
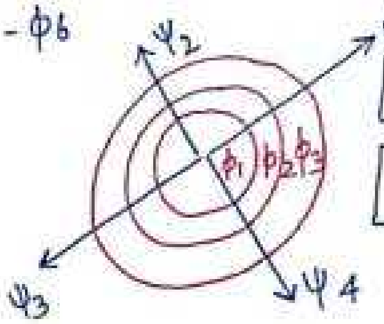
Vortex :- (irrotational) (free)





$\boxed{\phi = C \ln r + C_2}$

$\boxed{\psi = -C\theta + C_1}$

$v_\theta = 0$ source/sink

$v_\theta = \frac{C}{r}$

$-\frac{\partial \phi}{r\,\partial \theta} = v_\theta$

$-\frac{\partial \phi}{\cancel{r}\,\partial \theta} = \frac{C}{\cancel{r}}$

$\boxed{\phi = -C\theta + C_1}$

$\frac{\partial \psi}{\partial r} = v_\theta$

$\frac{\partial \psi}{\partial r} = \frac{C}{r}$

$\boxed{\psi = C \ln r + C_2}$

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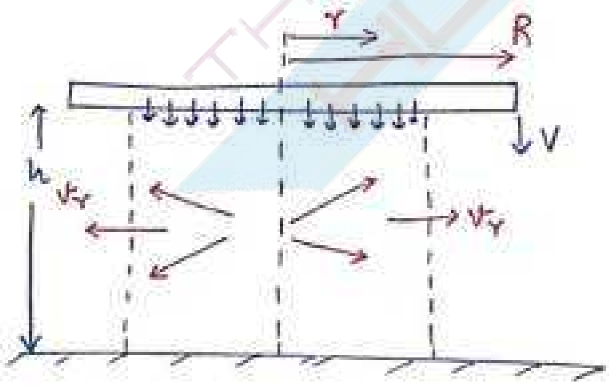
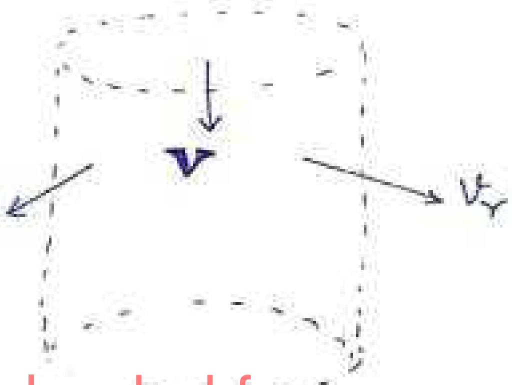
GateBook
Gate 2008
Pg 245 3.22



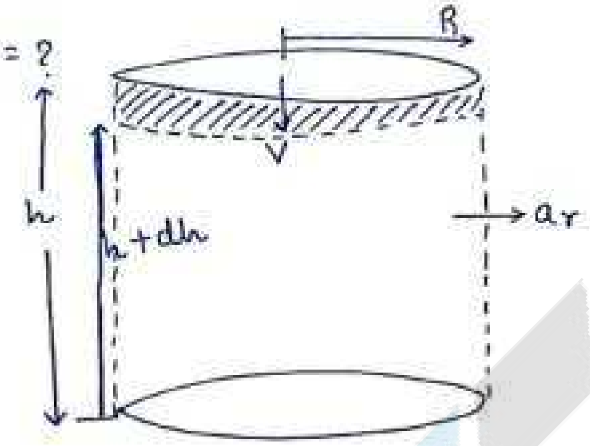


(i.) $\dot{m}_h = \dot{m}_r$

$\rho(\pi r^2)V = \rho(2\pi r h) \cdot V_r$

$$\boxed{V_r = \frac{V \cdot r}{2h}}$$

(ii.) $a_r = ?$



$V_r = f(r, h)$

$$dV_r = \frac{\partial V_r}{\partial r} dr + \frac{\partial V_r}{dh} dh$$

$$\frac{dV_r}{dt} = \frac{\partial V_r}{\partial r}\left(\frac{dr}{dt}\right) + \frac{\partial V_r}{dh}\left(\frac{dh}{dt}\right) \quad -ve$$

$$= V_r \frac{\partial V_r}{\partial r} + (-V)\frac{\partial V_r}{\partial h}$$

$h + dh < h$

$dh < 0$

$\frac{dh}{dt} < 0$

$-\left(\frac{dh}{dt}\right) > 0$

$$a_r = \frac{Vr}{2h}\left(\frac{V}{2h}\right) + (-V)\left[\frac{-Vr}{2h^2}\right]$$

$$a_r = \frac{V^2 r}{4h^2} + \frac{V^2 r}{2h^2}$$

$$\boxed{a_r = \frac{3}{4}\frac{V^2 r}{h^2}}$$

NOTE :- FLOWNET from theory Book

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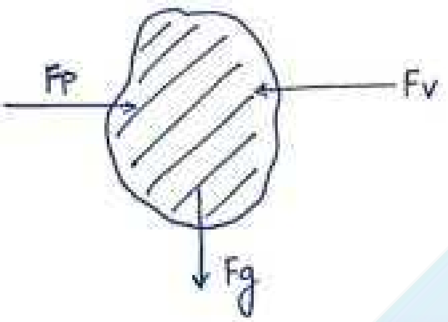
CHAPTER-6

FLUID DYNAMICS

Study of motion of the fluid with reference of forces and moments is known as fluid

In fluid flow, there are different types of forces occurs in the flow like viscous forces, gravitational forces, pressure forces, elastic forces, surface tension forces, eddy forces (turbulent forces) and diffr. type of other forces, etc.



$\Sigma F_{net} = ma$ Newton's eqn. (momentum eqn.)

$\boxed{F_p + F_g + F_v = F_i}$ Navier-Stokes eqn (laminar flow)

$F_v = 0$ Non Viscous flow → (viscosity is negligible)

$\boxed{F_p + F_g = F_i}$ Euler's eqn. (1755)

Integral form → Energy eqn. (B.E. eqn) (1738)

After Derivation

↓

Final Equation :→

$$\boxed{-dP - \rho g\,dz = \rho\,ds\left[V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right]}$$

Applicable to Both Incompressible, compressible, also unsteady flow

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$Re = \frac{F_i}{F_v}$ $\quad F_r = \sqrt{\frac{F_i}{F_g}}$ $\quad We = \sqrt{\frac{F_i}{F_s}}$ $\quad E_v = \sqrt{\frac{F_i}{F_p}}$ $\quad Ma = \sqrt{\frac{F_i}{F_e}}$

$$Re = \frac{ma}{\tau \times A} = \frac{\rho(\text{vol})\, \overset{\text{velocity}}{V}/t}{\mu \frac{V}{h} \times A}$$

$$\boxed{\begin{aligned} \text{vol.} &= L_c^3 \\ A &= L_c^2 \\ L, h, d &= L_c \end{aligned}}$$

$$F_i = \rho \cdot L_c^3 \frac{V}{t} = \rho \cdot L_c^2 \cdot \frac{L_c}{t} \cdot V$$

$$\boxed{F_i = \rho V^2 L_c^2} \text{ — (1)}$$

$$F_v = \mu \frac{V}{h} A = \frac{\mu V L_c^2}{L_c} = \mu V L_c$$

$$\boxed{F_v = \mu V L_c} \text{ — (2)}$$

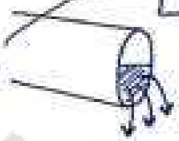
$$Re = \frac{F_i}{F_v}$$

$$Re = \frac{\rho V^{\cancel{2}} L_c^{\cancel{2}}}{\mu \cancel{V} \cancel{L_c}}$$

$$\boxed{Re = \frac{\rho V L_c}{\mu}}$$

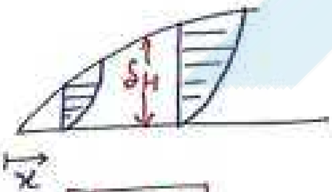
L_c

$$\boxed{R = L_c = \frac{A_{cs}}{P}}$$



open channel

Plate flow



$$\boxed{L_c = x}$$

$$\boxed{Re = \frac{\rho V x}{\mu}}$$

Pipe flow

$L_c = D$

$$\boxed{Re = \frac{\rho V D}{\mu}}$$

$Re \leq 2000$ laminar

$Re \geq 4000$ turbulent

V = mean velocity

$$D_h = \frac{4 \frac{\pi}{4} D^{\cancel{2}}}{\pi \cancel{D}}$$

$$\boxed{D_h = D}$$

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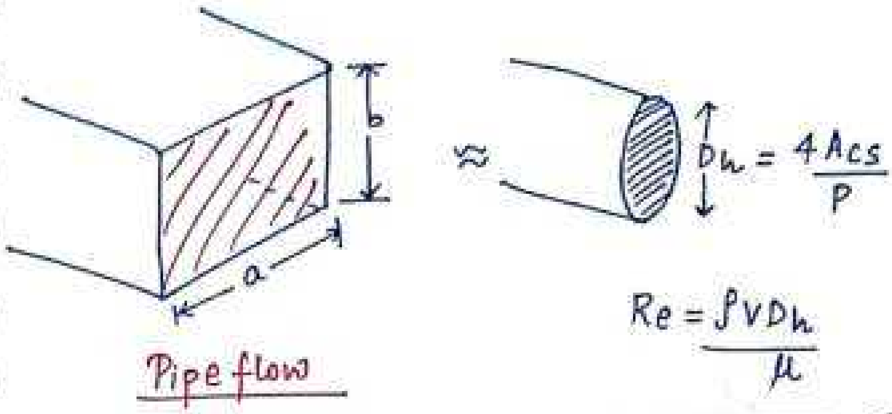

$D_h = \dfrac{4A_{cs}}{P}$

$Re = \dfrac{\rho V D_h}{\mu}$

$\boxed{D_h = \dfrac{2ab}{a+b}} \rightarrow \dfrac{4(ab)}{2a+2b}$

$\dfrac{2ab}{a+b} = \dfrac{\cancel{4}^{2}ab}{\cancel{2}(a+b)}$

* $F_r = \sqrt{\dfrac{F_i}{F_g}} = \sqrt{\dfrac{\rho V^2 L_c^2}{\rho L_c^3 g}} = \dfrac{V}{\sqrt{L_c g}}$

$\boxed{K = \dfrac{dP}{\left(\frac{-dV}{V}\right)} \; N/m^2}$

Bulk modulus

$Ma = \sqrt{\dfrac{F_i}{F_e}} = \sqrt{\dfrac{\rho V^2 L_c^2}{K \cdot A}} \approx \sqrt{\dfrac{\rho V^2 \cancel{L_c^2}}{K \cancel{L_c^2}}}$

$Ma = \dfrac{V}{\sqrt{K/\rho}} = \dfrac{V}{c}$

Bernoulli's Eqn. Assumptions :-

1. steady flow ⟶ properties → no change w.r.t time
2. Incompressible flow → the density doesn't changes w.r.t time
3. Non - viscous flow → there is no viscosity in the flow
4. Irrotational flow → No Rotationality there
5. flow along streamline.

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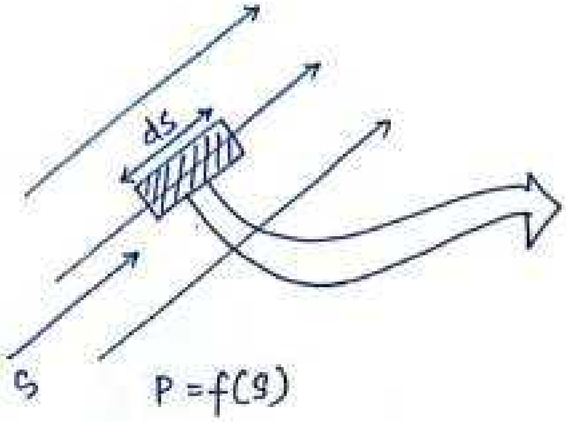
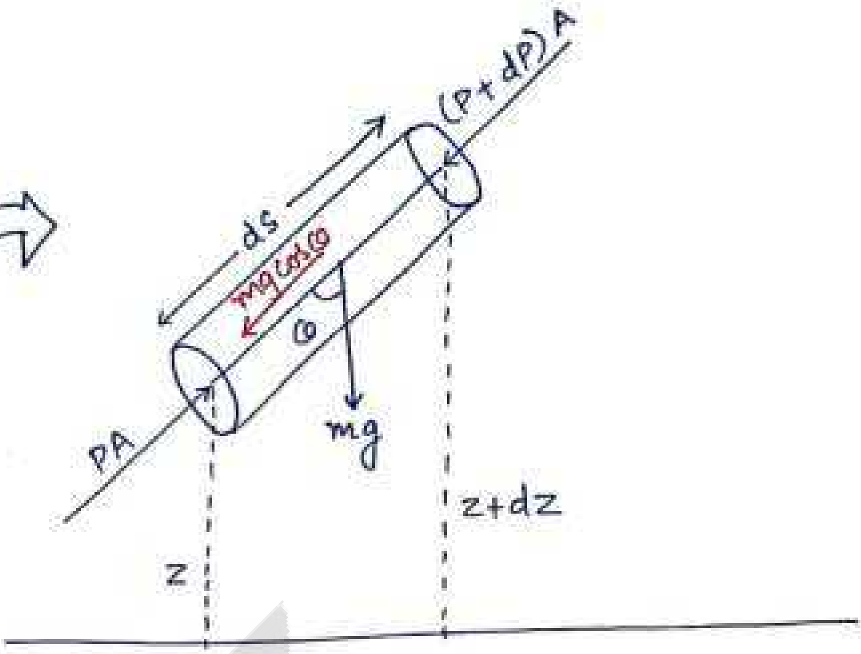




$P = f(s)$

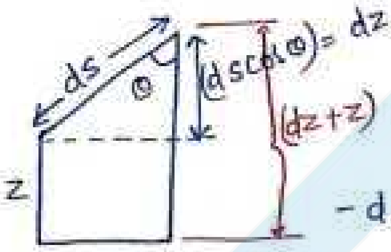
$dP = \frac{\partial P}{\partial s} ds$

$z = f(s)$

$dz = \frac{\partial z}{\partial s} ds$

$$F_p + F_g = m\left[v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right] \rightarrow \text{Euler's eqn}$$

(Non viscous flow)



$$PA - (P+dP)A - mg\cos\theta = m\left[v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]$$

$$-dP\cancel{A} - \rho g\cancel{A}\,(ds\cos\theta) = \rho(\cancel{A}\,ds)\left[v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]$$

*** $$\boxed{-dP - \rho g\,dz = \rho\,ds\left[v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right]}$$ *** → Euler's eqn. for unsteady flow

steady flow $\frac{\partial v}{\partial t} = 0$

*** $$\boxed{dP + \rho\left[v\cdot\frac{\partial v}{\partial s}\right]ds + \rho g\,dz = 0}$$ ***

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$$\left[\begin{aligned} mg\cos\theta &= \rho g A\cos\theta\, ds \\ &\quad \frac{m}{\cancel{V}}\, g\, \underbrace{(A\times ds)}_{\cancel{V}}\cos\theta = mg\cos\theta \end{aligned}\right]$$

$$\left[\rho A\,ds = \frac{m}{V}\underbrace{(A\,ds)}_{V} = m\right]$$

$$V\frac{\partial V}{\partial s} = \frac{1}{2}\frac{\partial V^2}{\partial s}$$ Irrotational flow

$$\frac{dP}{\rho} + \frac{1}{2}\frac{\partial V^2}{\partial s}ds + g\,dz = 0$$

$$\frac{1}{\rho}\int\frac{dP}{\partial \rho}ds + \frac{1}{2}\int\frac{\partial V^2}{\partial s}ds + g\int\frac{dz}{\partial s}ds = \int 0$$

$$\boxed{\frac{P}{\rho} + \frac{V^2}{2} + gz = C}$$ → stream line constant → $\frac{\text{Energy}}{\text{mass}}$ → $J/kg \cong \frac{m^2}{sec^2}$ → Dimens. analysis

or

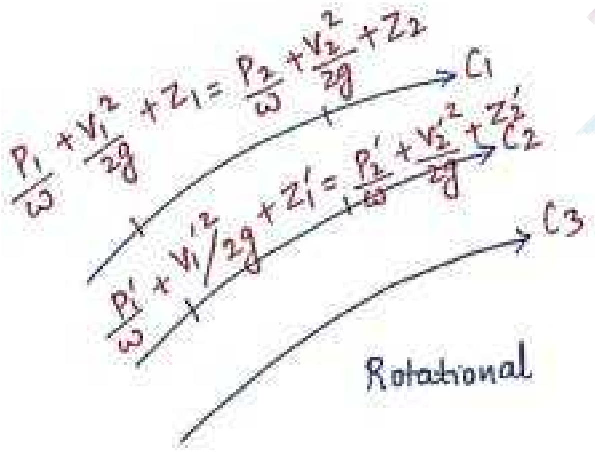
$$\boxed{\frac{P}{\omega} + \frac{V^2}{2g} + Z = C}$$ → $\frac{\text{Energy}}{\text{weight}}$

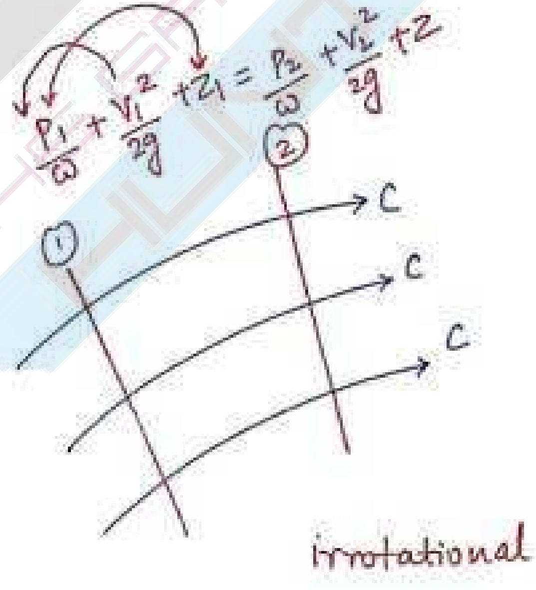
Incompressible flow

$\rho \neq f(P)$

$$K.E. = \frac{1}{2}mV^2$$

$$\frac{K.E.}{mg} = \frac{V^2}{2g}\left(\frac{\text{Energy}}{\text{wt}}\right)$$





for rotational flow, the energy constant C is diff for all stream lines so the expression $\left(\frac{P}{\omega}+\frac{V^2}{2g}+Z\right)$ is applicable only along the stream line. but for irrotational flow C is same for all stream lines

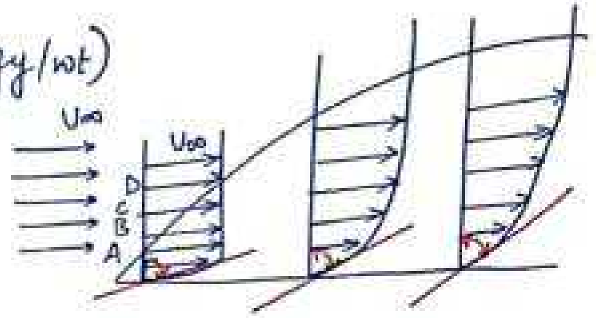
So the expression

$\left(\frac{P}{w} + \frac{V^2}{2g} + z\right)$ is applicable over the entire section.

$\frac{P}{\omega} + \frac{V^2}{2g} + z = C \longrightarrow$ (Energy/wt)

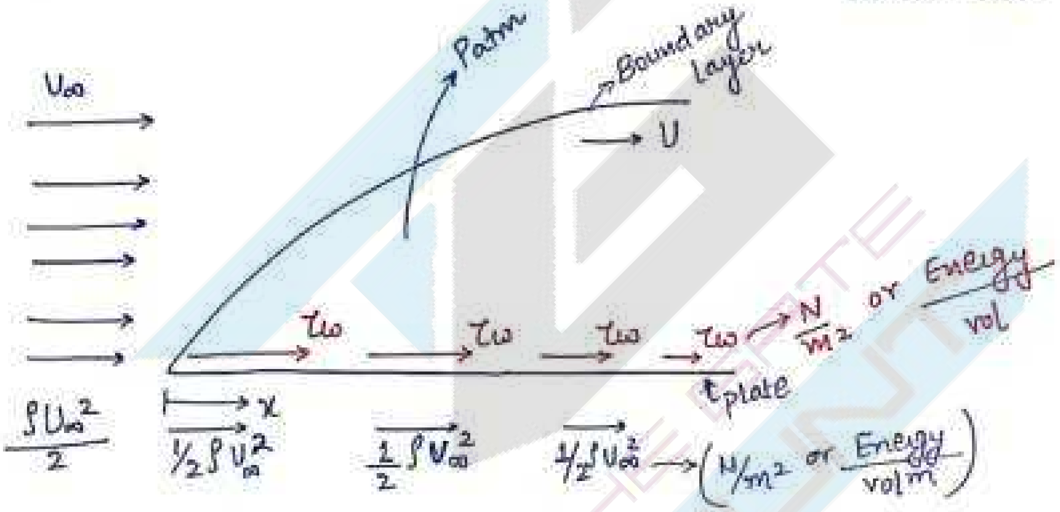
$\frac{P}{\rho} + \frac{V^2}{2} + gz = C \longrightarrow$ (Energy/mass)

$\underbrace{P}_{\text{static pressure}} + \underbrace{\frac{\rho V^2}{2}}_{\text{Dynamic Pressure}} + \rho g z = C \longrightarrow \left(\frac{N}{m^2} \text{ or } \frac{\text{Energy}}{\text{vol}^m}\right)$



$\mu \frac{d\phi_1}{dt}\Big|_{y=0} > \mu \frac{d\phi_2}{dt}\Big|_{y=0} > \mu \frac{d\phi_3}{dt}\Big|_{y=0}$

$\tau_{\omega_1} > \tau_{\omega_2} > \tau_{\omega_3}$



$x\uparrow \rightarrow \tau_\omega \downarrow \rightarrow C_{fx}\downarrow$

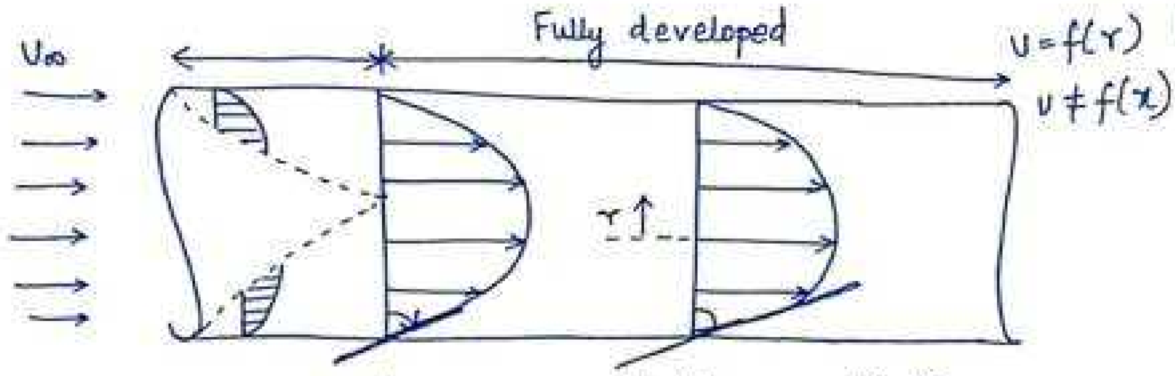
$$\boxed{C_{fx} = \frac{\tau_\omega \rightarrow N/m^2}{\frac{1}{2}\rho U_\infty^2 \rightarrow N/m^2}}$$

skin friction coefficient

- $U_\infty \rightarrow$ Free stream velocity

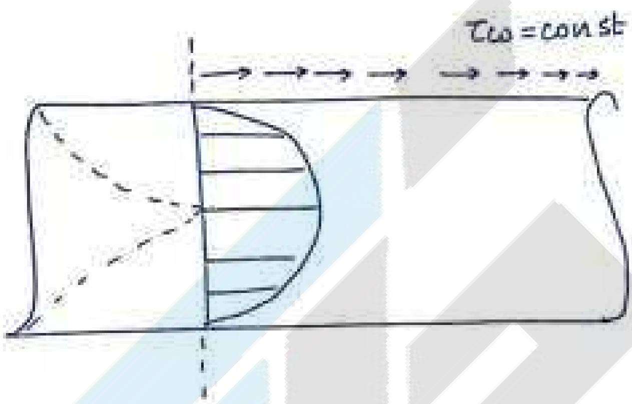
- $\frac{1}{2}\rho U_\infty^2 \rightarrow$ momentum

- $\tau_\omega \rightarrow$ wall shear stresses



$$\frac{\mu\, d\phi_1}{dt} = \frac{\mu\, d\phi_2}{dt} \qquad V_1 = V_2$$

$$\boxed{\tau_1 = \tau_2}$$



$$\boxed{f' = \frac{\tau_w}{\frac{1}{2}\rho V^2}}$$

mean velocity

$V \to$ Avg velocity

Darcy's friction coefficient

$$\boxed{f = 4f'}$$

friction factor

$$h_f = \frac{4f' L V^2}{2g D_h}$$

$$h_f = \frac{f L V^2}{2g\, D_h}$$

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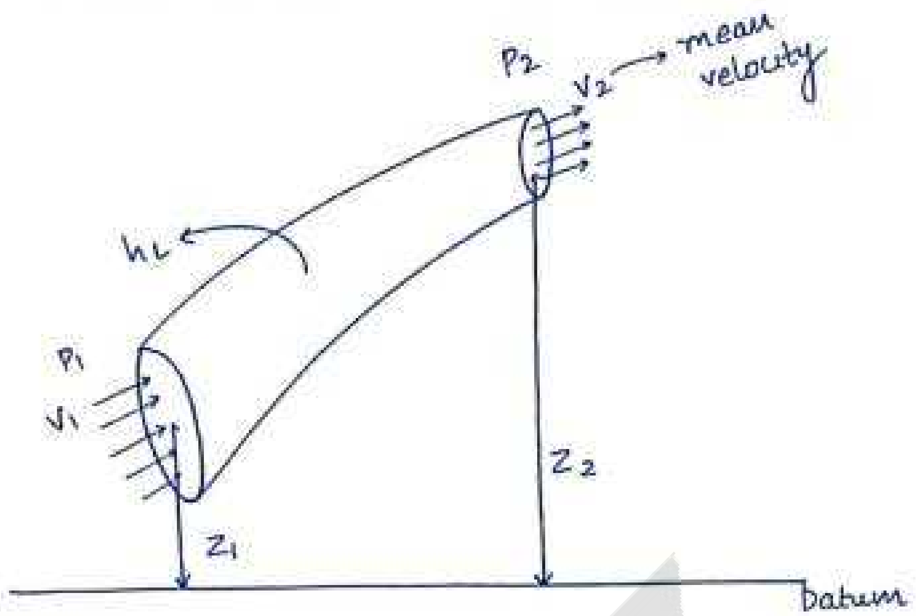
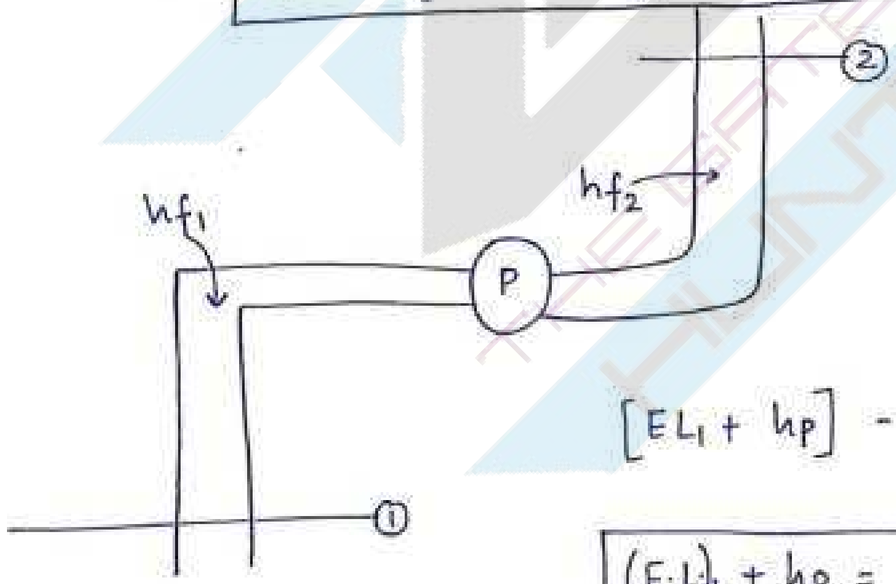
Modified Form of Bernoulli's equation :-



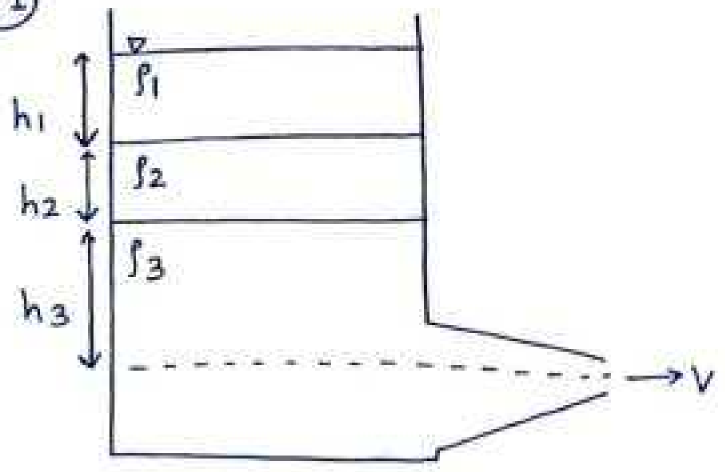
$$\Rightarrow \boxed{\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 + h_L}$$

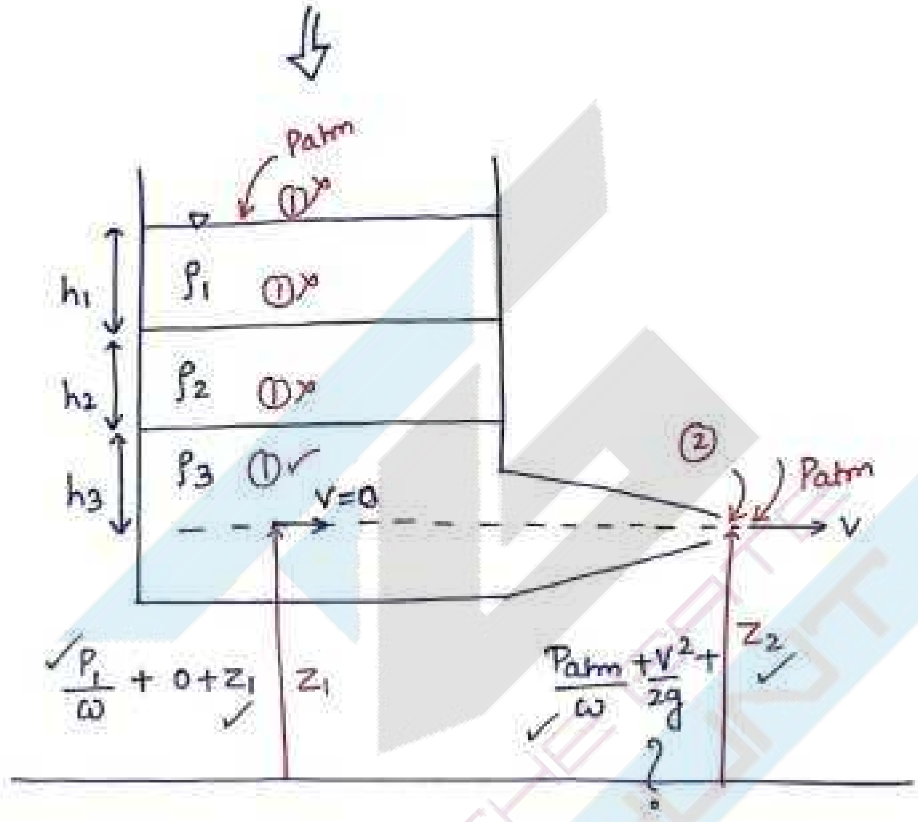
$$\Rightarrow \boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L}$$



$$[EL_1 + h_p] - (EL)_2 = h_f$$

$$\boxed{(E.L)_1 + h_p = (E.L)_2 + h_f}$$





insviscid fluids
↓
non viscous
$h_1 = h_2 = h_3$
(negligible change)

$$P_1 = P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3$$

1 & 2

$$\frac{\cancel{P_{atm}} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3}{\rho_3 g} + 0 + \cancel{z_1} = \frac{\cancel{P_{atm}}}{\cancel{\rho_3 g}} + \frac{V^2}{2g} + \cancel{z_2}$$

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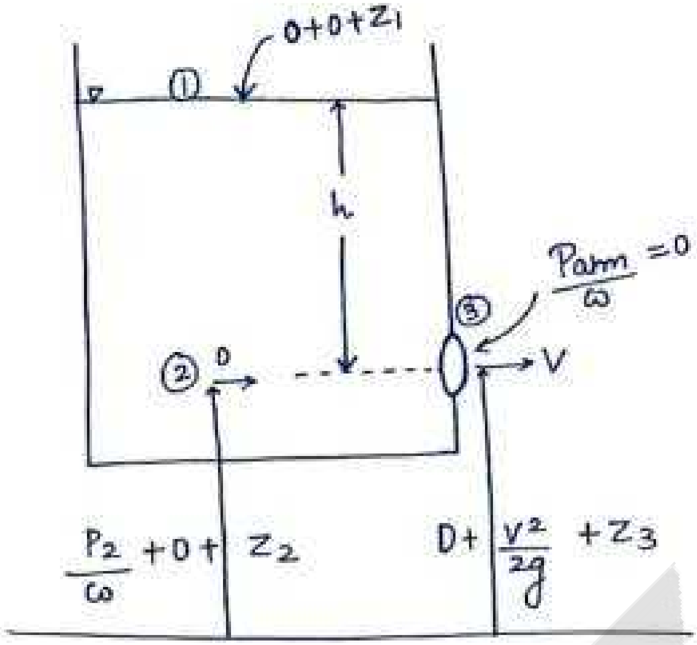
Note :-



① & ②

$$0+0+z_1 = \frac{P_2}{\omega} + 0 + z_2$$

$$\frac{P_2}{\omega} = (z_1 - z_2)$$

$$\boxed{\frac{P_2}{\omega} = h}$$

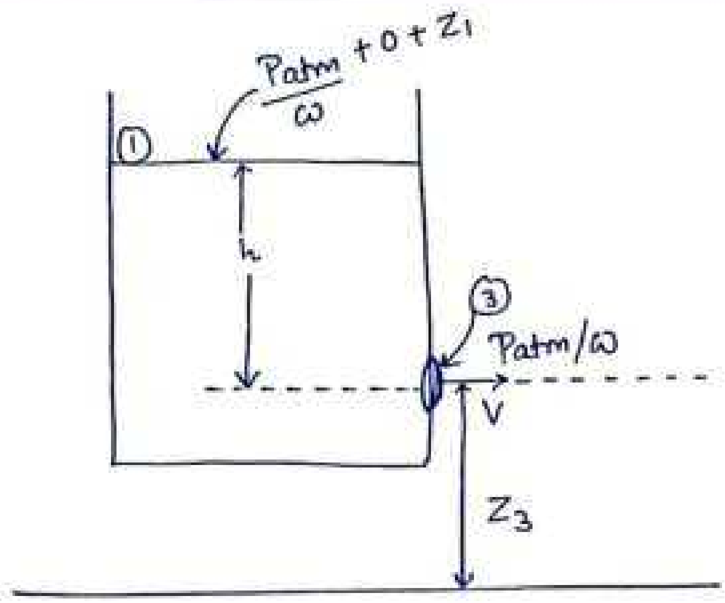
② & ③

$$\frac{P_2}{\omega} + 0 + \cancel{z_2} = 0 + \frac{v^2}{2g} + \cancel{z_3}$$

$$\frac{P_2}{\omega} = \frac{v^2}{2g}$$

$$h = \frac{v^2}{2g}$$

$$\boxed{V = \sqrt{2gh}}$$

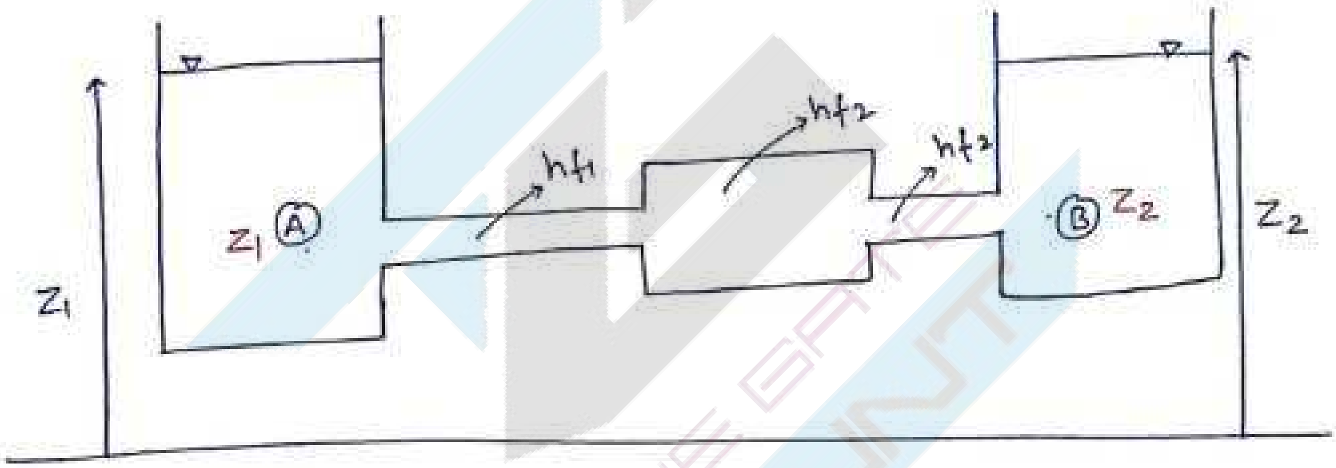


① & ③

$$\frac{\cancel{P_{atm}}}{\cancel{\omega}} + 0 + Z_1 = \frac{\cancel{P_{atm}}}{\cancel{\omega}} + \frac{V^2}{2g} + Z_3$$

$$\frac{V^2}{2g} = h$$

$$\boxed{V = \sqrt{2gh}}$$



$$EL_A = EL_B + h_{f_1} + h_{f_2} + h_3$$

$$z_1 = z_2 + h_f$$

$$\boxed{Z_1 - Z_2 = h_f}$$

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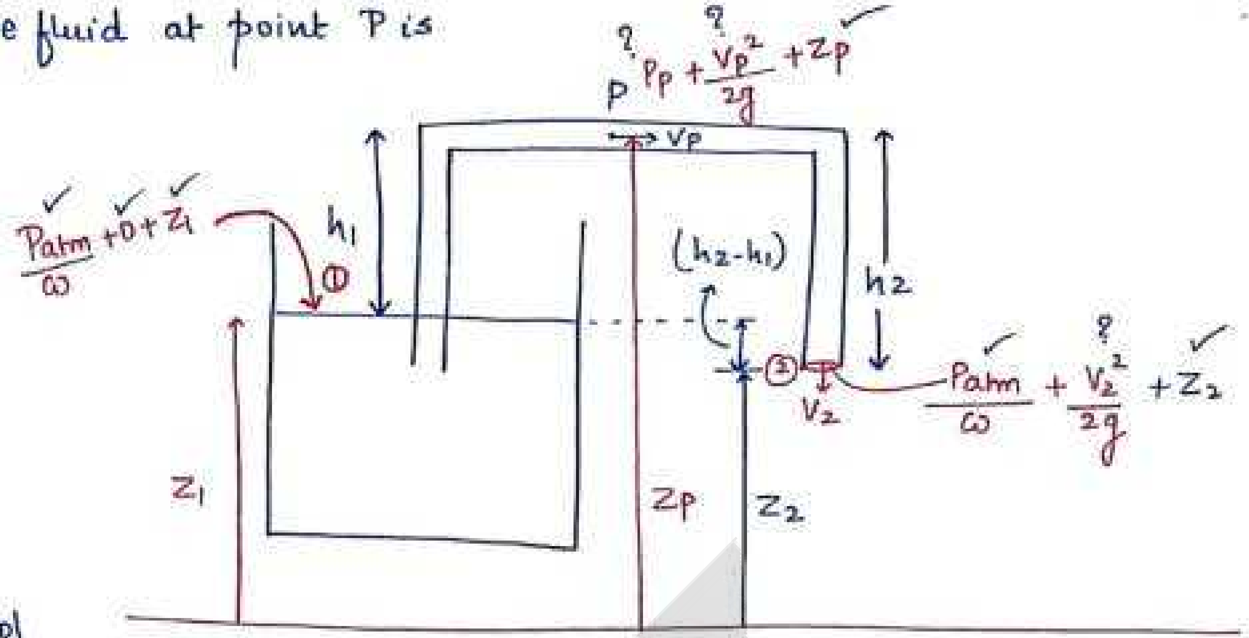
Q Pipe draws water from a Reservoir. for ideal fluid, the velocity of the fluid at point P is



Sol

① & ②

$$\frac{\cancel{P_{atm}}}{\cancel{\omega}} + 0 + Z_1 = \frac{\cancel{P_{atm}}}{\cancel{\omega}} + \frac{V_2^2}{2g} + Z_2$$

$$V_2 = \sqrt{2g(Z_1 - Z_2)}$$

$$V_2 = \sqrt{2g(h_2 - h_1)}$$

$$\cancel{A_p} V_p = \cancel{A_2} V_2$$

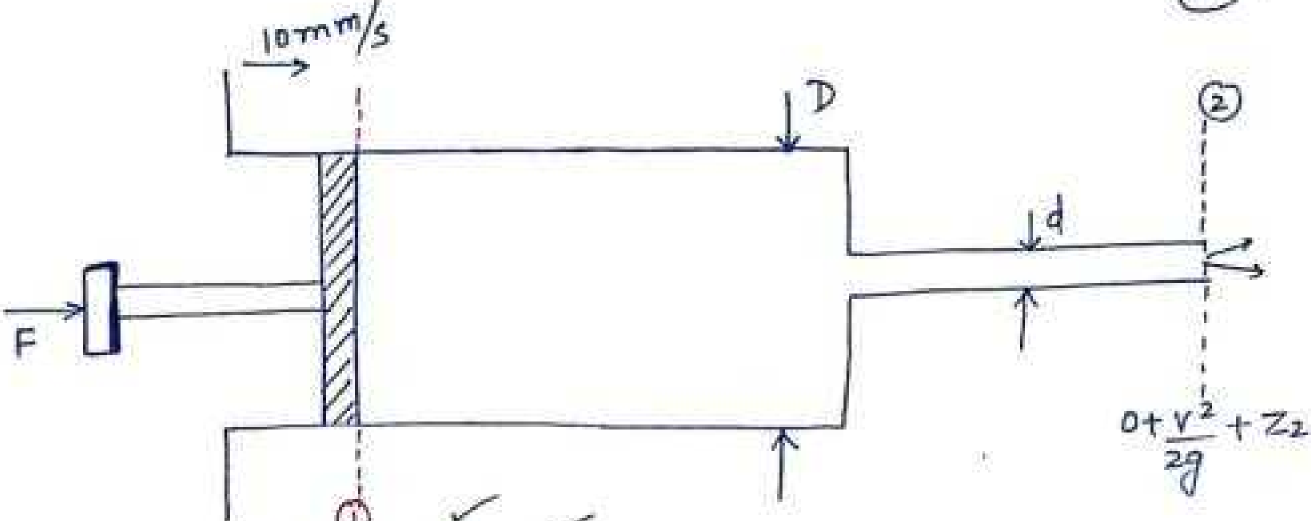
$$\boxed{V_p = \sqrt{2g(h_2 - h_1)}}$$

Pg 44

Q16

" N.P. "

MOHIT CHOUKSEY



$F = P_1 \times A$

$?\ \frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1$

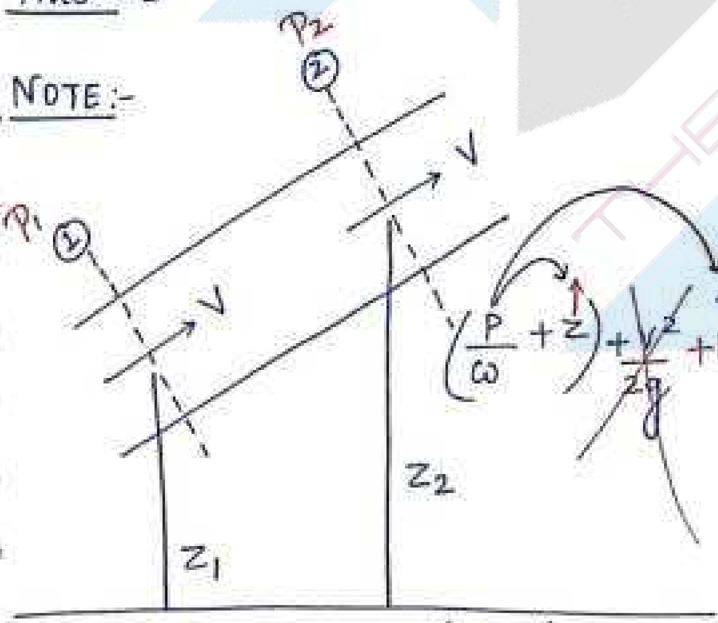
$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = P_2^{=0} + \frac{V_2^2}{2g} + Z_2$$

$P_1 = ?$

$$\boxed{F = P_1 A}$$

Ans → b

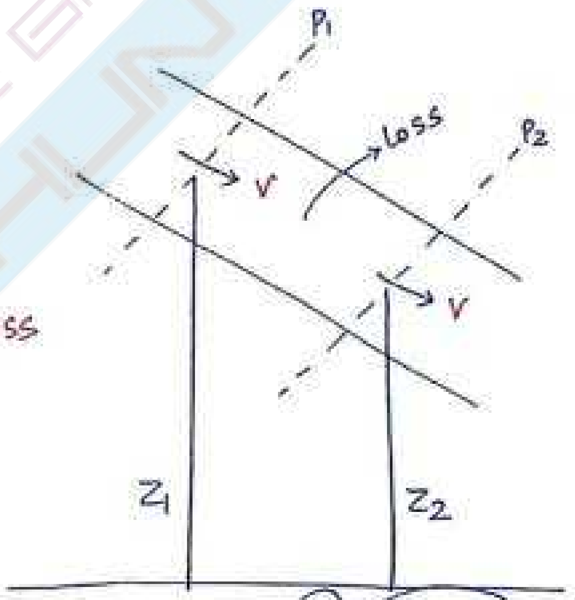
NOTE :-



$$\left(\frac{P}{\omega} + Z\right) + \cancel{\frac{V^2}{2g}} + \text{loss}$$

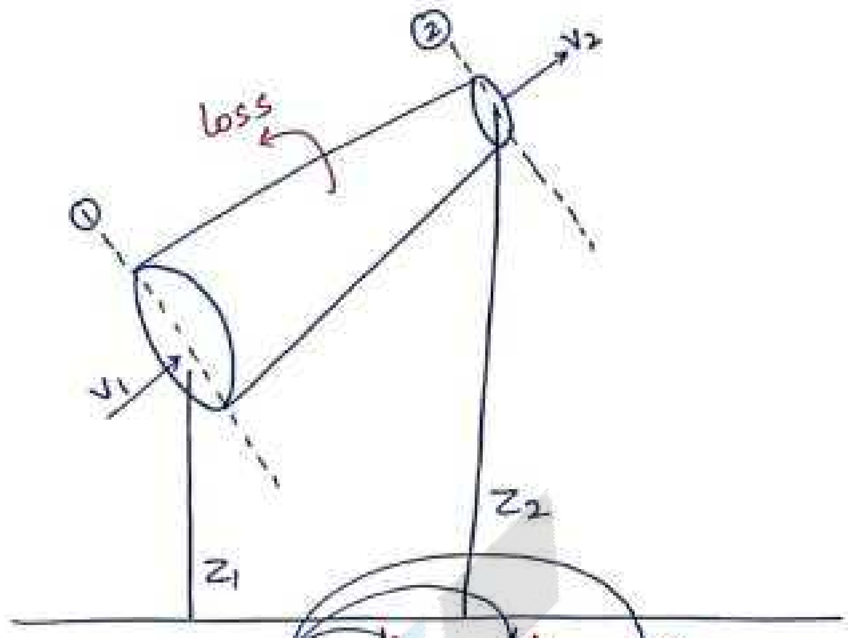
$\cancel{A_1} V_1 = \cancel{A_2} V_2$

$$\boxed{V_1 = V_2}$$

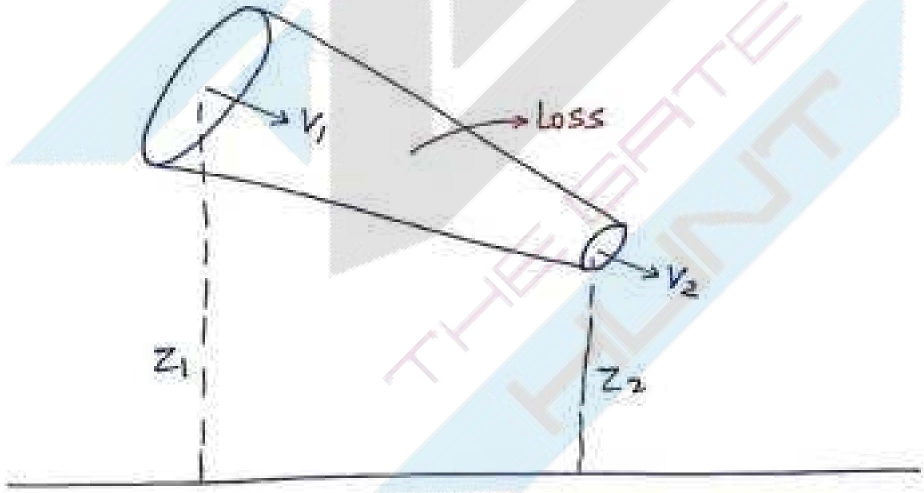


$$\left(\frac{P}{\omega} + Z\right) + \cancel{\frac{V^2}{2g}} + \text{loss}$$

MOHIT CHOUKSEY



$$\left(\frac{P}{\omega} + Z\right) + \frac{V^2}{2g} + \text{loss}$$



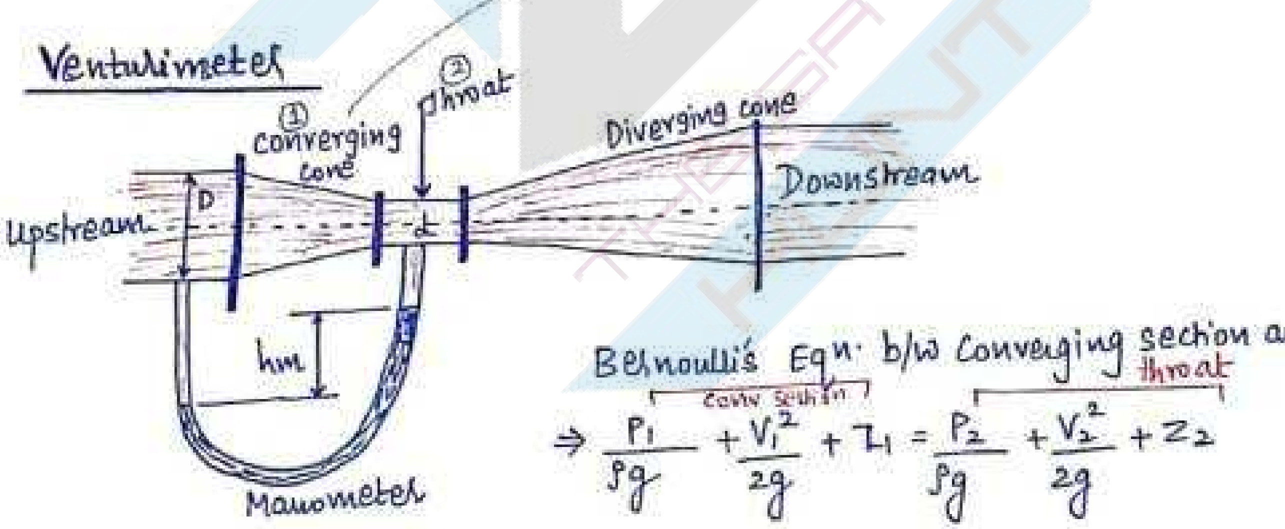
$$\left(\frac{P}{\omega} + Z\right) + \frac{V^2}{2g} + \text{loss}$$

If the expressions are in piezometric head, the expressions are independent of orientation of the pipe.

APPLICATION OF B.E.

- Discharge measurement
  - Venturimeter
  - Orificemeter
  - flow nozzle
- Velocity measurement
  - Pitot tube
- Pipeflow ✓
- Free vortex ✓

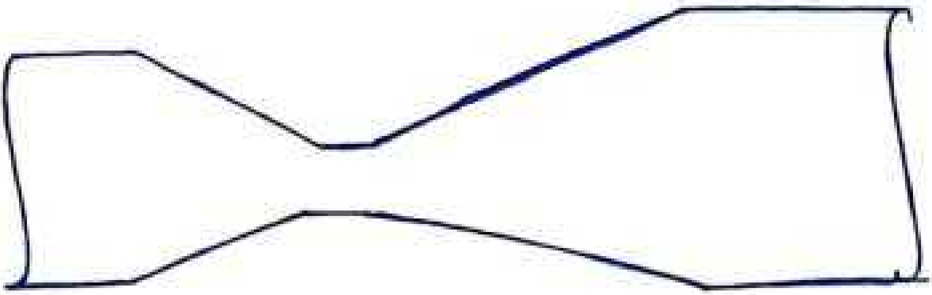
Venturimeter



Bernoulli's Eqn. b/w Converging section and ~~cone~~ throat,

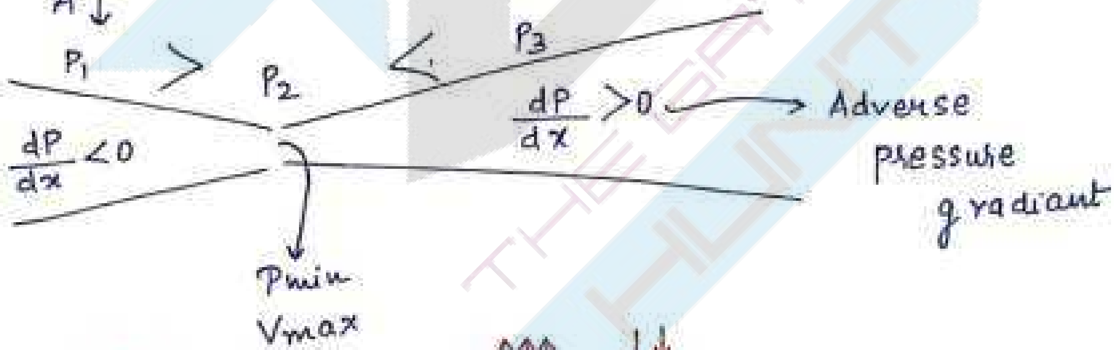
$$\Rightarrow \underbrace{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1}_{\text{conv section}} = \underbrace{\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2}_{\text{throat}}$$

$$\Rightarrow \underbrace{\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)}_{\text{Piezometric head difference}} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$



FLOW SEPARATION :-

$\uparrow V \propto \frac{1}{A \downarrow}$ $\quad\quad\quad\quad$ $\downarrow V \propto \frac{1}{A \uparrow}$

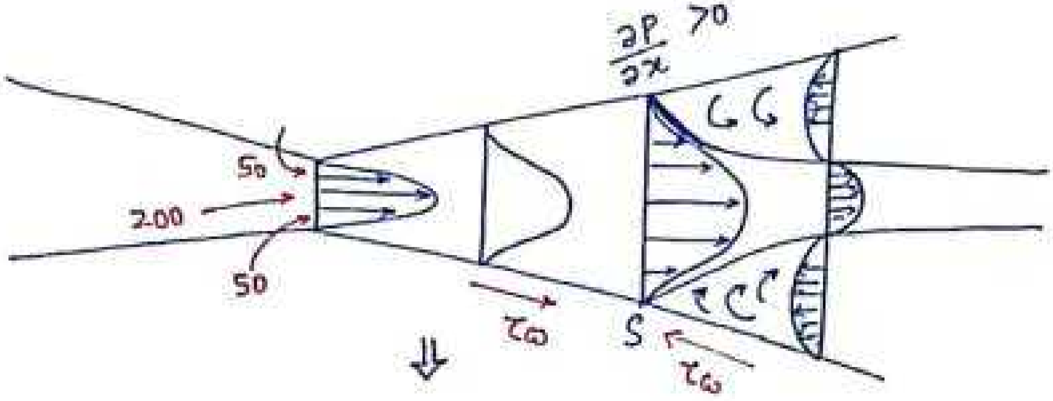


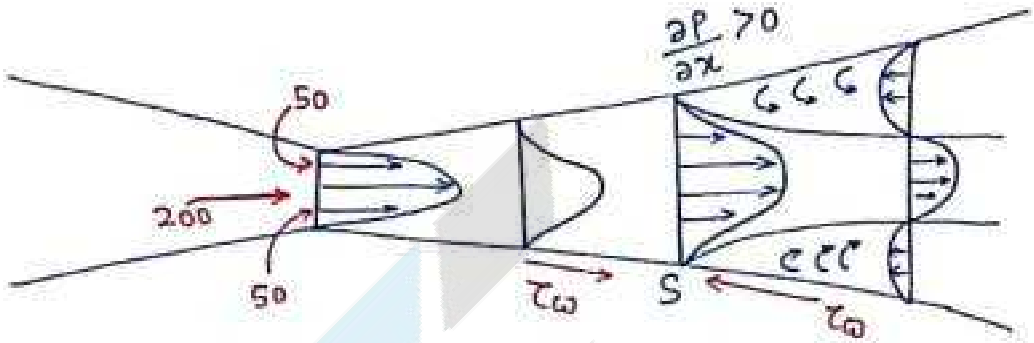
$\Rightarrow \overset{\downarrow\downarrow}{\frac{P}{\omega}} + \overset{\uparrow\uparrow}{\frac{V^2}{2g}} = C$ $\quad\quad\quad\quad$ $\Rightarrow \overset{\uparrow\uparrow}{\frac{P}{\omega}} + \overset{\downarrow\downarrow}{\frac{V^2}{2g}} = C$

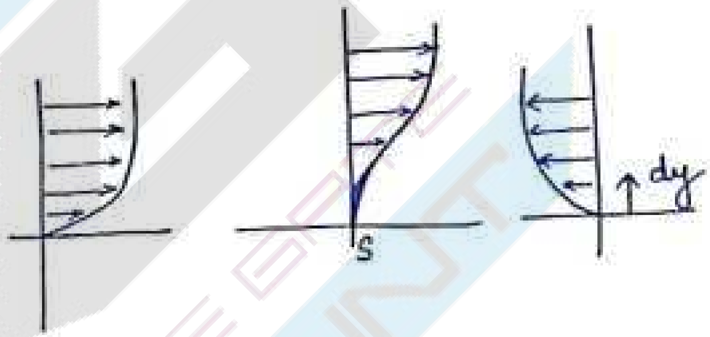
$\Rightarrow dP = P_2 - P_1$ $\quad\quad\quad\quad$ $\Rightarrow dP = P_3 - P_2$

$\Rightarrow dP < 0$ $\quad\quad\quad\quad$ $\Rightarrow dP > 0$

MOHIT CHOUKSEY

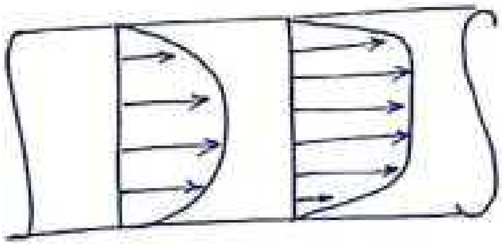


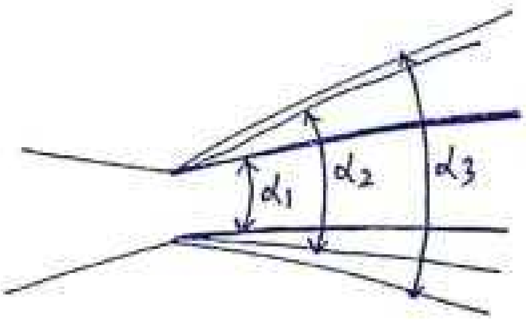


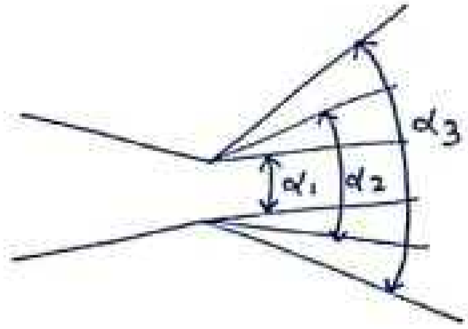


$\frac{du}{dy} > 0$ $\qquad$ $\frac{du}{dy} = 0$ $\qquad$ $\frac{du}{dy} < 0$

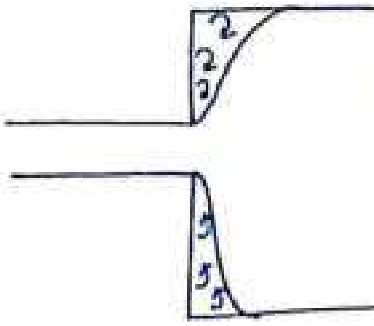
$\mu \frac{du}{dy}\Big|_{y=0} > 0$ $\qquad$ $\mu \frac{du}{dy}\Big|_{y=0} = 0$ $\qquad$ $\mu \frac{du}{dy}\Big|_{y=0} < 0$

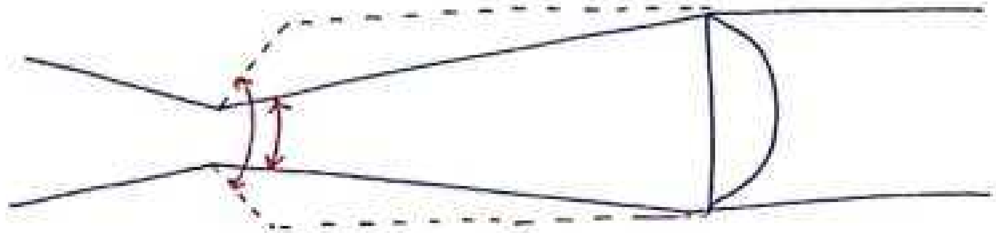


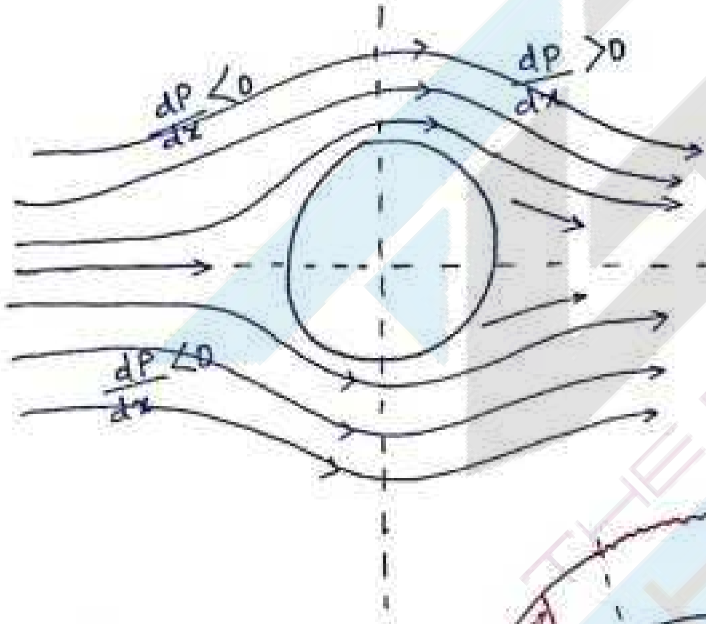




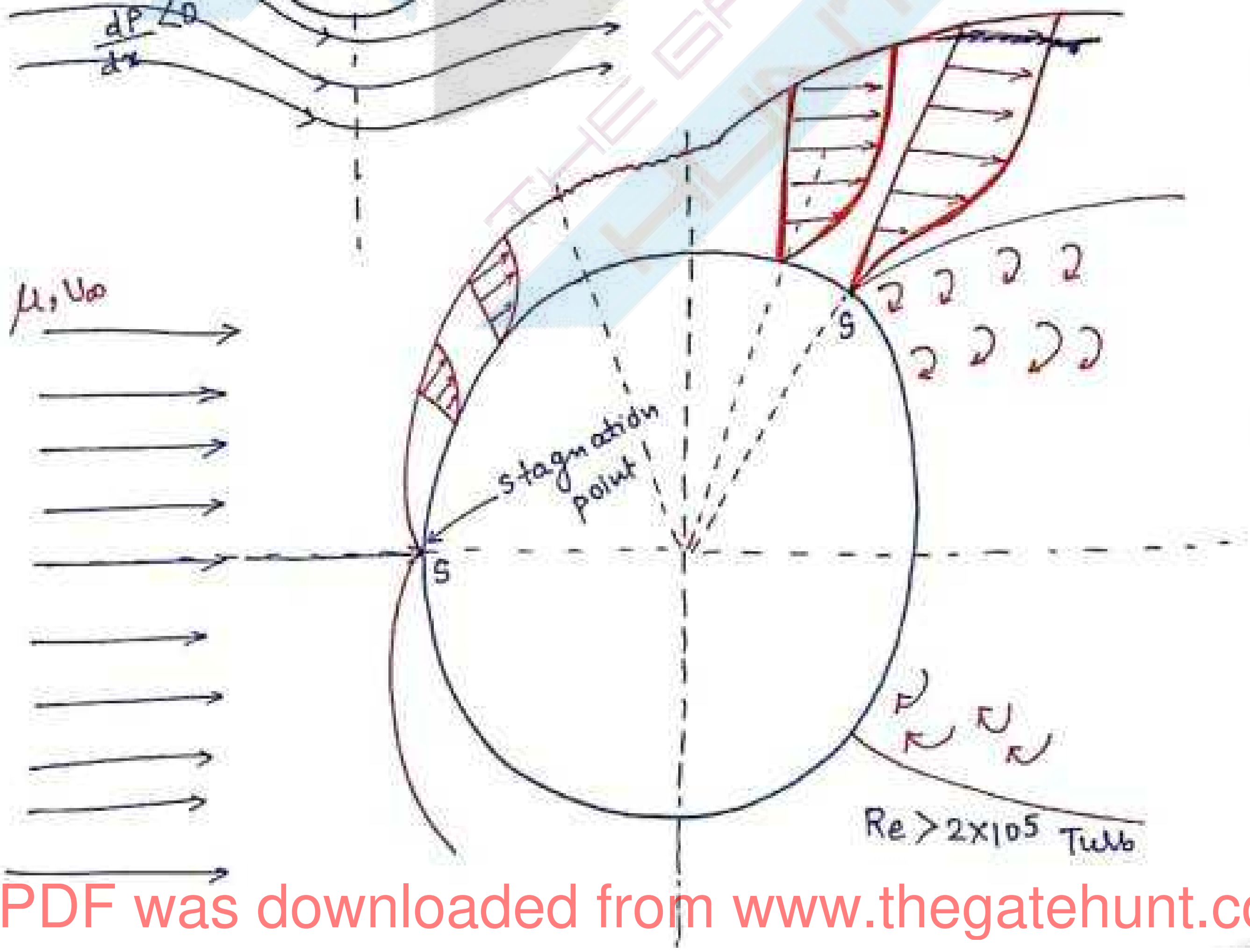
α1
α2
α3





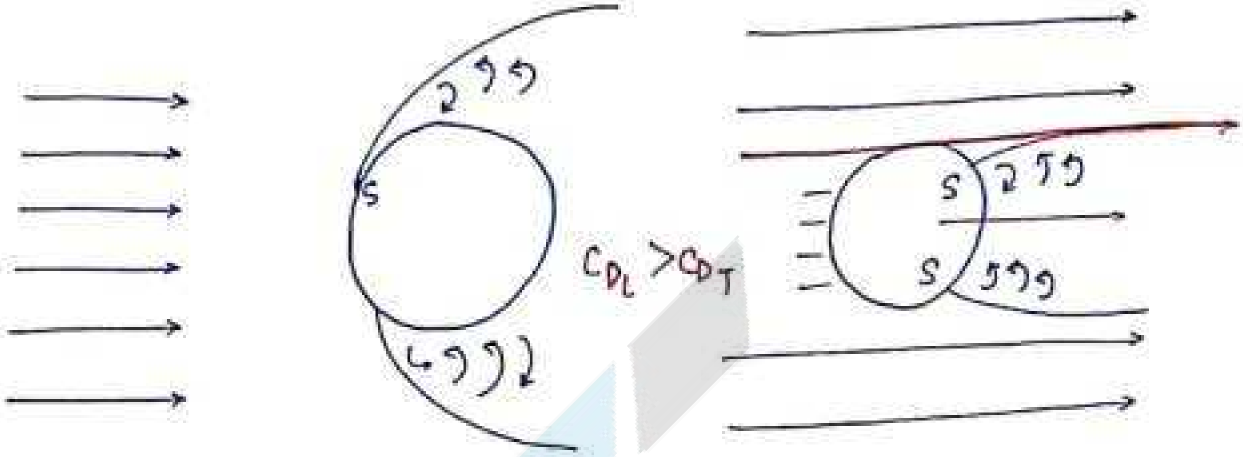


dP/dx < 0
dP/dx > 0
dP/dx < 0

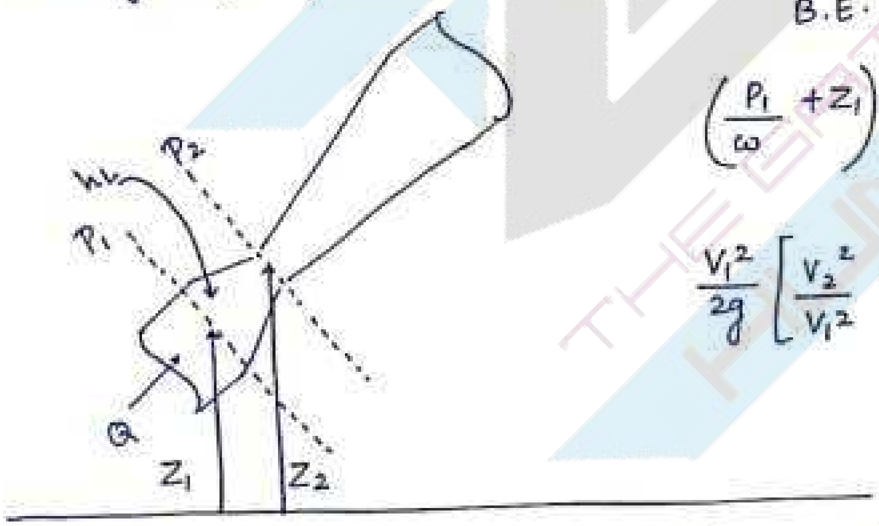


μ, U∞
Stagnation point
S
S
Re > 2x10^5 Turb

When fluid flows under adverse pressure condition, fluid looses its momentum near the surface, the particles has less momentum so they looses their momentum fasetly at the point where momentum ~~(?)~~ of the fluid particle become 0 and after which near the surfau, fluid moves under adverse pressure condition that is in ~~Reverse adverse~~ reverse ~~pressure~~ direction, that point is known as point of separation.



Discharge Eqn. for Venturimeter



B.E. ① & ②

$$\left(\frac{P_1}{\omega}+z_1\right)+\frac{V_1^2}{2g} = \left(\frac{P_2}{\omega}+z_2\right)+\frac{V_2^2}{2g}+h_L$$

$$\frac{V_1^2}{2g}\left[\frac{V_2^2}{V_1^2}-1\right] = \left(\frac{P_1}{\omega}+z_1\right)-\left(\frac{P_2}{\omega}+z_2\right) - h_L$$

$$\frac{V_1^2}{2g}\left[\frac{A_1^2}{A_2^2}-1\right] = h - h_L$$

Piezometric head difference

$\because A_1V_1 = A_2V_2$

$\Rightarrow \frac{V_2}{V_1} = \frac{A_1}{A_2}$

$\Rightarrow \frac{V_2^2}{V_1^2} = \frac{A_1^2}{A_2^2}$

$\boxed{Q = \frac{dV}{A}}$

Actual Velocity $V_1 = \frac{A_2\sqrt{2g(h-h_L)}}{\sqrt{A_1^2 - A_2^2}}$

$$A_1V_1 = \frac{A_1A_2\sqrt{2g(h-h_L)}}{\sqrt{(A_1^2-A_2^2)}}$$

$$\boxed{Q_{act} = \frac{A_1A_2\sqrt{2g(h-h_L)}}{\sqrt{A_1^2-A_2^2}}}$$

if $h_L \rightarrow$ dont use C_d

$h_L = 0$

$$\boxed{Q_{theo} = \frac{A_1A_2\sqrt{2gh}}{\sqrt{A_1^2-A_2^2}}}$$

This expression is independent of orientation of Venturimeter.

$$\boxed{C_d = \frac{Q_{act}}{Q_{th}}}$$

coefficient of discharge

$$\boxed{Q_{act} = \frac{C_d A_1A_2\sqrt{2gh}}{\sqrt{A_1^2-A_2^2}}}$$

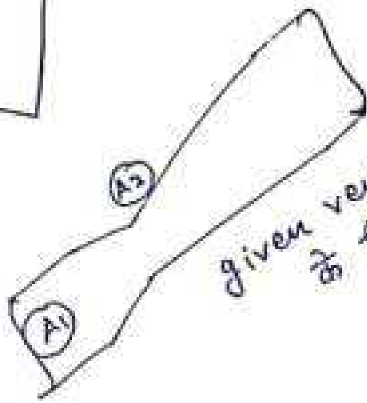
if $C_d \rightarrow$ dont use h_L

[$C_d = 0.96 - 0.98$ more accuate]

$$\frac{C_d \cancel{A_1A_2}\sqrt{\cancel{2g}h}}{\cancel{\sqrt{A_1^2-A_2^2}}} = \frac{\cancel{A_1A_2}\sqrt{\cancel{2g}(h-h_L)}}{\cancel{\sqrt{A_1^2-A_2^2}}}$$

$$\boxed{C_d = \sqrt{\frac{(h-h_L)}{h}}}$$

$$Q_{th} = \frac{A_1A_2\sqrt{2gh}}{\sqrt{A_1^2-A_2^2}}$$



given venturimeter के लिए A_1, A_2 constant

$$\boxed{Q_{th} = c\sqrt{h}}$$

$c \rightarrow$ constant of venturimeter

$$C_d = \frac{Q_{act}}{Q_{th}}$$

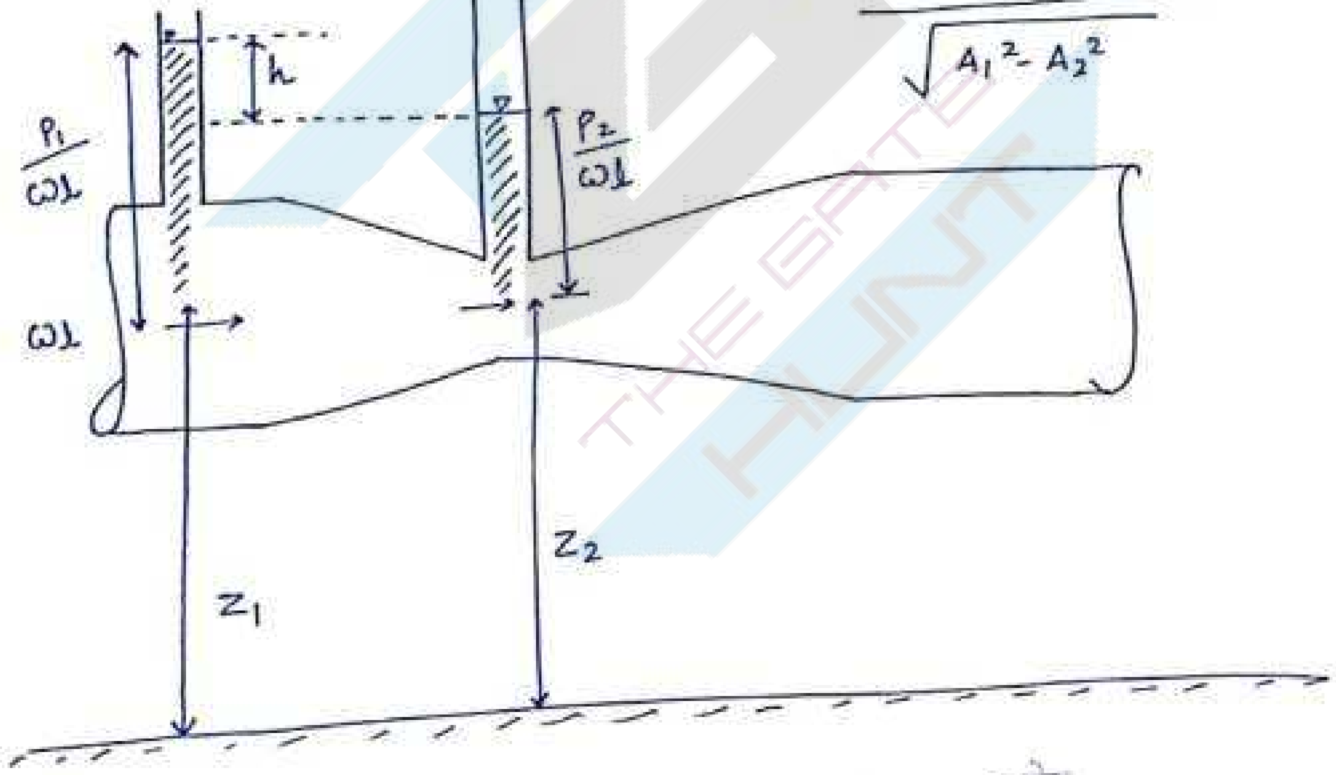
$$\boxed{Q_{act} = C_d \cdot C \cdot \sqrt{h}} \longrightarrow \text{Piezometric head difference}$$

$$C_d \longrightarrow \begin{cases} Re \\ \text{surface Roughness} \\ \frac{A_1}{A_2} > 1 \ \text{(area Ratio)} \end{cases}$$

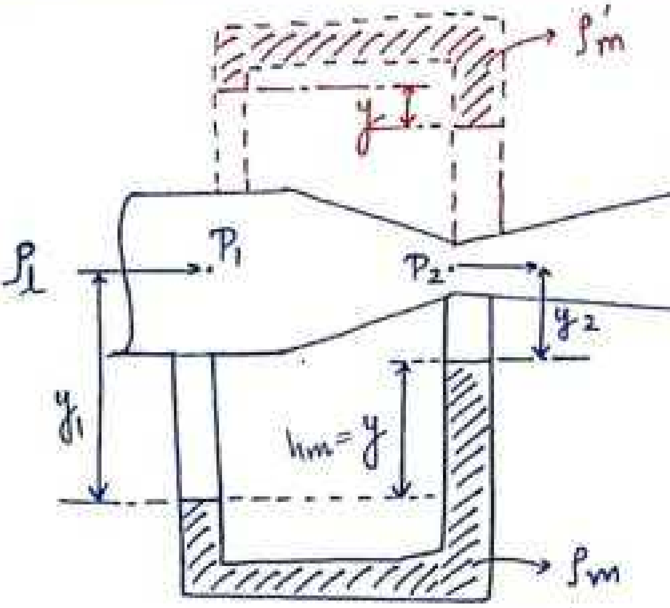
$\rightarrow Re \uparrow \longrightarrow$ adverse $P \uparrow \rightarrow \downarrow\downarrow$?? $\rightarrow$ discharge ability $\uparrow\uparrow$

* Relationship between Piezometric head difference and manometric Deflection :-

$$Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$



ये diagram Piezometric head difference को
दिखाता है सिर से



$$\boxed{h = \left(1 - \frac{\rho_m'}{\rho_l}\right) y}$$

$z_1 = z_2$

Case 1 $\rho_m > \rho_l$

$$P_1 + \rho_l g y_1 - \rho_m g y - \rho_l g y_2 = P_2$$ Pascal's law लगाया है ~~Energy/volume (N/m²)~~

$$P_1 - P_2 = \rho_m g y - \rho_l g [y_1 - y_2]$$

$$\Rightarrow P_1 - P_2 = \left(\frac{\rho_m g y}{\rho_l g} - 1\{\underbrace{y_1 - y_2}_{y}\}\right) \rho_l g$$

$$\frac{P_1}{\rho_l g} - \frac{P_2}{\rho_l g} = \left(\frac{\rho_m}{\rho_l} - 1\right) y$$

$$\Rightarrow \frac{P_1}{\rho_l g} - \frac{P_2}{\rho_l g} = \left(\frac{\rho_m}{\rho_l} y - y\right) = \left(\frac{\rho_m}{\rho_l} - 1\right) y$$

$$\left(\frac{P_1}{\omega_l} + z_1\right) - \left(\frac{P_2}{\omega} + z_2\right) = \left(\frac{\rho_m}{\rho_l} - 1\right) y$$

$$\boxed{h = \left(\frac{\rho_m}{\rho_l} - 1\right) y}$$ → manometric deflection

Piezometric head difference

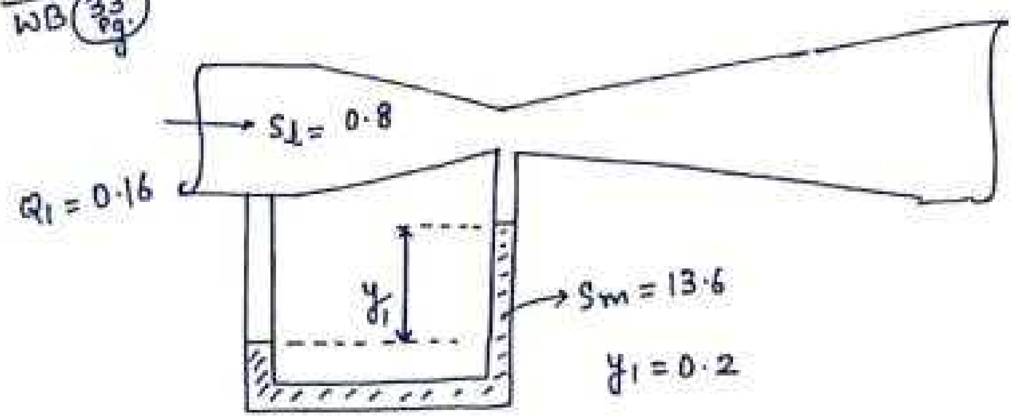
$$\boxed{h = \left(\frac{\omega_m}{\omega_l} - 1\right) y} \quad \text{OR} \quad \boxed{h = \left(\frac{S_m}{S_l} - 1\right) y}$$

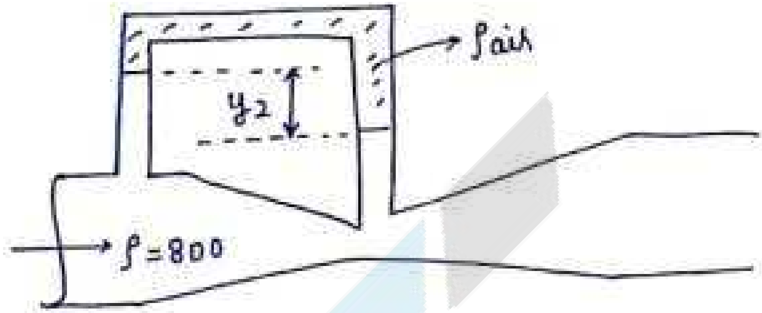
Case 2 $\rho_m < \rho_l$

$$h = \left(1 - \frac{\rho_m}{\rho_l}\right) h_m = \left(1 - \frac{\rho_m}{\rho_l}\right) y$$

$$h = \left(1 - \frac{S_m}{S_l}\right) h_m = \left(1 - \frac{\omega_m}{\omega_l}\right) y = \left(1 - \frac{S_m}{S_l}\right) y = \left(1 - \frac{\omega_m}{\omega_l}\right) h_m$$

T2
WB (33 pg.)





$Q_2 = 0.08\ m^3/s$

$Q_1 = 2Q_2$

$$\frac{\cancel{C_d}\,\cancel{A_1A_2}\sqrt{2gh_1}}{\cancel{\sqrt{A_1^2-A_2^2}}} = \frac{2\cancel{C_d}\,\cancel{A_1A_2}\sqrt{2gh_2}}{\cancel{\sqrt{A_1^2-A_2^2}}}$$

$$h_1 = 4h_2$$

$$\left(\frac{S_m}{S_L}-1\right)y_1 = 4\left[\left(1-\frac{\rho_{air}}{\rho_L}\right)\right]y_2$$

$$\left(\frac{13.6}{0.8}-1\right)\times 0.2 = 4\left[1-\frac{1.23}{800}\right]y_2$$

$$y_2 = ?$$

2. Vent. is installed in a horizontal pipe line of 400 mm diameter. The throat diam. is 1/3rd of pipe diam. Water flows through the pipe, the pressure in the pipeline is 1.405 kgf/cm² and the

vacuum in the throat is 37.5 cm of Hg. if 4% of differential head is lost between gauges then find the discharge (155)

Sol

$D_1 = 400\,mm = 0.4\,m$

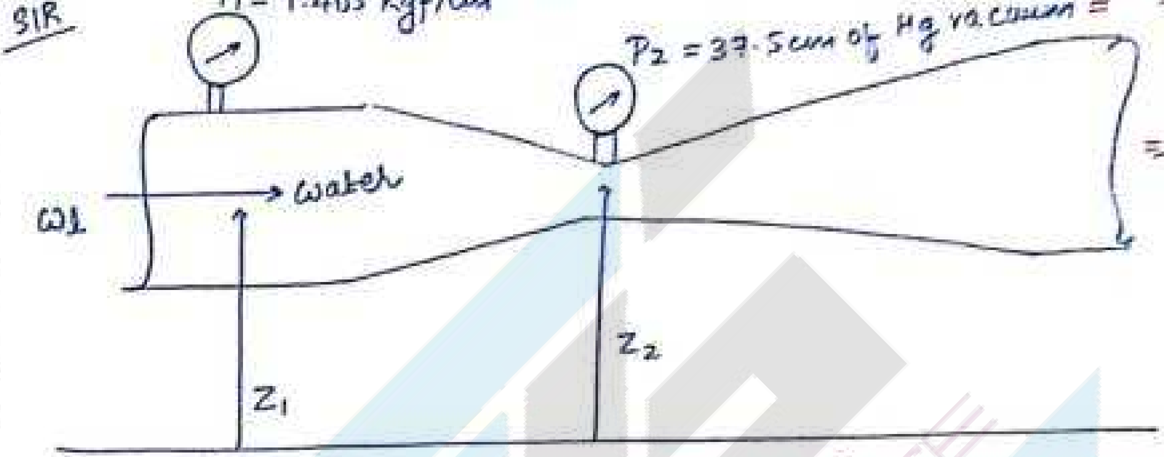
$A_1 V_1 = A_2 V_2$

$D_2 = \frac{1}{3} D_1$

$P_1 = 1.405\ kgf/cm^2$

$P_2 = 37.5\,cm$ of Hg

SIR



no need to make absolute pressure (conversion time pass)

$A_1 = \frac{\pi}{4} d_1^2$ $\qquad$ $A_2 = \frac{\pi}{4} d_2^2$

$$Q = \frac{A_1 A_2 \sqrt{2g(h - h_L)}}{\sqrt{A_1^2 - A_2^2}}$$

$h_L = 0.04h$

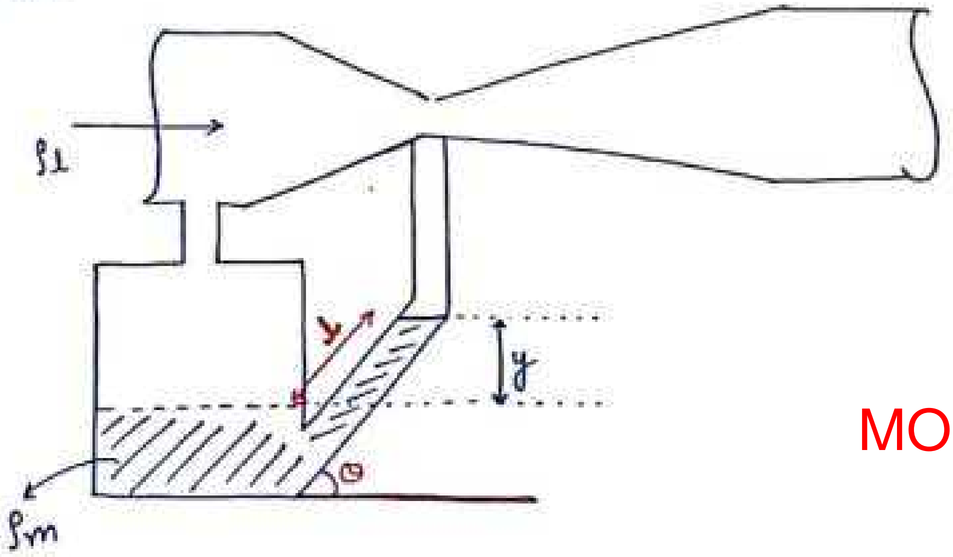
MOHIT CHOUKSEY

$$h = \frac{P_1}{\omega l} - \frac{P_2}{\omega l}$$

$$h = \frac{1.405 \times 10^5 - (-37.5 \times 10^{-3} \times 13.6 \times 9810)}{9810}$$

$h \rightarrow$ $\boxed{0.258 = Q \ m^3/s}$ Ans

Note :-

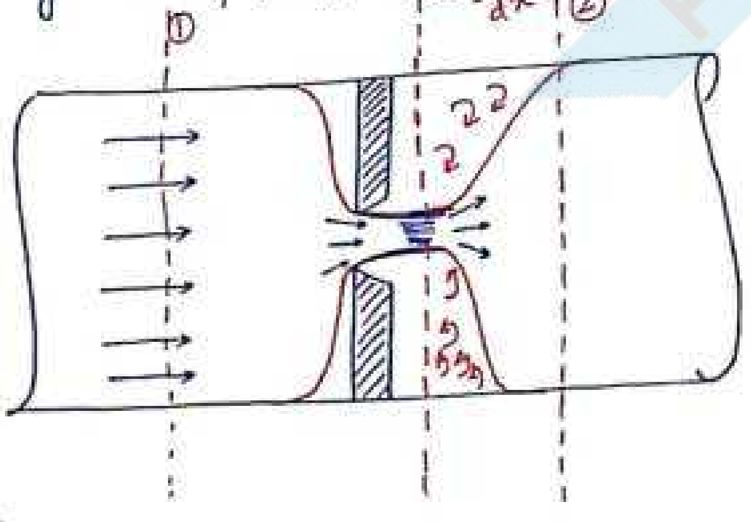


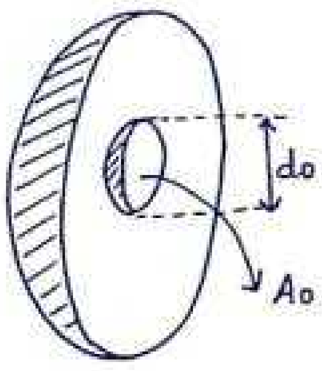
$$Q_{act} = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$\boxed{h = \left(\frac{\rho_m}{\rho_l} - 1\right) y} \qquad \boxed{y = l \sin\theta}$$

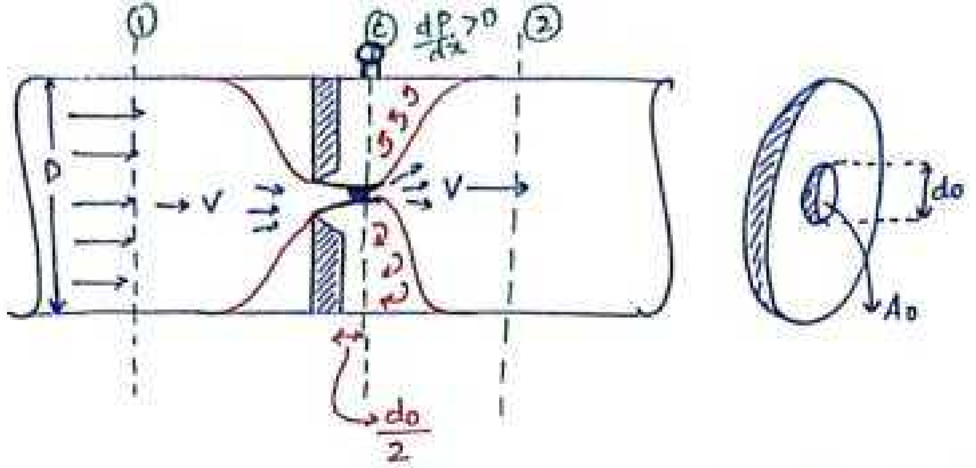
less accurate than Venturimeter (vent. much expensive device than orificemeter)

Orificemeter :- It is a circular plate with concentric sharp edge hole at the centre, due to formation of vena contracta, high losses takes place so less values of Cd (0.62 to 0.64). It is used in tank to openings and pipe flow





we can apply the hydrostatic law in flow fields when streamlines are parallel



no loss in velocity from ① to ②, datum same
hence loss compensated by Pressure

B.E. ① and ②

$$\frac{V_1^2}{2g}\left[\frac{A_1^2}{A_c^2} - 1\right] = \left[h - h_L\right]$$

$$\Rightarrow (h - h_L) = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_c}{\rho g} + z_c\right) = \frac{V_c^2}{2g} - \frac{V_1^2}{2g}$$

$$\boxed{A_1 V_1 = A_c V_c}$$

$$\Rightarrow V_1 = \frac{\sqrt{2g(h - h_L)}}{\sqrt{\frac{A_1^2}{C_c^2 A_0^2} - 1}}$$

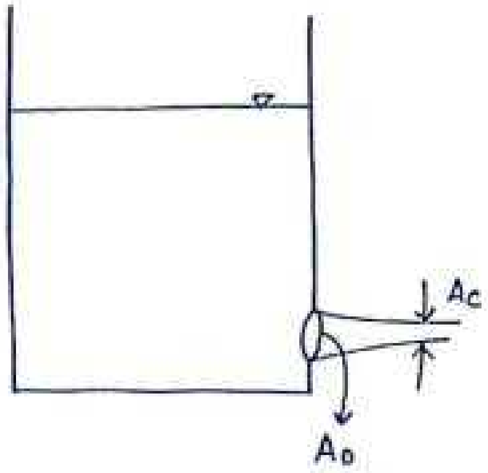
$$A_1 V_1 = \frac{A_1 A_0 \sqrt{2g}\sqrt{h}}{\sqrt{A_1^2 - A_0^2}} \quad \frac{\sqrt{h - h_L}}{\sqrt{h}} \quad \frac{\sqrt{A_1^2 - A_0^2}}{\sqrt{\frac{A_1^2}{C_c^2} - A_0^2}}$$

C ⇒ coeff. of orifice

$$C \approx C_{do}$$

$$\boxed{Q_{act} = \frac{C_{do} A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}}$$

A_0 → area of orificeplate diameter

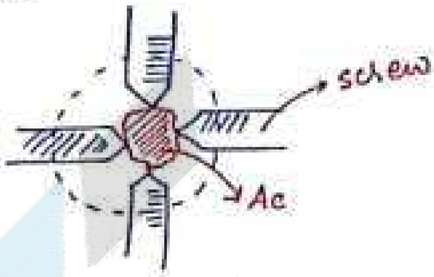


- $C_v = \dfrac{V_{act}}{V_{th}}$
- $C_c = \dfrac{A_c}{A_{th}}$
- $C_d = C_c \times C_v = \dfrac{V_{act}\,A_c}{A_{th}\,V_{th}}$

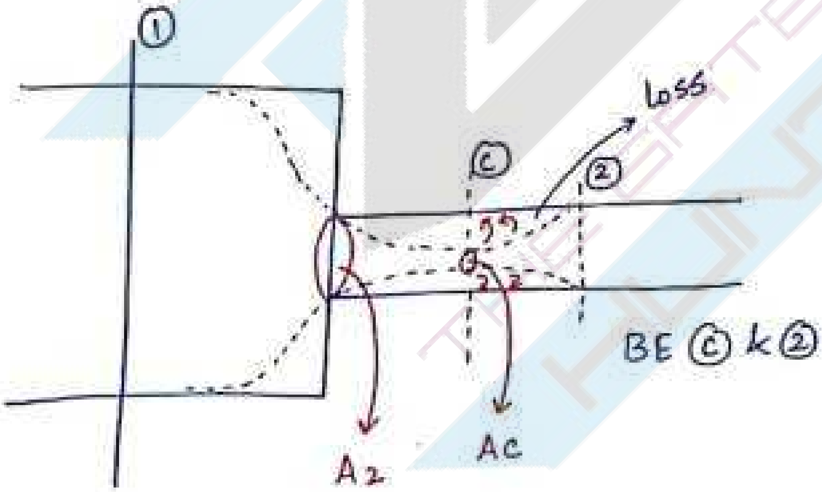
also $C_d = \dfrac{Q_{act}}{Q_{theo}}$

$C_c = \dfrac{A_c \rightarrow \text{contraction}}{A_0 \rightarrow \text{orifice}}$

coeff. of contraction



$A_c = C_c A_0$



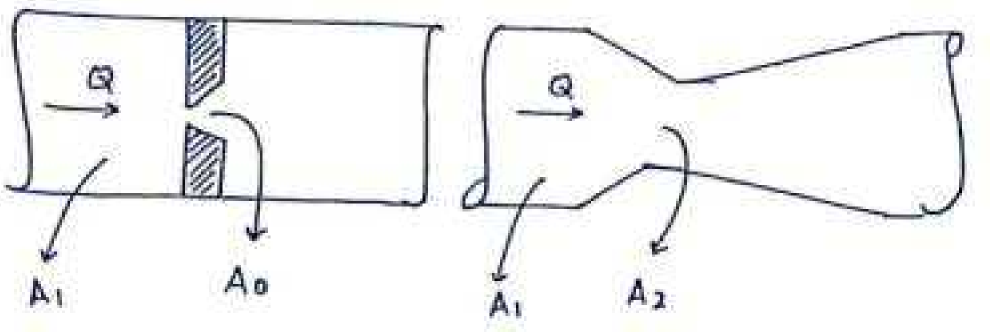
$$\boxed{C_c = \frac{A_c}{A_2}}$$

MOHIT CHOUKSEY

WB
(17) Pg 30

$C_d = 0.61 \rightarrow$ O.M.

$C_d = 0.98 \rightarrow$ V.M.



$C_{d_0} = 0.61$ $\quad$ $A_0 = A_2$ $\quad$ $C_{d_V} = 0.98$

$$Q_V = Q_0$$

$$\frac{C_{dV}\, \cancel{A_1 A_2}\sqrt{2gh_V}}{\cancel{\sqrt{A_1^2 - A_2^2}}} = \frac{C_{d0}\, \cancel{A_1 A_0}\sqrt{2gh_0}}{\cancel{\sqrt{A_1^2 - A_0^2}}}$$

$$\frac{h_V}{h_0} = \left(\frac{C_{d0}}{C_{dV}}\right)^2 = \left(\frac{0.61}{0.98}\right)^2$$

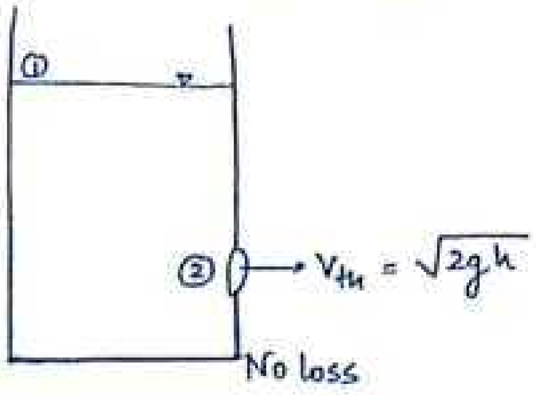
$$\frac{\left[\left(\frac{P_1}{\omega} + Z_1\right) - \left(\frac{P_2}{\omega} + Z_2\right)\right]_V}{\left[\left(\frac{P_1}{\omega} + Z_1\right) - \left(\frac{P_2}{\omega} + Z_2\right)\right]_0} = \frac{(0.61)^2}{(0.98)^2}$$

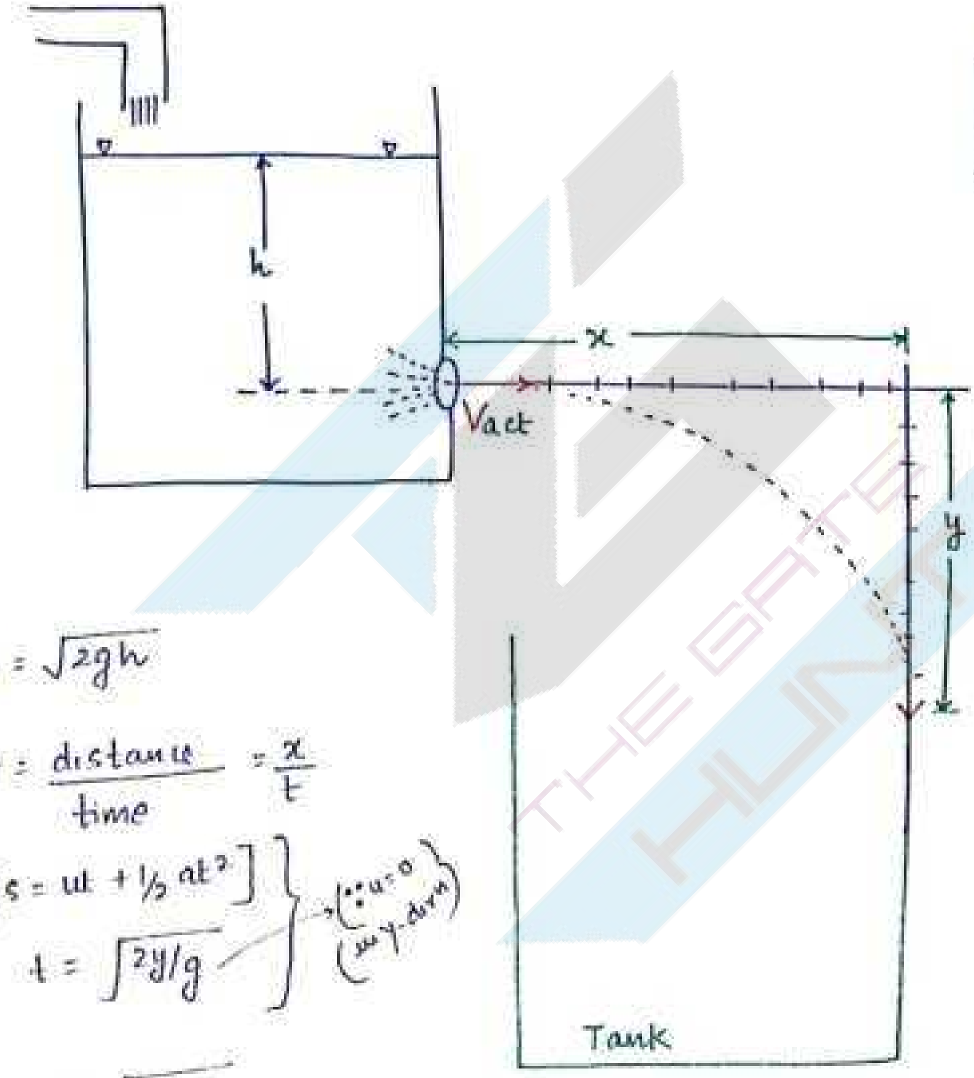
$Z_1 = Z_2$ $\quad$ Horizontal pipeline

$$\frac{(P_1 - P_2)_V}{(P_1 - P_2)_0} = \frac{(0.61)^2}{(0.48)^2}$$

Read Qn. 31

* Measurement of coefficient of velocity of orifice meter :-





$$\boxed{C_V = \frac{V_{act}}{V_{th}}}$$

$V_{th} = \sqrt{2gh}$

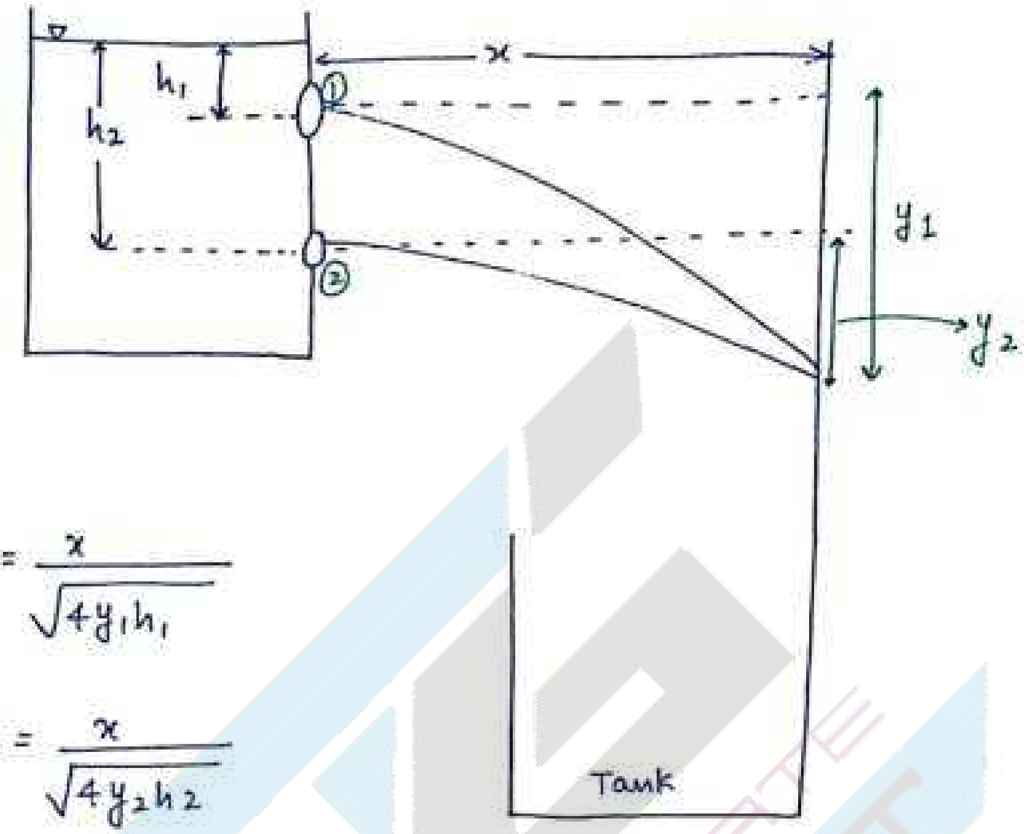
$V_{act} = \dfrac{\text{distance}}{\text{time}} = \dfrac{x}{t}$

$$\left\{ \begin{array}{l} \left[s = ut + \tfrac{1}{2} a t^2 \right] \\ t = \sqrt{2y/g} \end{array} \right\} \left(\begin{array}{l} \because u = 0 \\ \text{in y-dir}^n \end{array} \right)$$

$V_{act} = x \sqrt{g/2y}$

$$C_V = \frac{x\sqrt{g/2y}}{\sqrt{2gh}} = \frac{x}{\sqrt{4yh}}$$

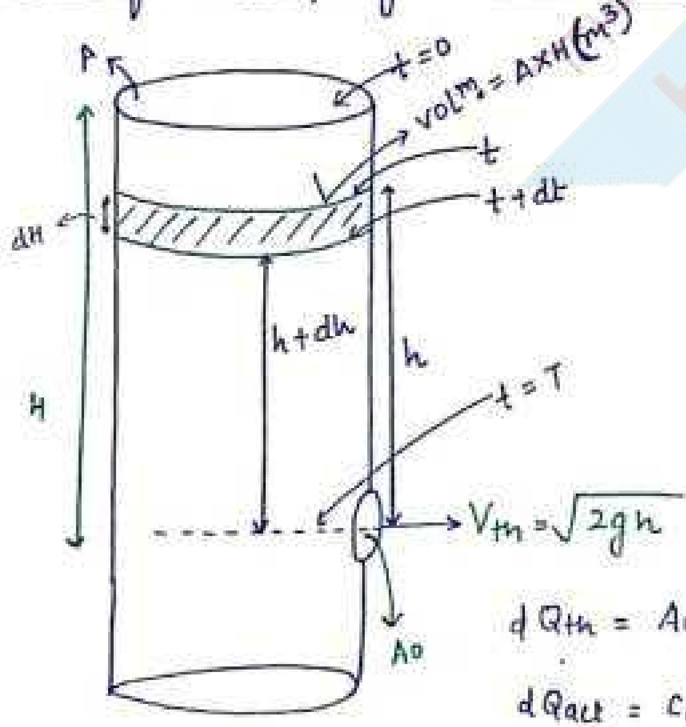
$$\boxed{C_V = \frac{x}{\sqrt{4yh}}}$$



$$C_{V_1} = \frac{x}{\sqrt{4y_1h_1}}$$

$$C_{V_2} = \frac{x}{\sqrt{4y_2h_2}}$$

* Time of ~~...~~ empting a tank through orificemeter :→



volm of liquid displaced in tank in time 'dt' = $-Adh$ —(1)

(+ve) $\quad$ $-(-ve)$

$$dQ_{th} = A_o\sqrt{2gh} \ m^3/s$$

$$dQ_{act} = C_d \, Q_{th} = C_d A_o \sqrt{2gh}$$

$h + dh < h$

$dh < 0$

volm. of liq. displaced in tank in time 'dt' $= -Adh$ —(1)

(+ve) $\qquad$ $-(-ve)$

discharge rate through orifice

$$dQ = C_d A_0 \sqrt{2gh} \quad m^3/s$$

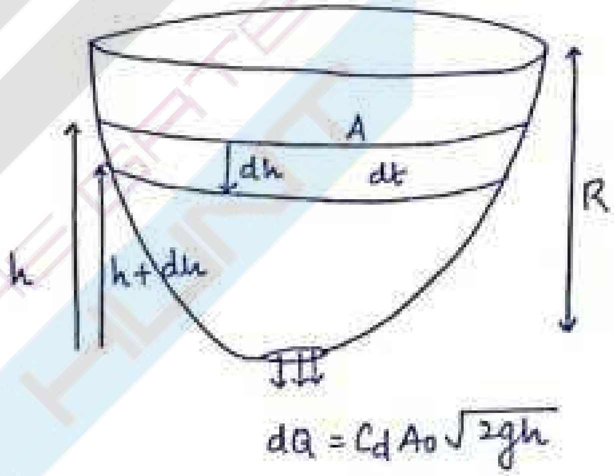
Displaced fluid by orifice in time

$$\text{'}dt\text{'} = dQ \cdot dt \quad m^3$$

$$C_d A_0 \sqrt{2gh}\, dt = -Adh$$

$$\int_0^T dt = \frac{-A}{C_d A_0 \sqrt{2g}} \int_H^0 \frac{dh}{\sqrt{h}}$$

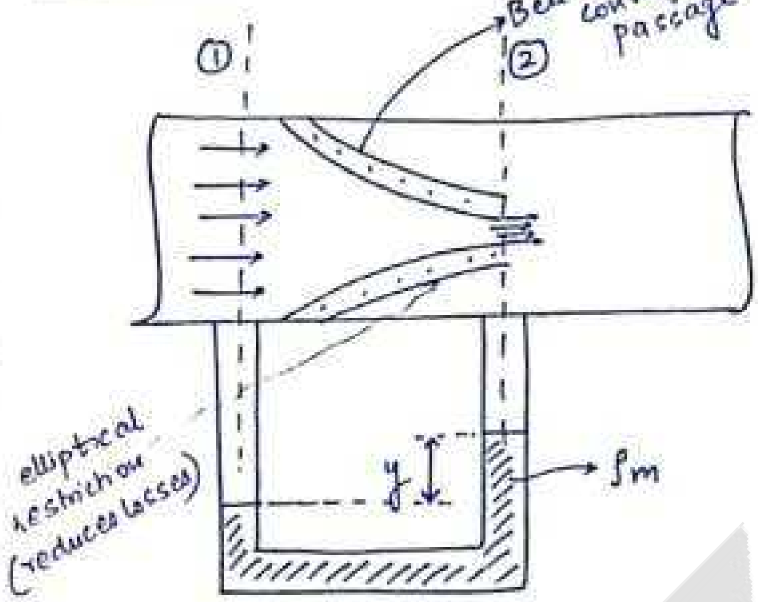
$$\boxed{\begin{array}{l} dQ \times dt = -Adh \\ C_d A = \sqrt{2gh} \end{array}}$$



$$\Rightarrow \boxed{T = \frac{2A}{C_d A_0 \sqrt{2g}} \sqrt{h}}$$

MOHIT CHOUKSEY

* FLOW NOZZLE :- → Gradual conversion of the flow



$$Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

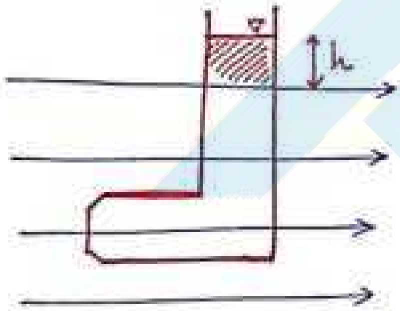
$C_d = 0.72$ to 0.74

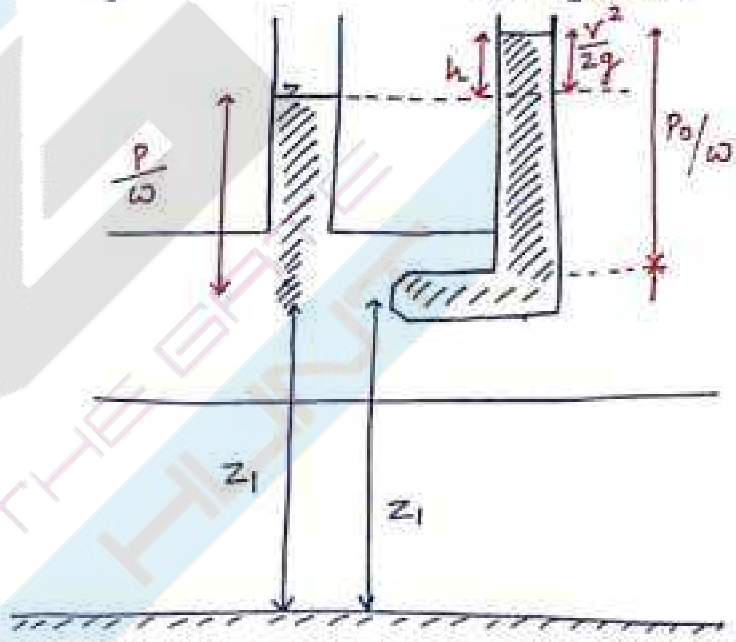
$$\boxed{h = \left(\frac{\rho_m}{\rho_l} - 1\right) y}$$

* PITOT TUBE :- used to velocity measurement (local velocity) at any point

(Most accurate velocity device)

$\boxed{C_v = 0.99}$





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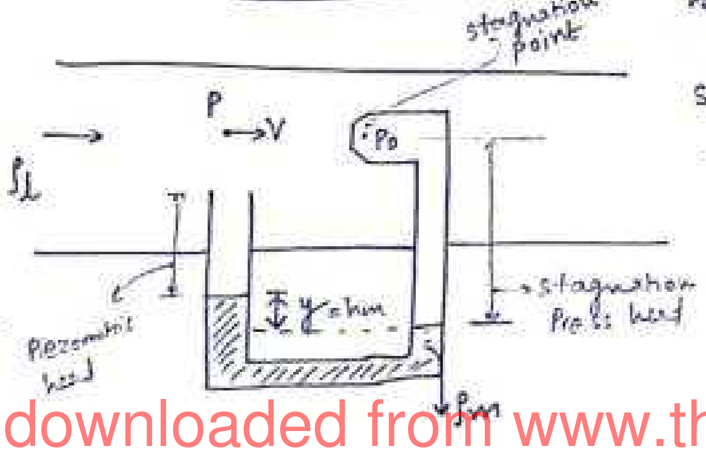
$\boxed{V = C_v \sqrt{2gh}}$

$$\boxed{h = \left(\frac{\rho_m}{\rho_l} - 1\right) y}$$

Piezometric head
↓
Sum of the Pressure head and datum head

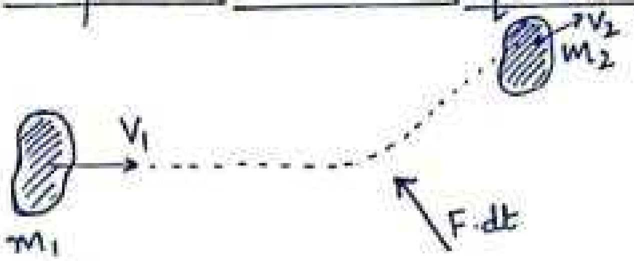


Stagnation Pressure head = Sum of Piezometric + datum + Velocity head

stagnation Press head - Piezometric head = h_m

see Q 16 → 12.8✓ and Q 33

* Impulse Momentum Equations ⇒



→ if the fluid is changing its direction, now suddenly there will be change in the momentum of the fluid because the direction is changing and momentum is changing, so momentum is a vector, so momentum is changing. we always know that when momentum changes, there will always be action of the impulse, so there must be impulse.

V ⇒ magnitude + dirn.

$$\boxed{F.dt = m_2V_2 - m_1V_1}$$

$$F = \frac{d}{dt}(m_2V_2 - m_1V_1)$$

$$\Sigma F_{net} = \dot{m}_2 V_2 - \dot{m}_1 V_1$$

$$\underbrace{\Sigma F_{net}}_{\text{on the Body}} = \underbrace{\dot{m}_f V_f}_{\text{final}} - \underbrace{\dot{m}_i V_i}_{\text{initial}}$$

$\dot{m}$ → mass flow Rate

$\dot{m} = \rho A V$ (A → area, V → velocity)

$= \rho Q$ → discharge

$\frac{m}{V} \times AV$

$\frac{kg}{m^3} \times m^2 \times \frac{m}{sec}$ (kg → m, m^2 → A, $\frac{m}{sec}$ → V; m^3 → V)

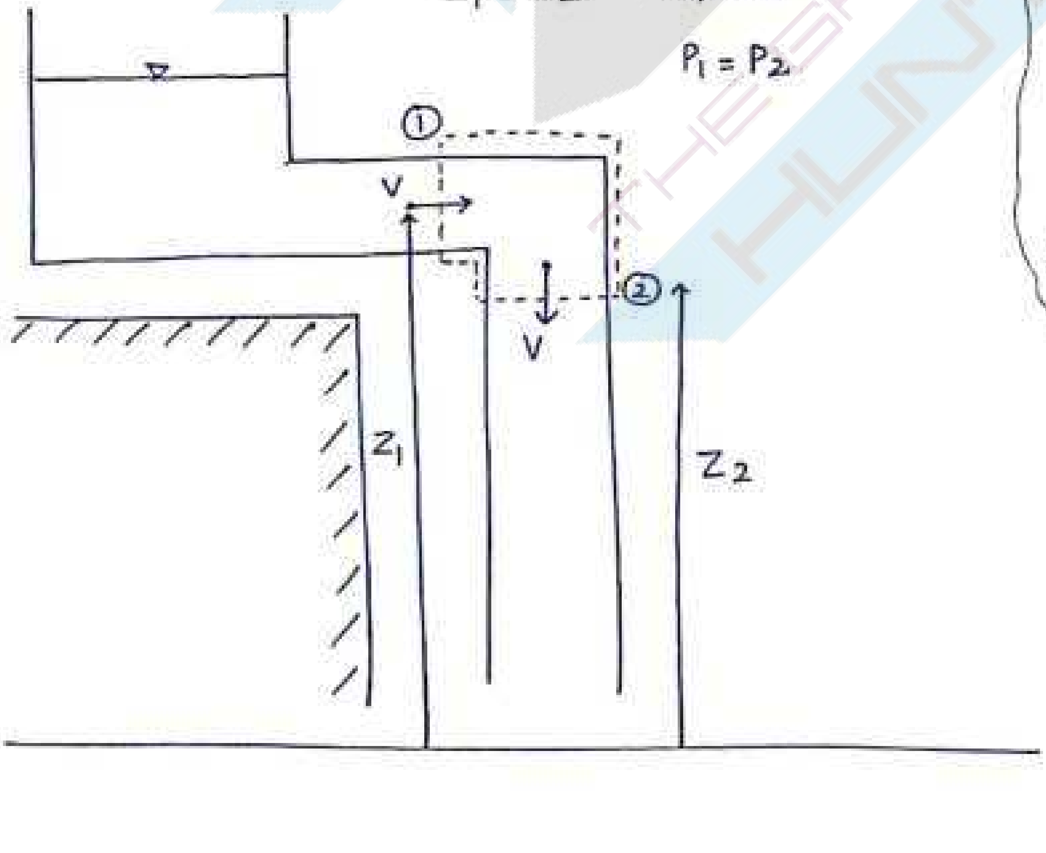
$\frac{kg}{sec}$ ✓

"or"

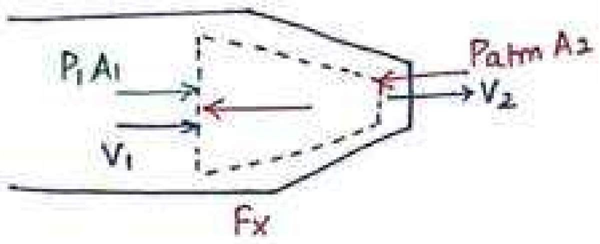
$\frac{kg}{m^3} \times \frac{m^3}{sec}$ (kg → m, $\frac{m^3}{sec}$ → Q; m^3 → V) ✓

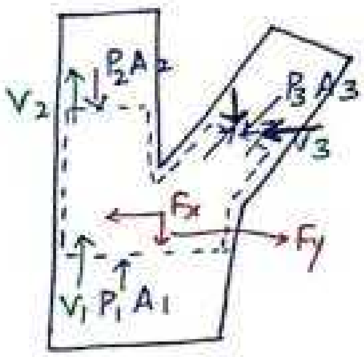
$Z_1 = Z_2 \quad V_1 = V_2$

$P_1 = P_2$



Pressure force हमेशा compressive लगेगा $P_1A_1, P_{atm}A_2$





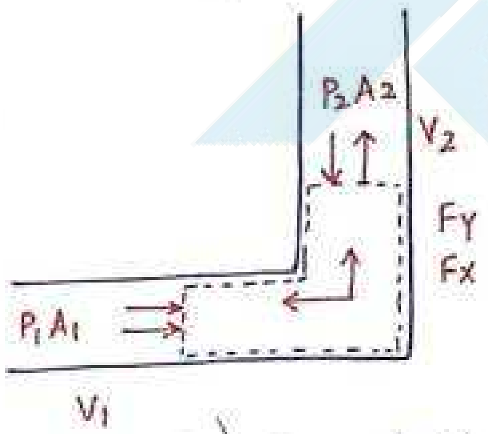
$$\Sigma F_{net_x} = \dot{m}_f V_f - \dot{m}_i V_i$$

on the fluid

$$P_1A_1 - F_x - P_{atm}A_2 = \rho Q V_f - \rho Q V_i$$

$$\boxed{F_x = P_1A_1 - P_{atm}A_2 - \rho Q V_f + \rho Q V_i}$$

on the fluid by the bend



~~$P_1A_1 - F_x = \rho Q V_{fx} - \rho Q V_{ix}$~~

~~$P_1A_1 - F_x = 0 - \rho Q V_1$~~

~~$\boxed{F_x = P_1A_1 + \rho Q V_1}$~~

~~Bend force on the fluid by the bend~~

Horizontal

$$\Sigma F_{net_x} = (\dot{m}_f V_f)_x - (\dot{m}_i V_i)_x$$

$$P_1A_1 - F_x + 0 = (\rho Q V_f)_x - (\rho Q V_i)_x$$

$$P_1A_1 - F_x = \cancelto{0}{\rho Q V_{fx}} - \rho Q V_{ix}$$

$$P_1A_1 - F_x = 0 - \rho Q V_1$$

$$\boxed{F_x = P_1A_1 + \rho Q V_1}$$

Bend force on the fluid by the bend

Vertical

$$\Sigma F_{net_y} = (\dot{m}_f V_f)_y - (\dot{m}_i V_i)_y$$

$$0 + F_y - P_2A_2 = (\rho Q V_f)_y - (\rho Q V_i)_y$$

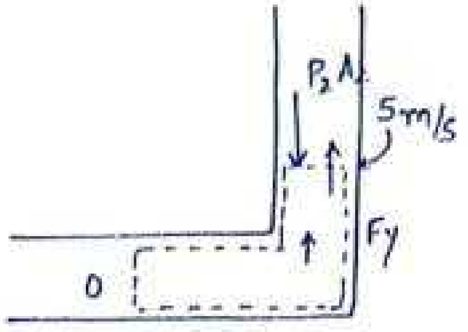
$$F_y = P_2A_2 + \rho Q V_{fy} - \cancelto{0}{\rho Q V_{iy}}$$

$$\boxed{F_y = P_2A_2 + \rho Q V_2}$$

on the fluid

WB (23)

d = 0.3



$$0 + F_y - P_2A_2 = (\rho Q V_f)_y$$

$$F_y = P_2A + \rho Q V$$

$$F_y = 4\times10^3\left(\frac{\pi}{4}\right)(0.3)^2 + 1000\times\frac{\pi}{4}\times0.3^2\times5^2$$

"CHAPTER-7"

* Laminar Pipe Flow/ Viscous Pipe flow →

→ laminar flow is characterized by smooth streamlines and highly ordered motion observed in highly viscous fluids in small pipes or narrow passages.

→ Turbulent flow is characterized by velocity fluctuations and highly disordered motion

→ Features of Laminar Pipe Flow :-

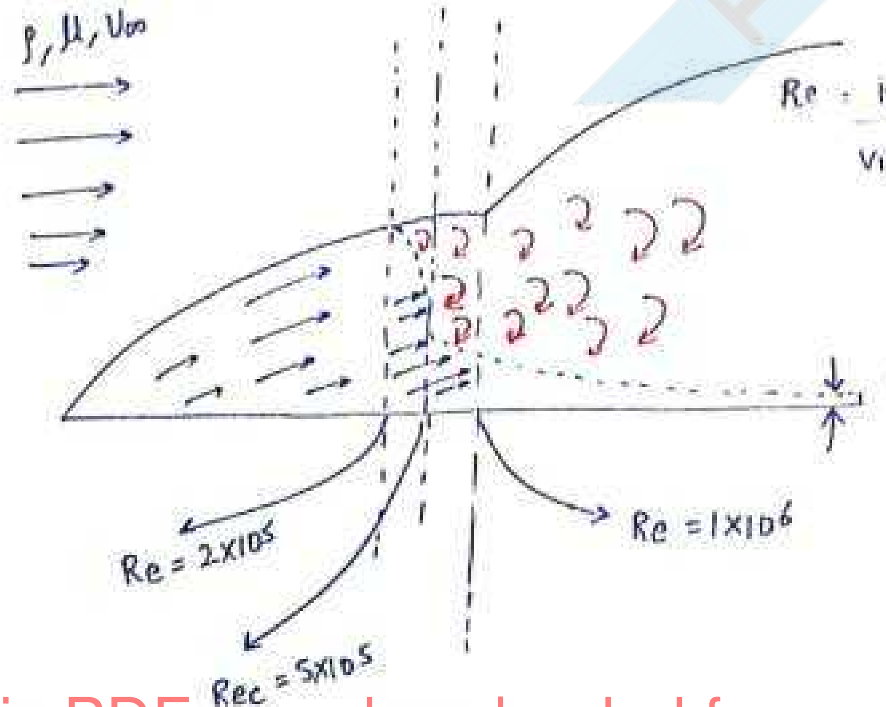
① Follow newton's law of viscosity

② negligible mixing between layers.

③ head loss due to friction is proportional to 1^{st} power of velocity

④ No slip at the surface (velocity profile approaches at the surface).

→ Critical Reynolds no :- The Re. no. below which flow is assumed to be fully developed laminar and after which fully developed turbulent known as Critical Rey. no.



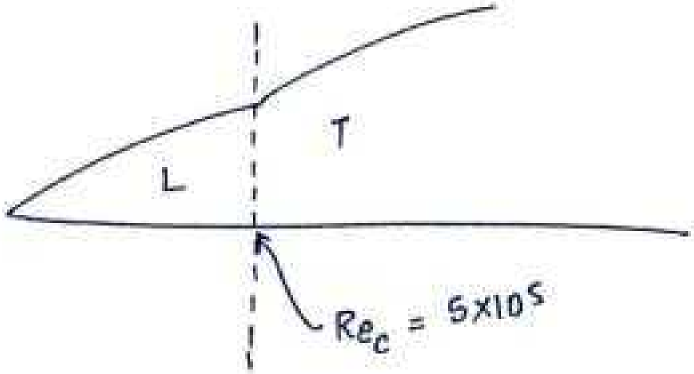
laminar sublayer

$$Re = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{DV_{avg}}{\nu}$$

$$Re = \frac{\rho D V_{avg}}{\mu}$$

$$\delta' = \frac{11.6\nu}{\sqrt{\frac{\tau_w}{\rho}}}$$

→ Transition from laminar to turbulent flow occurs over a region in which the flow fluctuates



For non-circular pipes hydraulic Diameter for Re

$$D_h = \frac{A/c \text{ of pipe}}{\text{wetted perimeter}} = \frac{A_c}{P}$$

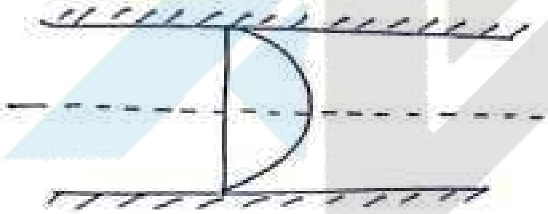
Pipeflow/ Internal flow

Re $\leq$ 2000 laminar $\rightarrow Re_c = 2300$

Re $\geq$ 4000 turb.

$\mu \rightarrow$ Dynamic Viscosity $\rightarrow \frac{N\text{-}s}{m^2} = Pa\text{-}s = \frac{kg}{m\text{-}s}$

$\nu \rightarrow$ Kinematic Viscosity



Flow b/w plates

$Re_c = 1000$

open channel flow $Re_c = 500$

Navier stokes eqn. is the fundamental eqn. for viscous flow.

* Application of Navier Stokes Eqn :-

① Viscous pipe flow (circular) [Hagen-Poiseuille's flow] — laminar flow in circular pipe

② Viscous flow b/w parallel plates (stationary) [Plane Poiseuille's flow] — both plates

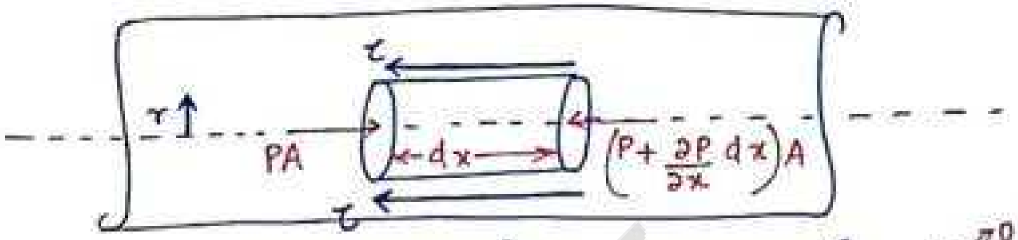
③ Viscous flow b/w Parallel plates [Relative motion] [Couette flow] — plates move

④ Flow b/w concentric cylinder

* Shear stress distribution in circular pipe flow :-

Assumptions

1. Uniform flow
2. S.F.
3. L.F. or T.F.



$$F_p + \cancel{F_g}^{\,0} + F_v = m\left[\cancel{v\frac{\partial v}{\partial s}}^{\,0} + \cancel{\frac{\partial v}{\partial t}}^{\,0}\right]$$

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x}dx\right)\pi r^2 - \tau \times 2\pi r\,dx = 0$$

$$\boxed{\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}}$$

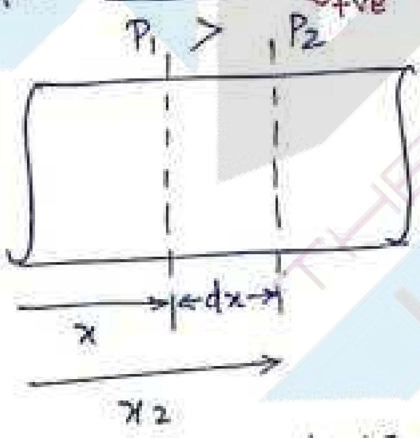
laminar

Turbulent

+ve

+ve

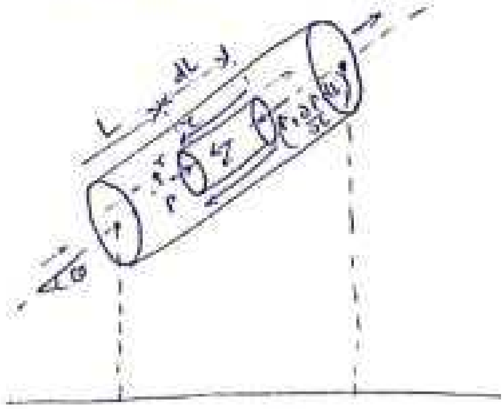
$P_1 > P_2$



$dx > 0 \qquad dP < 0$

$$\frac{dP}{dx} < 0 \qquad \left(-\frac{dP}{\partial x}\right) > 0$$

Inclined Pipe



$$F_p + F_g + F_v = ma_c$$

$$\tau = -\left[\left(\frac{\partial P}{\partial L}\right) + \frac{\omega \sin\theta}{A\,dL}\right]\frac{r}{2}$$

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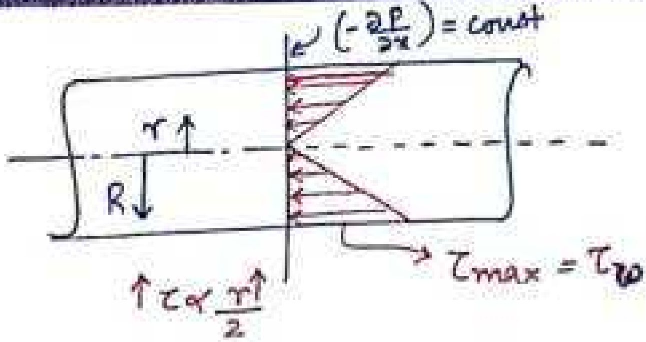
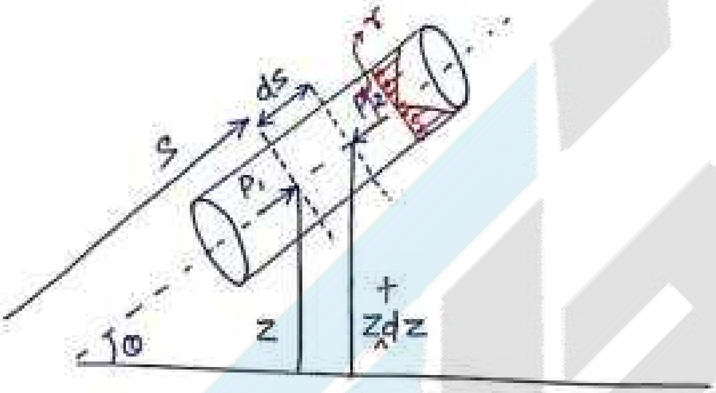


$r = R \;,\; \tau = \tau_{max}$

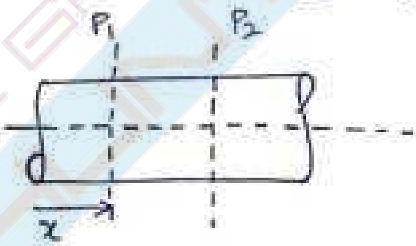
$$\boxed{\tau_{max} = \left(-\frac{\partial P}{\partial x}\right)\frac{R}{2}}$$



$$\boxed{\tau = -\frac{\partial}{\partial s}\left[P + \rho g z\right]\cdot\frac{r}{2}}$$

$$\tau = -\frac{\left[(P_2 + \rho g z_2) - (P_1 + \rho g z_1)\right]}{(s_2 - s_1)}\frac{r}{2}$$

$$\tau = \frac{\left[(P_1 + g\rho z_1) - (P_2 + \rho g z_2)\right]}{(s_2 - s_1)}\frac{r}{2}$$



$z_1 = z_2$

$$\tau = \frac{(P_1 - P_2)}{(x_2 - x_1)}\cdot\frac{r}{2}$$

Types of Flow	Critical Reynolds No.
* ① Flow over plate	5×10^5
* ② Pipe Flow	2300
③ Through open channel	500
④ Between ‖ plates	1000
⑤ Over a small sphere	1

MOHIT CHOUKSEY

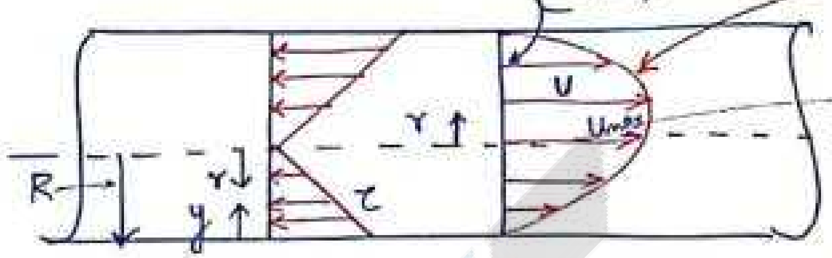
$\tau \rightarrow v$ (actual) $\rightarrow Q$

$v \rightarrow v_m$ (mean) $\rightarrow Q$, $v_m \rightarrow h_f$

* Velocity Distribution in Laminar flow through Circular Pipe →

Assumptions

Uniform flow, steady flow, laminar flow



r → taken from centre line of pipe

y → taken from wall of the pipe

$r + y = R$

$dr + dy = 0$

$dy = -dr$

$$\tau = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2} \quad \text{—(i)}$$

$$\tau = \mu \frac{dv}{dy} \quad \text{—(ii)}$$

$$\Rightarrow \mu \frac{dv}{dy} = \left(-\frac{\partial P}{\partial x}\right)\frac{r}{2}$$

$$\Rightarrow \boxed{dv = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right) r\,dy}$$

r is taken from the center of the pipe and y is taken from the wall of the pipe

$$\Rightarrow \int dv = \frac{+1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\int r\,dy$$

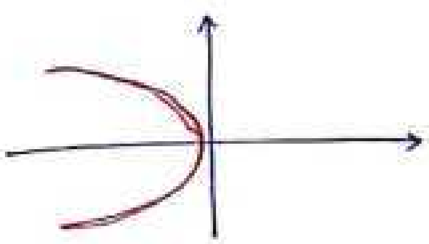
$-dr$

-ve

$$\Rightarrow \int dv = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\int r\,dr$$

$$\Rightarrow v = -\frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) r^2 + C$$

$$\Rightarrow v \propto -r^2$$



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$r = R, \; V = 0$

$$\Rightarrow C = +\frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)R^2$$

$$\Rightarrow \boxed{V = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2)}$$

local velocity

$r = 0 \quad V = U_{max}$

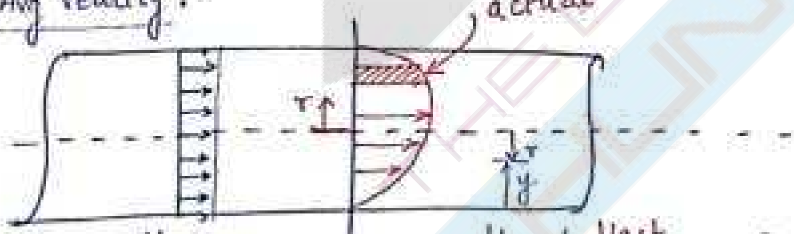
$$*** \Rightarrow \boxed{U_{max} = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)R^2} \; ***$$

$$\Rightarrow V = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)R^2\left[1 - \frac{r^2}{R^2}\right]$$

$$*** \Rightarrow \boxed{V = U_{max}\left[1 - \frac{r^2}{R^2}\right]} \; ***$$

→ velocity at any of the Radial distance from the centre.

* Mean/Avg Velocity :-



$U_m \to$ mean or avg velocity

$Q_m = Q_{mean} = \pi R^2 \times U_m$

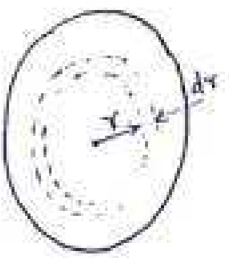
$$\boxed{Q_{avg} = Q_{act}}$$

Qmean

$\int dQ = \int_0^R U \times 2\pi r\, dr$

$Q_{act} = 2\pi\int_0^R U \times r\, dr$

also $Q_m = Q_{act}$



$$U_m A = \int_A U\, dA$$

$$U_m \cancel{\pi} R^2 = \int_0^R U\, 2\cancel{\pi} r\, dr$$

→ laminar and turbulent

$$\boxed{U_m = \frac{2\int_0^R U \cdot r\, dr}{R^2}}$$

mean velocity

→ This eqn. is valid for to circular pipe having any Velocity profile

Laminar Pipe flow

$$U = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

$$U_m = \frac{2\int_0^R \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2)\cdot r\,dr}{R^2}$$

$$\boxed{U_m = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right)R^2}$$

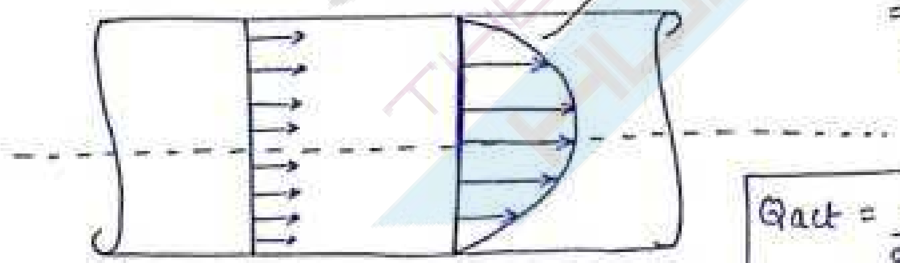
U_m
Mean & Avg Velocity
hypothetical constant velocity which gives the same rate of discharge as obtained from actual velocities at a given section

$$\frac{U_m}{U_{max}} = \frac{\cancel{1/8\mu\left(-\frac{\partial P}{\partial x}\right)R^2}}{\cancel{1/4\mu\left(-\frac{\partial P}{\partial x}\right)R^2}}$$

$$\boxed{U_m = \frac{U_{max}}{2}}$$ Laminar pipe flow

Umean

Discharge :-



$$Q_{act} = U_m \pi R^2$$

$$= \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right)R^2 \pi R^2$$

$$\boxed{Q_{avg} = Q_{act}}$$

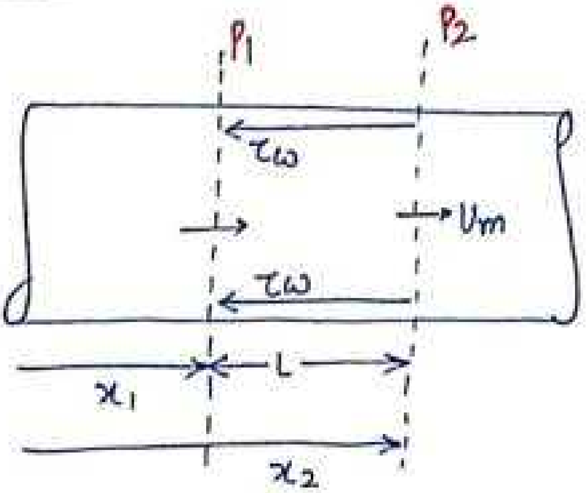
$$\boxed{Q_{act} = \frac{\pi}{8\mu}\left(-\frac{\partial P}{\partial x}\right)R^4}$$

or $R = D/2$

$$\boxed{Q_{act} = \frac{\pi}{128\mu}\left(-\frac{\partial P}{\partial x}\right)D^4}$$

This eqn. is also known as Hagen - Poiselles eqn.

Head loss :→



$P_1 > P_2$

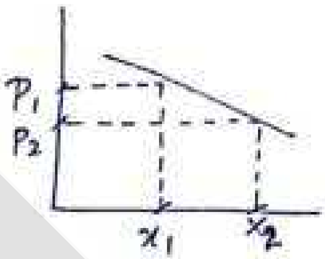
i.e. $P_2 - P_1 < 0$ $(\because P_2 < P_1)$

$U_{mean} = V$

$$U_m = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right)R^2$$

$U_m = V$

$$\frac{8\mu V}{R^2} = \left[-\frac{(P_2 - P_1)}{(x_2 - x_1)}\right]$$



$$\boxed{P_1 - P_2 = \frac{8\mu V L}{R^2}}$$

$R = D/2$

$$\boxed{\frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{32\mu V L}{\omega D^2}}$$

$\omega = \rho g$

$z_1 = z_2$

$$\boxed{\left(\frac{P_1}{\omega} + z_1\right) - \left(\frac{P_2}{\omega} + z_2\right) = \frac{32\mu V L}{\omega D^2}}$$ → Laminar

$$\boxed{\left(\frac{P_1}{\omega} + z_1\right) - \left(\frac{P_2}{\omega} + z_2\right) = h_f}$$

$$\cancel{\boxed{\left(\frac{P_1}{\omega} + z_1\right) - \left(\frac{P_2}{\omega} + z_2\right) = h_f}}$$

$$\boxed{\left(\frac{P_1}{\omega}+z_1\right)-\left(\frac{P_2}{\omega}+z_2\right)=\frac{fLV^2}{2gD}}$$ → Darcy's eqn Laminar and turbulent

or $\frac{4f'LV^2}{2gD}$

h_f

Laminar

$$h_f = \frac{32\mu VL}{wD^2}$$

Lami or Turb

$$h_f = \frac{fLV^2}{2gD}$$

$$f = 4f'$$

$$4f' = \frac{64}{Re}$$

$$\boxed{f' = \frac{16}{Re}}$$

friction coefficient

$\boxed{\text{Laminar}}$

$$\frac{f\cancel{L}V^{\cancel{2}}}{2\cancel{g}D} = \frac{32\mu V\cancel{L}}{\cancel{\rho g}D^2}$$

$$\Rightarrow f = \frac{64}{\left(\frac{\rho VD}{\mu}\right)}$$

Laminar

$$\Rightarrow \boxed{f = \frac{64}{Re}}$$

friction factor

Laminar

$$f' = \frac{16}{Re}$$

$$\frac{\tau_w}{\frac{1}{2}\cancel{\rho} V^{\cancel{2}}} = \frac{16}{\frac{\cancel{\rho}\cancel{V}D}{\mu}}$$

$$\boxed{\tau_w = \frac{8\mu V}{D}}$$

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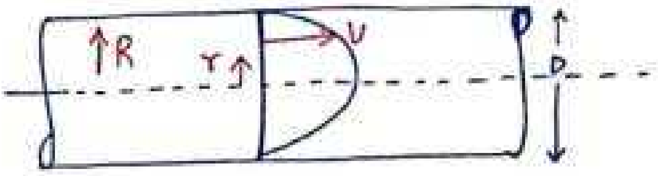
Conclusions

Pipeflow :-



$$v = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

$r = 0$ $\quad U_{max} = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right)R^2$

$$\boxed{U_m = \frac{U_{max}}{2}}$$ Pipe flow

$$U_m = \frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right)R^2$$

$$Q = U_m \pi R^2$$

$$\boxed{Q = \frac{\pi}{8\mu}\left(-\frac{\partial P}{\partial x}\right)R^4}$$

$$h_f = \frac{fLV^2}{2gD}$$

$$f = \frac{64}{Re}$$

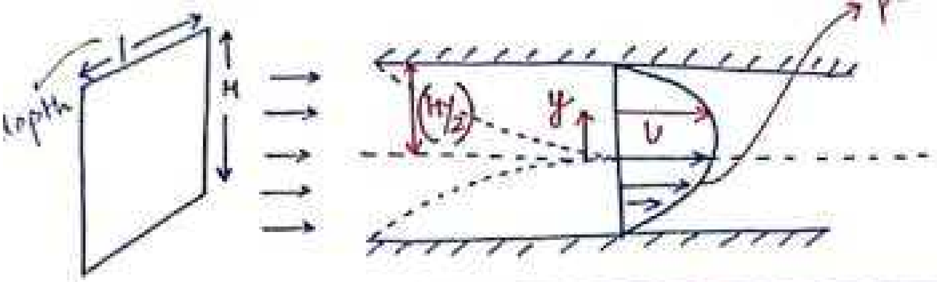
$$\boxed{h_f = \frac{32\mu VL}{\omega D^2}}$$

$$4f' = \frac{64}{Re} \qquad f' = \frac{16}{Re}$$

$$\boxed{\tau_w = \frac{8\mu V}{D}}$$

Plane - Poiseuille's :-

Plane poiseuille flow



$$\boxed{U = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left[\left(\frac{H}{2}\right)^2 - y^2\right]}$$

→ at $y = 0$, $U = U_{max}$

$$\boxed{U_{max} = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left(\frac{H}{2}\right)^2} = \boxed{\frac{1}{8\mu}\left(-\frac{\partial P}{\partial x}\right)H^2}$$

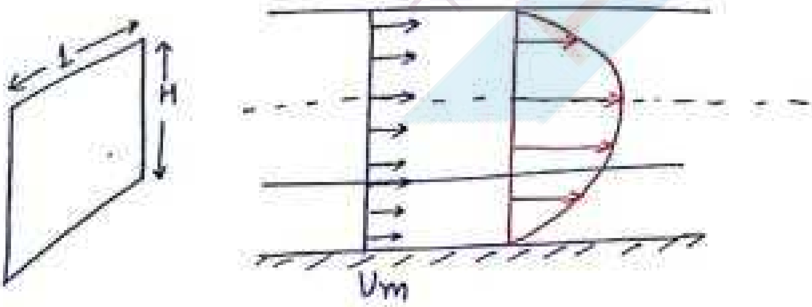
$$\boxed{U_m = \frac{2}{3}U_{max}}$$

U_{mean}

v.v.v.v. imp

$$\boxed{U_m = \frac{1}{3\mu}\left(-\frac{\partial P}{\partial x}\right)\left(\frac{H}{2}\right)^2}$$

considering unit depth inside the page



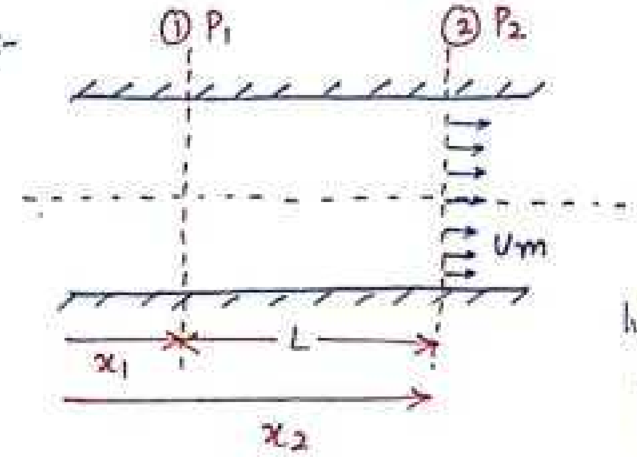
$$\boxed{Q = \frac{1}{12\mu}\left(-\frac{\partial P}{\partial x}\right)H^3}$$

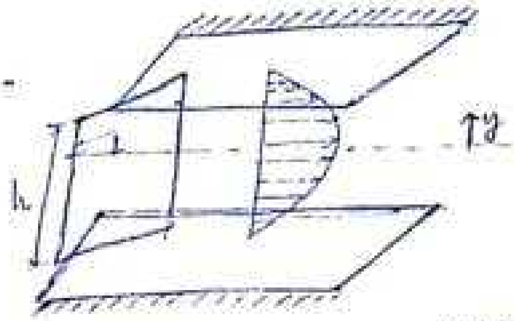
$Q_{act} = Q_{avg}$

$Q_{act} = U_m \cdot A$

$$\boxed{Q_{act} = \frac{1}{3\mu}\left(-\frac{\partial P}{\partial x}\right)\left(\frac{H^2}{2^2}\right)\cdot(H \times 1)}$$

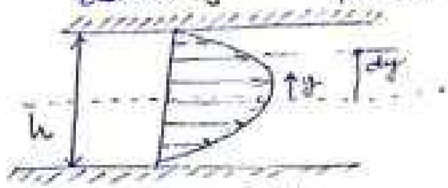
head loss:-



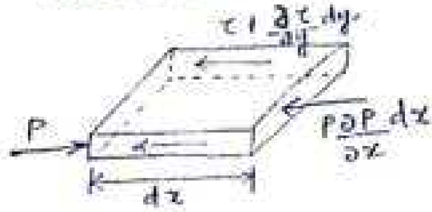


$$U_m = \frac{1}{3\mu}\left[\frac{P_1 - P_2}{x_2 - x_1}\right]\left(\frac{H}{2}\right)^2$$

"Take element dy at a distance from the center"



$$P_1 - P_2 = \frac{12\mu V L}{H^2}$$



$$\frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{12\mu V L}{\omega H^2}$$

$$\boxed{h_f = \frac{12\mu V L}{\omega H^2}}$$

Applying Navier stokes eqn.

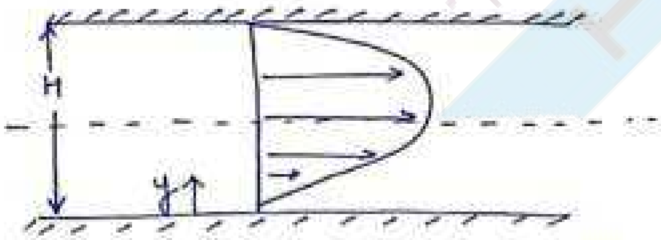
$$*** \boxed{\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y}} ***$$

(पीछे वाले Page पे)

Pressure gradient along the x direction equals to the change in shear stress moving in y-direction

$$\boxed{\tau_\omega = \frac{6\mu V}{H}}$$

*



$y = 0 \quad U = 0$

$y = H \quad U = 0$

$$\boxed{U = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left[Hy - y^2\right]}$$

Hall's formula

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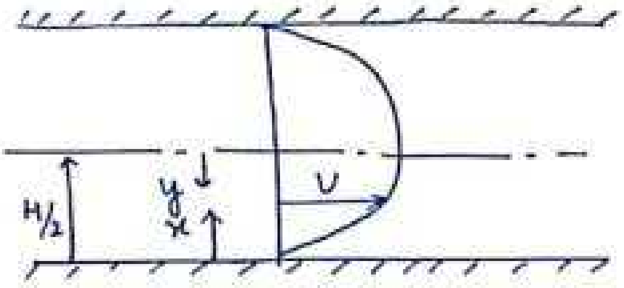
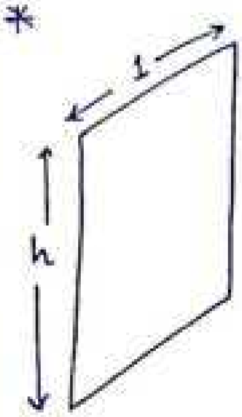
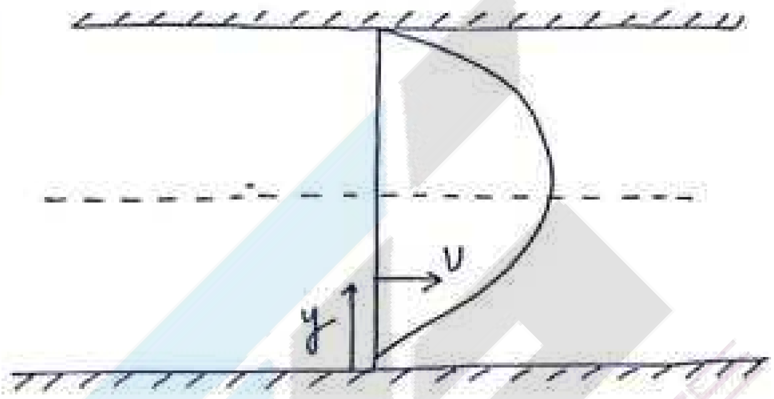


$$\boxed{V = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left[\left(\frac{H}{2}\right)^2 - y^2\right]}$$

*





$$\boxed{U = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(Hy - y^2\right)}$$

$$\tau_\omega = \mu \frac{du}{dy}\Big|_{y=0}$$

$$\tau_\omega = \cancel{\mu}\left[\frac{1}{2\cancel{\mu}}\left(-\frac{\partial p}{\partial x}\right)H\right]$$

$$\tau_\omega = \frac{1}{2}\left(-\partial p/\partial x\right)\cdot H$$

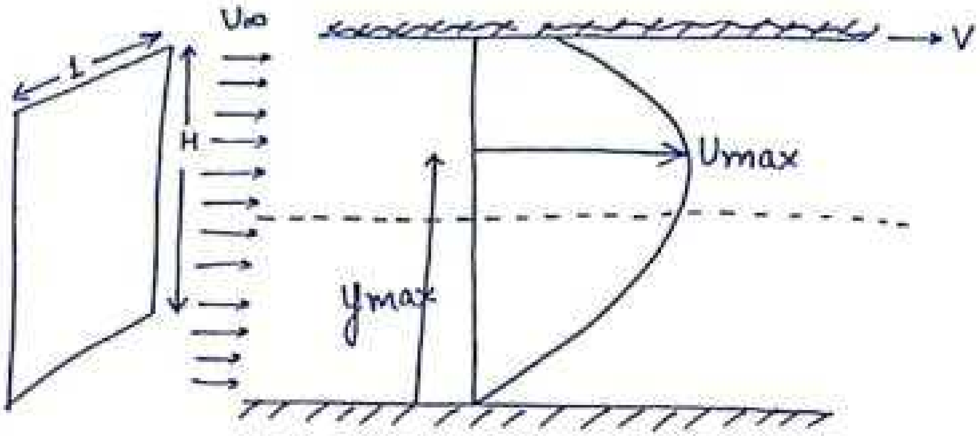
$$U_m = \frac{1}{3\mu}\left(-\frac{\partial p}{\partial x}\right)\left(\frac{H}{2}\right)^2$$

$$V = \frac{1}{6\mu}\times\left(\frac{1}{2}\left(-\frac{\partial p}{\partial x}\right)H\right)H$$

$$V = \frac{1}{6\mu}\tau_\omega \cdot H$$

$$\boxed{\tau_\omega = \frac{6\mu V}{H}}$$

③ Covette flow :-



$U_\infty > U$ $\quad y = 0,\ U = 0 \quad y = H,\ U = V$

$$\boxed{U = \frac{V}{H}y + \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left(Hy - y^2\right)}$$

WB

⑦ 0 → max

⑨ ✓

⑧ Fixed ‖ Plates

$U_{max} = 6$

$U_{mean} = \frac{2}{3}U_{max} = \frac{2}{3}\times 6 = 4\,m/s$

④ d ⑤ d

⑥ b

③

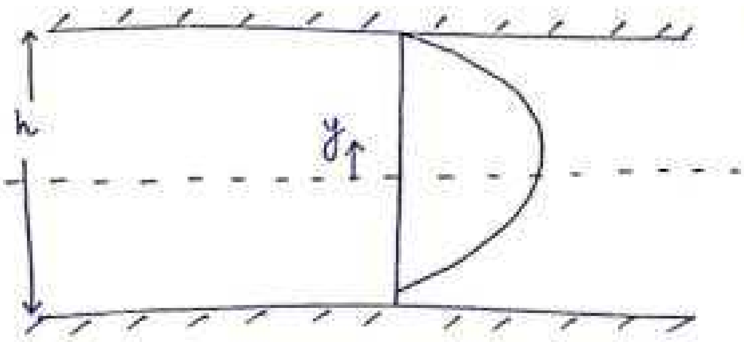
① $U_m = \frac{2}{3}U_{max}$

$\frac{U_{max}}{U_m} = 3/2 = 1.5$

$U = -\frac{h^2}{8}\left(\frac{\partial P}{\partial x}\right)\left[1 - \frac{4y^2}{h^2}\right]$

$U = \frac{1}{2\mu}\left(-\frac{\partial P}{\partial x}\right)\left(h/2\right)^2\left[1 - \frac{y^2}{(h/2)^2}\right]$

$U = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left[\left(h/2\right)^2 - \cancel{y^2}^{\,0}\right]$



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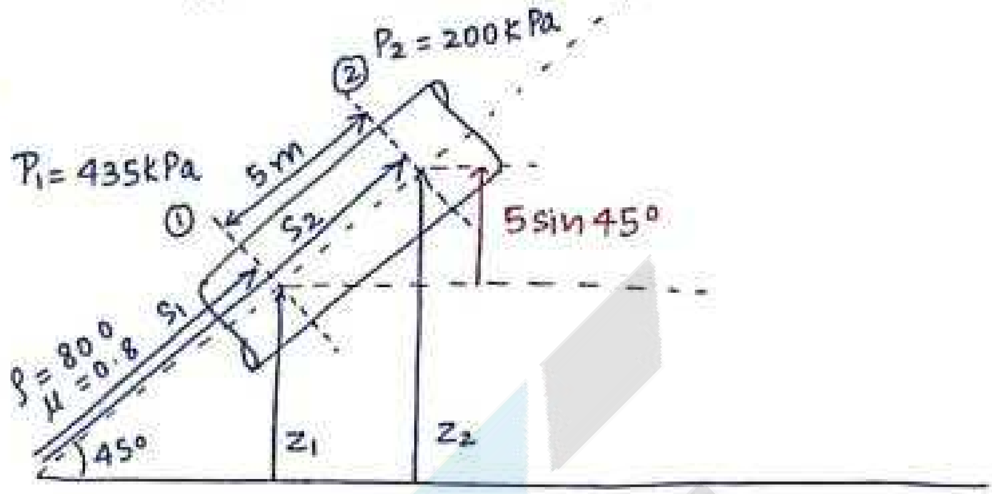
(9) $\tau_\omega = \left(\frac{-\partial P}{\partial x}\right)\frac{R}{2}$

$$\tau_\omega = \left[\frac{P_1 - P_2}{x_2 - x_1}\right]\frac{R}{2}$$

$$\tau_\omega = \frac{50\times10^3}{10} \times \frac{50\times10^{-3}}{2}$$ (b) ✓

(15)



$$Q = U_m \pi R^2$$

$$Q = \frac{U_{max}}{2}\pi R^2$$

$$= \frac{1}{2}\frac{1}{4\mu}\left(\frac{-\partial P}{\partial x}\right)R^2 \pi R^2$$

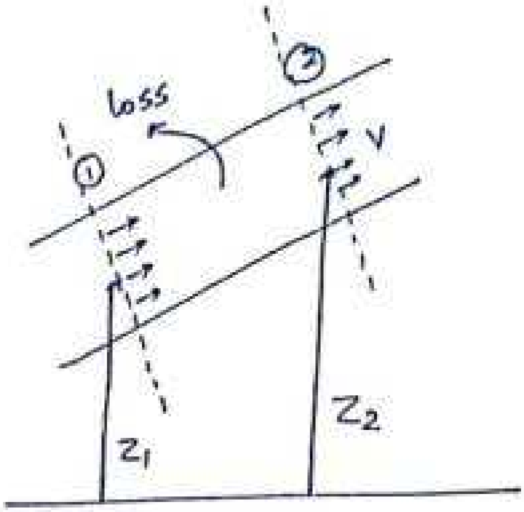
$$Q = \frac{\pi}{8\mu}\left(\frac{-\partial P}{\partial x}\right)R^4$$

$$Q = \frac{\pi}{8\mu}\left(\frac{-\partial}{\partial s}\left\{P + \rho g z\right\}\right)R^4$$

$$Q = \frac{\pi}{8\mu} - \frac{\left[\left(P_2 + \rho g z_2\right) - \left(P_1 + \rho g z_1\right)\right]}{(S_2 - S_1)} R^4$$

$$Q = \frac{\pi}{8\mu}\left[\frac{(P_1 - P_2) - \rho g\,(z_2 - z_1)}{(S_2 - S_1)}\right]R^4$$

$$Q = \frac{\pi}{8\mu}\left[\frac{(435-200)\times 10^3 - 800\times 4.815 \sin 45^\circ}{5}\right]\times (0.05)^4$$



$$\frac{P_1}{\omega} + \cancel{\frac{V^2}{2g}} + Z_1 = \frac{P}{\omega} + \cancel{\frac{V^2}{2g}} + Z_2 + h_{loss}$$

$$h_{loss} = \frac{P_1 - P_2}{\omega} - (Z_2 - Z_1)$$

$$\frac{32\mu V L}{\omega d^2} = \frac{(P_1 - P_2)}{\omega} - 5\sin 45^\circ$$

$\rho_0 g$ $\rho_0 g$

$V = ?$

$\boxed{Q = A \times V}$

$A_1 = A_2$

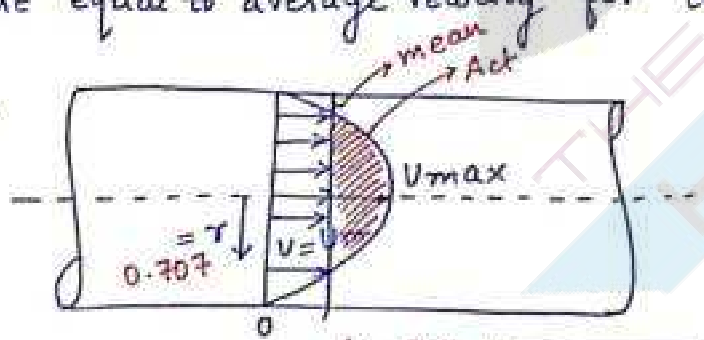
$V_1 = V_2$

MOMENTUM CORRECTION factor and ENERGY Correction Factor :-

(Kinetic energy correction factor)

Q Find the distance from the centre of the pipe where actual velocities are equal to average velocity for laminar flow

Sol



$\Rightarrow U = \frac{1}{4\mu}\left(-\frac{\partial P}{\partial x}\right) R^2 \left(1 - \frac{r^2}{R^2}\right)$ — Umax

$\Rightarrow V = U_{max}\left[1 - \frac{r^2}{R^2}\right]$

$\Rightarrow \boxed{V = U_m}$ (given)

Laminar

$\boxed{U_m = \frac{U_{max}}{2}}$

$U_m \neq U_{act}$

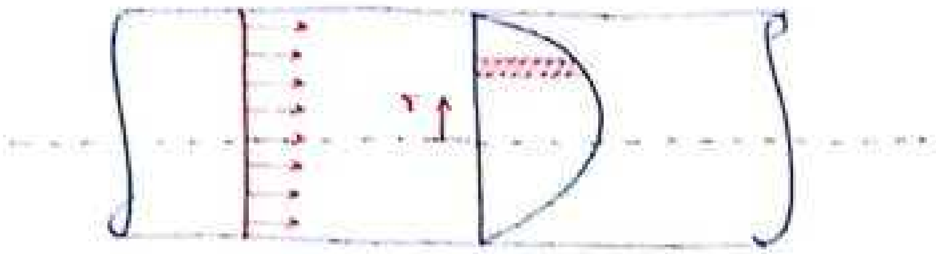
$\boxed{Q_m = Q_{act}}$

$\Rightarrow U_m = U_{max}\left[1 - \frac{r^2}{R^2}\right]$

$\Rightarrow \frac{\cancel{U_{max}}}{2} = \cancel{U_{max}}\left[1 - \frac{r^2}{R^2}\right]$

$\Rightarrow \boxed{r = 0.707 R}$

Pg 44
Q22



$V_{avg} < V_{act}$

$P = mV$

$\dot{P}_{avg} = \boxed{\dot{m}_{avg}} V_{avg}$ $\qquad$ $\dot{P}_{act} = \boxed{\dot{m}_{act}} V_{act}$

$\dot{P}_{avg} < \dot{P}_{act}$

$KE_{avg} = \frac{1}{2} \dot{m}_{avg} V_{avg}^2$ $\qquad$ $KE_{act} = \frac{1}{2} \dot{m}_{act} V_{act}^2$

$\beta \dot{P}_{avg} = \dot{P}_{act}$ $\qquad$ $\alpha \dot{K}_{avg} = \dot{K}_{act}$

$\boxed{\beta = \frac{\dot{P}_{act}}{\dot{P}_{avg}}} > 1$ $\qquad$ $\boxed{\alpha = \frac{KE_{act}}{KE_{avg}}} > 1$ $\qquad$ $\boxed{\alpha > \beta > 1}$

$\dot{P} = (\rho A V) \cdot V$

$\dot{P} = (\rho A V^2)$

$KE = \frac{1}{2}(\rho A V) V^2$

$= \frac{1}{2}(\rho A V^3)$

$$\beta = \frac{\int_A \rho v^2 \, dA}{\rho A V_m^2} \qquad \alpha = \frac{\cancel{\frac{1}{2}} \int_A \rho v^3 \, dA}{\cancel{\frac{1}{2}} \rho A V_m^3}$$

$$\boxed{\beta = \frac{\int_A v^2 \, dA}{V_m^2 A}} \qquad \boxed{\alpha = \frac{\int_A v^3 \, dA}{A V_m^3}}$$

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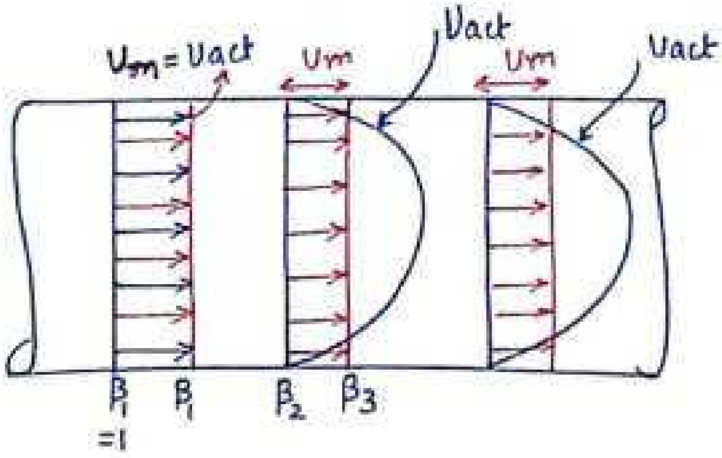
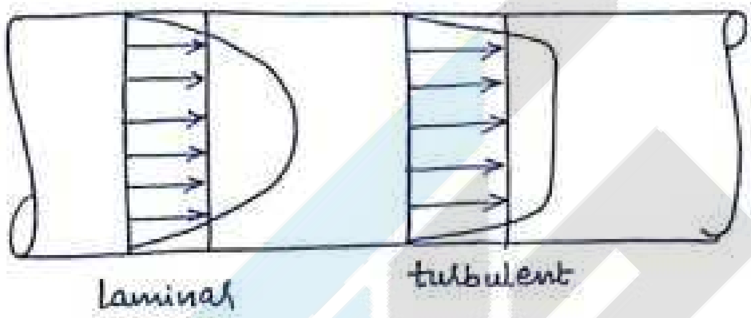


MOHIT CHOUKSEY

* $\boxed{V_m = 0.5 V_{max}}$ $\boxed{V_m = 0.82 V_{max}}$



* when fluid flows through the pipe, the actual velocities are more than avg. velocities, so the Rate of momentum transfer, Rate of K.E. transfer at the section, the actual momentum transfer rate and kinetic energy transfer rate are more than the avg. velocities than the calculated by avg. velo so "α & β" are used resp. for the correction/calculation in avg. calculations.

Q Find the momentum correction factor for $\frac{v}{V_{max}} = \left(1 - \frac{r^2}{R^2}\right)$

Sol

$$\beta = \frac{\int_A v^2 \, dA}{V_m^2 A}$$

$$\beta = \frac{\int_0^R v^2 (2\pi r \, dr)}{V_m^2 \pi R^2}$$

$$\beta = \frac{2}{\left(\frac{V_{max}}{2}\right)^2 R^2} \int_0^R \left[V_{max}\left(1 - \frac{r^2}{R^2}\right)\right]^2 . r dr$$

$$\beta = \frac{8}{R^2}\int_0^R \left(1+\frac{r^4}{R^4} - \frac{2r^2}{R^2}\right) r\,dr$$

$$\beta = \frac{8}{R^2}\left[\frac{r^2}{2} + \frac{r^6}{6R^4} - \frac{2r^4}{4R^2}\right]_0^R$$

$$\boxed{\beta = 1.33}$$

MOHIT CHOUKSEY

* $\frac{v}{U_{max}} = \left(1 - \frac{r^2}{R^2}\right)$

β — Laminar — 1.33

β — Tulbulent — 1.03

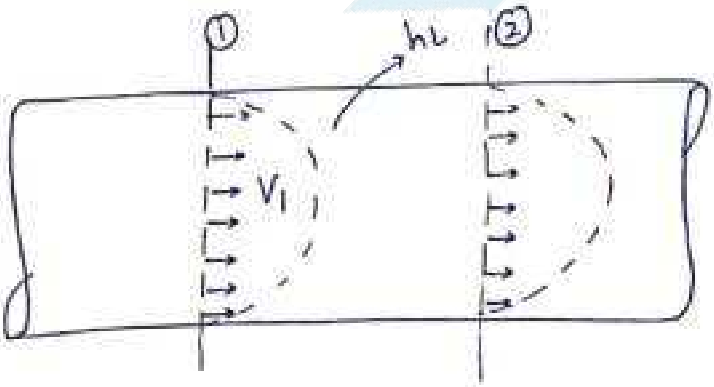
$\frac{v}{U_{max}} = \left(1 - \frac{r}{R}\right)^{1/7}$

α — $\frac{v}{U_{max}} = \left(1 - \frac{r^2}{R^2}\right)$ laminar — 2

(21) HW Pg 44

α — Tulbulent — 1.33

$$\frac{v}{U_{max}} = \left(1 - \frac{r}{R}\right)^{1/7}$$



$$\frac{P_1}{\omega} + \alpha_1 \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \alpha_2 \frac{V_2^2}{2g} + Z_2 + h_L$$

FLOW THROUGH PIPES ← CHAPTER-8

A flow is said to be pipe flow if the flow is under full depth.

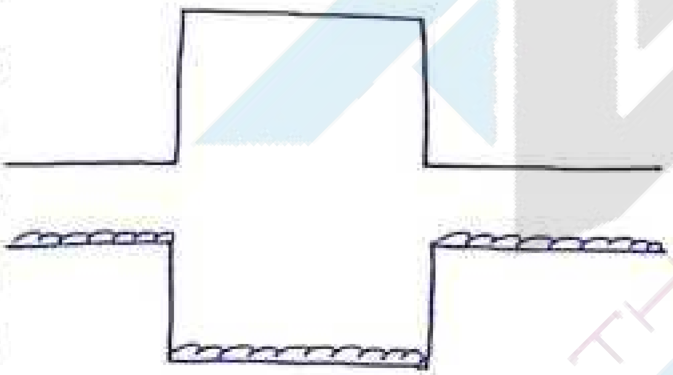
In pipe flow, different types of losses takes place :-

① Major losses

② Minor losses

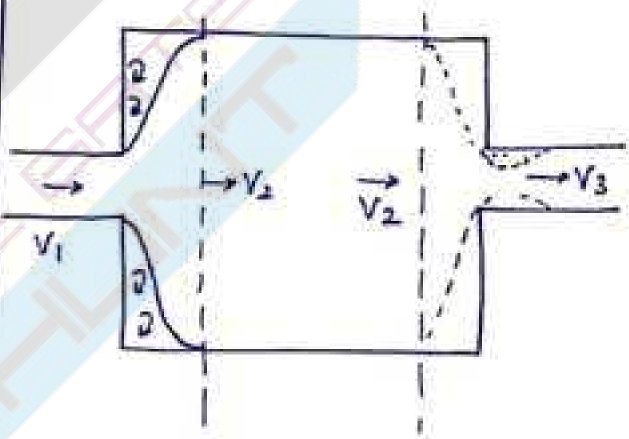
MAJOR LOSSES

① These losses are due to viscous resistance through the valve wall. These losses are having high magnitude as well as their occurence is more. ② also known as constant velocity loss

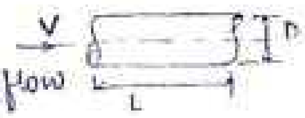


MINOR LOSSES

① These losses are due to change in momentum of the fluid that is either velocity due to magnitude or direction or both. There occurence is less as well as magnitude is less.



(1) Head loss in viscous pipe flow (laminar flow)



- steady & uniform flow

→ $\boxed{h_f = \frac{32 \mu V L}{\rho g D^2}}$ → due to friction

→ $\boxed{h_f = \frac{f L V^2}{2 g D}}$ → Darcy weisbach equation

f → friction factor

f' → friction coefficient (C_{fx})

$\boxed{f = 4f'}$ (laminar flow, [illegible]

Ex:- sudden expansion loss,

— " — exit — " — ,

— " — contraction — " —,

Bend loss, entrance loss,

and diff. pipe fitting losses

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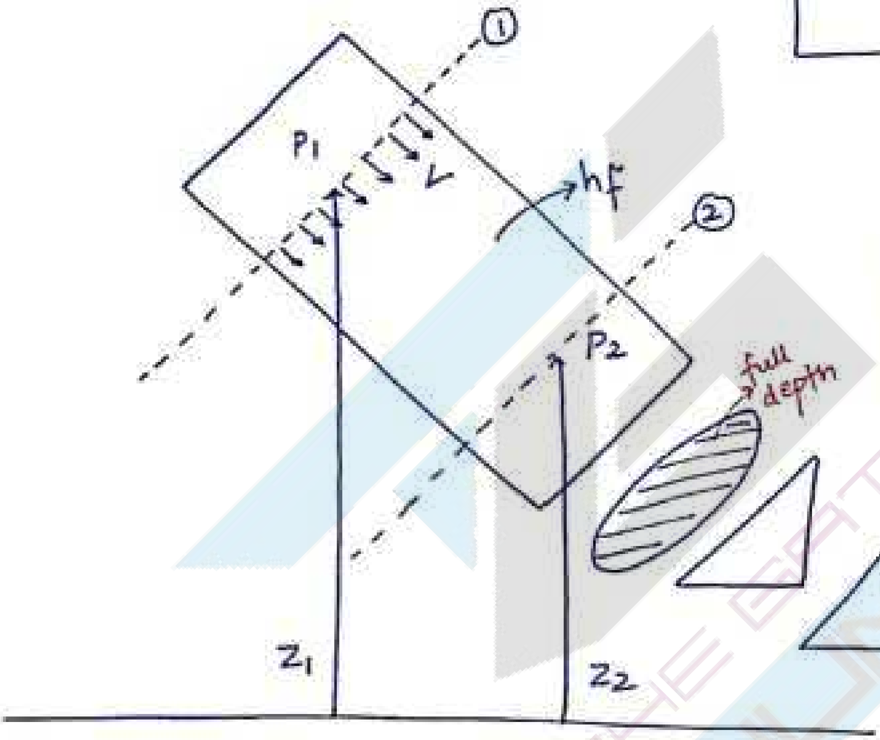
Major losses

Darcy's Weisbach equation :- → full depth

Assumptions → ① Steady flow.
② Uniform flow.
③ mean Velocity.



f' → friction coefficient

$$\boxed{\left(\frac{P_1}{\omega}+Z_1\right)-\left(\frac{P_2}{\omega}+Z_2\right)=\frac{4f'LV^2}{2gD_h}}$$

$$f'=\frac{\tau_\omega}{\frac{1}{2}\rho V^2}$$

$$\boxed{\left(\frac{P_1}{\omega}+Z_1\right)-\left(\frac{P_2}{\omega}+Z_2\right)=\frac{fLV^2}{2gD_h}}$$

$\boxed{f=4f'}$ friction factor

$$\boxed{\left(\frac{P_1}{\omega}+Z_1\right)-\left(\frac{P_2}{\omega}+Z_2\right)=h_f}$$

(2) Head loss in turbulent flow:
- Darcy weisback eqn to be used.
- steady and uniform flow

How to find friction factor ?
- empirical relations
- experimental chart : Moody chart

$$h_f=\frac{fLV^2}{2gD_h}=\frac{fLV^2}{2gd}$$

$$\boxed{Q=\frac{\pi}{4}d^2V}$$

$$h_f=\frac{fLQ^2}{\left[2\times9.81\times\frac{\pi^2}{16}\right]d^5}$$

$$** \boxed{h_f=\frac{fLQ^2}{12.1d^5}} **$$

(197)

→ Gives experimental result to find the velocity including losses for steady flow.

→ **Chezy's eqn** :- ——→ for open channel flow

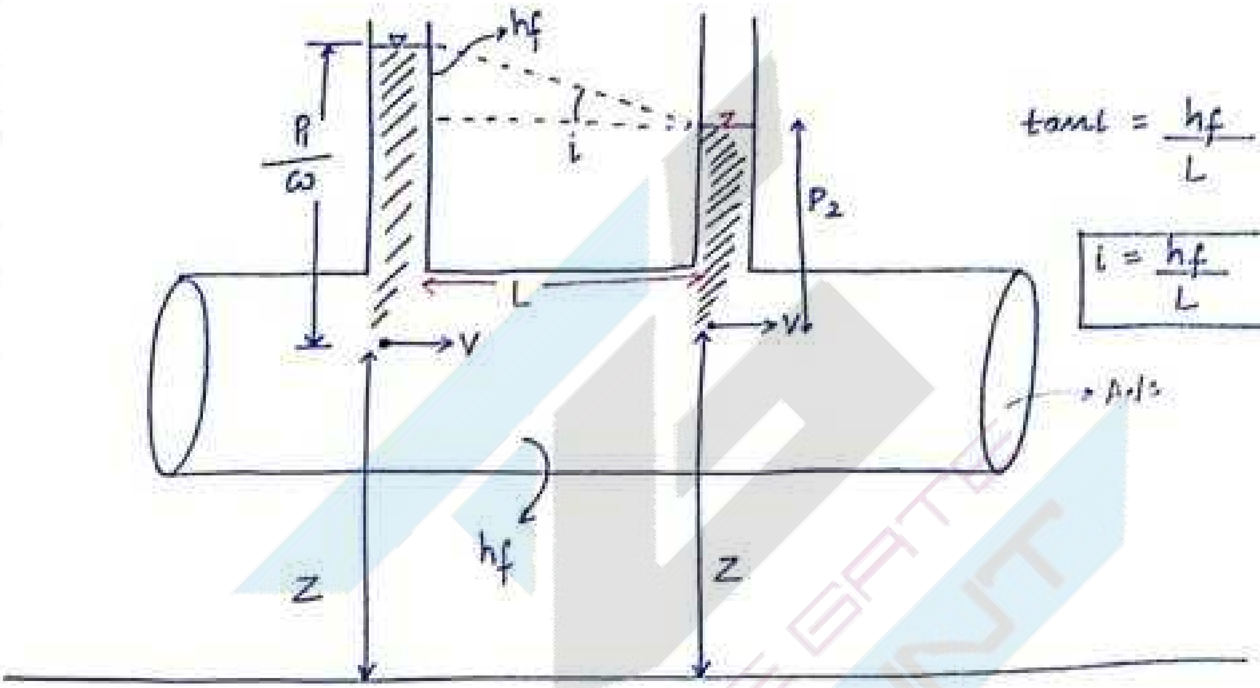
$$\boxed{V = C\sqrt{mi}}$$

V → avg velocity

C = chezy's coefficient

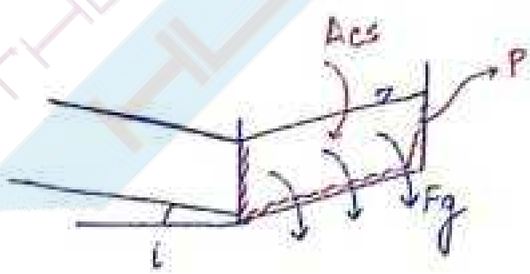
m = hydraulic mean depth = $\frac{A_{cs}}{P}$ P → Perimeter

i = ——"—— "—— slope = $\frac{h_f}{L}$



$\tan i = \frac{h_f}{L}$

$$\boxed{i = \frac{h_f}{L}}$$



$r_c = \frac{A_{cs}}{P}$

* Generally in all the cases we are neglecting the minor losses and only considering the major losses because as compared to major losses, the minor losses are negligible. But sometimes, in the flow of varying cross section over small length, minor losses become significant.

CH→6
WB

① $V = C\sqrt{mi}$

$$V^2 = C^2 \left[\frac{A_{cs}}{P}\right]\left(\frac{h_f}{L}\right)$$

$$\cancel{V^2} = C^2 \left[\frac{A\cancel{cs}}{\cancel{P}}\right] \left[\frac{f\cancel{L}\cancel{V^2}}{2g\left(\frac{4A_{cs}}{P}\right)}\right] \frac{1}{\cancel{L}}$$

$$C^2 = \frac{8g}{f}$$

$$\boxed{C = \sqrt{\frac{8g}{f}}}$$

Q find the relation b/w head loss due to friction and diameter of the pipe for laminar flow

ⓐ $1/d$

ⓑ $1/d^2$

ⓒ $1/d^4$

ⓓ $1/d^5$

Sol $h_f = \frac{fLQ^2}{12.1 d^5}$ $< \begin{matrix} \to L \\ \to T \end{matrix}$

Laminar

$$f = \frac{64}{\frac{\rho V d}{\mu}} \Rightarrow V = \frac{Q}{\frac{\pi}{4}d^2} \quad V \propto \frac{1}{d^2}$$

$$f \propto \frac{1}{\left(\frac{1}{d^{\cancel{2}}} \times \cancel{d}\right)}$$

$$\boxed{f \propto d}$$

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$h_f \propto \frac{f}{d^5}$ $\qquad$ $h_f = \frac{32 \mu V L}{w d^2}$

laminar

$h_f \propto \frac{1}{d^5}$ $\qquad$ $h_f = \frac{32 \mu Q L}{w \frac{\pi}{4} d^2 \cdot d^2}$

$\boxed{h_f \propto \frac{1}{d^4}}$ $\qquad$ $\boxed{h_f \propto \frac{1}{d^4}}$

Pg 42
Q2 Laminar flow

2h ✓ $\qquad h_f = ?$

$h_f = \frac{12 \mu V L}{w H^2}$ $\qquad$ $V = \frac{Q}{A}$

$V = \frac{Q}{H \times 1}$ $\qquad$ $V = \frac{Q}{H}$

$h_f = \frac{12 \mu Q L}{w \cdot H \cdot H^2}$

$\boxed{h_f \propto \frac{1}{H^3}}$

* Relationship b/w Darcie's friction factor and shear stress at the wall

$f' = \frac{\tau_w}{\frac{1}{2} \rho V^2}$ $\qquad$ $\boxed{\tau_w = \frac{f}{8} \rho V^2}$ V.V.V. Imp

$4f' = f$ $\qquad$ $\frac{\tau_w}{\rho} = \frac{f}{8} V^2$

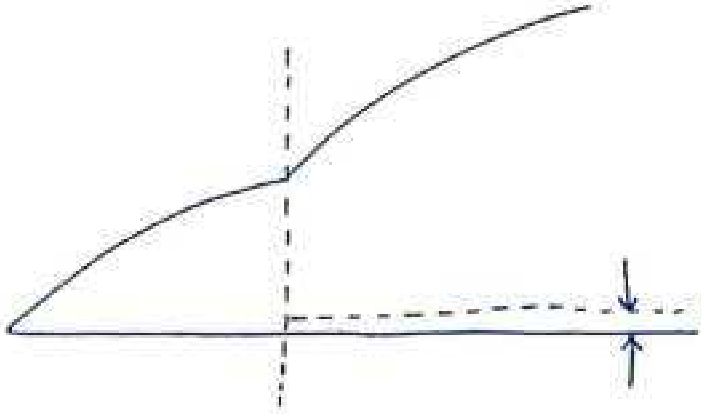
$\frac{f}{4} = \frac{\tau_w}{\frac{1}{2} \rho V^2}$ $\qquad$ $\sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{f}{8}} \cdot V$ (m/s)

mean velocity

shear velocity or friction velocity

$\boxed{V_* = \sqrt{\frac{f}{8}} \cdot V}$

Pg 44 → Sec ③



$$\delta' = \frac{11.6\nu}{\sqrt{\frac{\tau_\omega}{\rho}}}$$

$$\boxed{\delta' = \frac{11.6\nu}{V_*}}$$

Pg 37
③ 25%

$$\underbrace{\left(\frac{P_1}{\omega}+Z_1\right)-\left(\frac{P_2}{\omega}+Z_2\right)}=\frac{fLQ^2}{12.1d^5}$$

$$fQ^2 = \text{const}$$

$$\ln f + 2\ln Q = \ln C$$

$$\frac{df}{f} + 2\frac{dQ}{Q} = 0$$

$$dQ = -\frac{1}{2}\frac{df}{f}$$

$$= -\frac{1}{2}\times(+25\%)$$

$$= -12.5\%$$

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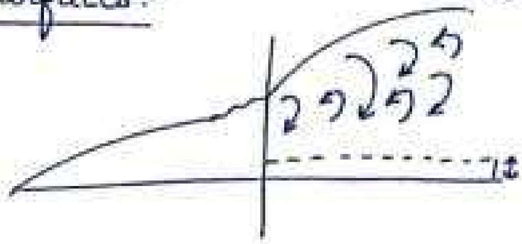
* Hydrodynamically smooth & Rough surfaces :-

Re ↓↓

Laminar



$$\boxed{Re = \frac{VL_c}{\nu}}$$

This concept is used for Turb. flow bec. Lami. flow is independent of surface Roughness.

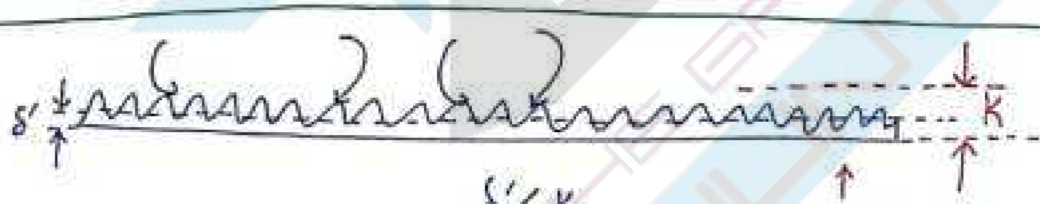
$\rightarrow \dfrac{K}{\frac{11.6\nu}{V_*}} < 0.25$

Surface Roughness

$$\boxed{\frac{K}{\delta'} < 0.25}$$

$K < \delta'$

K = Avg roughness



$\delta' < K$

$$\boxed{\frac{K}{\delta'} > 6}$$

$\rightarrow \delta' \; \dfrac{K}{\frac{11.6\nu}{V_*}} > 6$

$\nu \rightarrow$ Kinematic viscosity

S	R
$\dfrac{V_* K}{\nu} < 0.25 \times 11.6$	$6 \times 11.6 < \dfrac{V_* K}{\nu}$
$\boxed{\dfrac{V_* K}{\nu} < 3}$	$\boxed{70 < \dfrac{V_* K}{\nu}}$
Roughness Reynolds number	

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* Calculation of friction factor :-

Co-Relations

f

- $f = \phi[Re]$ — laminar
  - $Re \le 2000$
  - $$\boxed{f = \frac{64}{Re}}$$
  - $xy = c$
  - $\downarrow y = \frac{c}{x\uparrow}$
- $4000 \le Re$, $f = \phi[Re, K]$ — Turbulent
  - $f = \phi[Re]$ — Smooth Pipes
    - $4000 \le Re \le 10^5$
    - $$\boxed{f = \frac{0.316}{Re^{1/4}}}$$
    - Smooth surface $\left[\frac{K}{\delta'} < 0.25\right]$
  - $f = \phi[K]$ — Rough pipes
    - $$\boxed{\frac{1}{\sqrt{f}} = 1.74 + 2\log_{10}\left(\frac{R}{K}\right)}$$
    - radius of pipe $= R$
    - $K =$ surface Roughness
    - Rough Surface $\left[\frac{K}{\delta'} > 6\right]$

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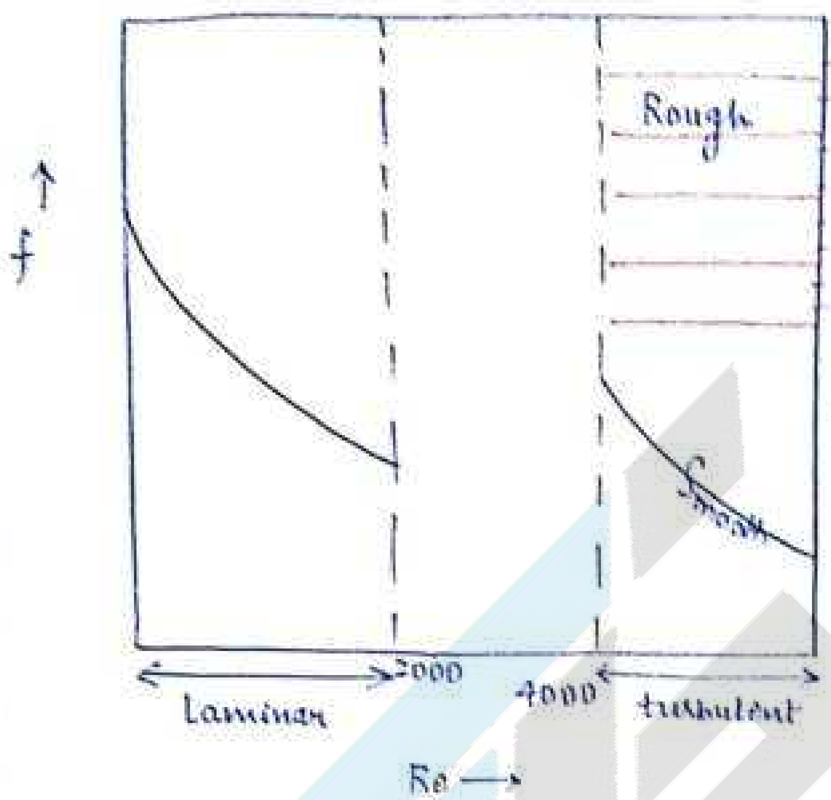
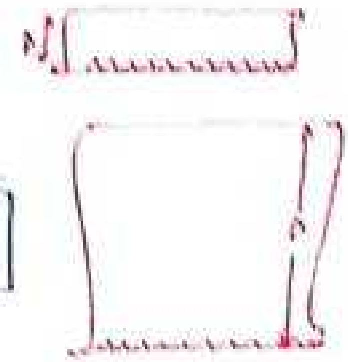
Stanton - Moody's chart



$\boxed{\epsilon = \frac{k'}{R}}$

Relative roughness



Gate Q28 Pg 45

WB Pg 45 Q26 Gate 2008

$\rho = 1000$

$\nu = 10^{-6}$

$k = 0.12 mm$

$\tau_w = 600$

$\frac{k}{\delta'} = ?$ $\quad \delta' = \frac{11.6\nu}{V_*}$

$$\delta' = \frac{11.6\nu}{\sqrt{\frac{\tau_w}{\rho}}}$$

$$= \frac{11.6 \times 10^{-6}}{\sqrt{\frac{600}{1000}}}$$

Q A Rough pipe of diameter 0.1m carries water at 20°C at the rate of 50 litre/sec. If the average height of roughness projection on the pipe surface is 0.15mm Determine :- (a) friction factor

(b) shear stress at the surface

(c) shear velocity

(d) maxm velocity

Kinematic viscosity, $\nu = 10^{-6}\ m^2/sec$

$\rho = 1000\ kg/m^3$

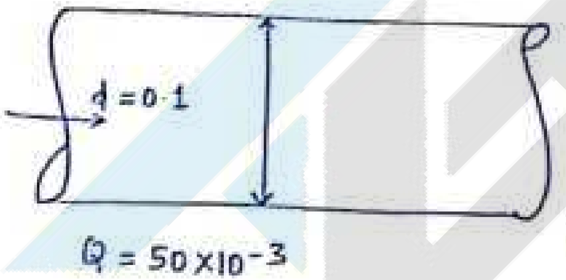
Sol

$D = 0.1m$

$Q = 50 \times 10^{-3}\ m^3/sec$

$h_{av} = 0.15mm$

S/R



$V = \dfrac{Q}{\frac{\pi}{4} d^2}$

$V = 6.36\ m/s$

$Re = \dfrac{VD}{\nu}$

$= \dfrac{6.36 \times 0.1}{10^{-6}}$

$= 6.36 \times 10^5 > 4000$

(turb)

Rough pipe

(1) $\dfrac{1}{\sqrt{f}} = 1.74 + 2\log_{10}\left(R/K\right)$

$\boxed{f = 0.0217}$

(ii) $f' = \dfrac{\tau_\omega}{\frac{1}{2}\rho V^2}$

$$\frac{f}{4} = \frac{\tau_\omega}{\frac{1}{2}\rho V^2}$$

$$\tau_\omega = \frac{f}{8}\rho V^2$$

$$= \frac{0.0217 \times 1000 \times 6.36^2}{8}$$

$$\tau_\omega = 109.8 \ N/m^2$$

(iii) $V_* = \sqrt{\dfrac{\tau_\omega}{\rho}}$

$$= \sqrt{\frac{109.8}{1000}}$$

$$V_* = 0.33 m/s$$

(iv) Rough and Smooth Pipes (turbulent flow)

imp

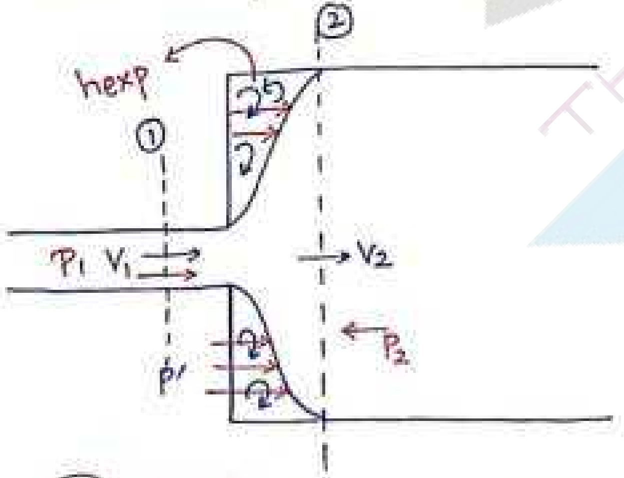
$$\boxed{\frac{U_{max}}{U_m} = 1 + 1.33\sqrt{f}}$$

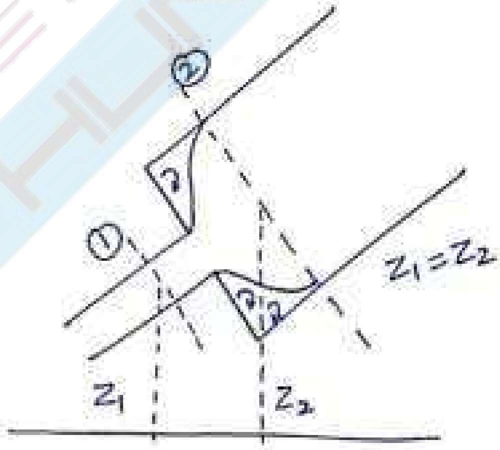
$$\boxed{U_m = 0.82 U_{max}}$$

$$\boxed{U_m = 7.61 m/s}$$

* Minor Losses

① Sudden Expansion loss :-





B.E. ① & ② & Momentum eqn.

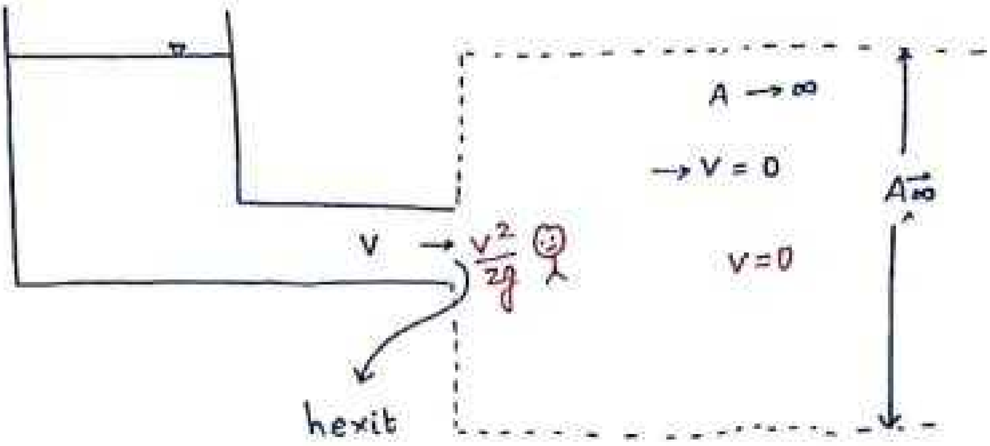
$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2 + h_{exp}$$

Assumption :-

$$\boxed{P_1 = P'}$$

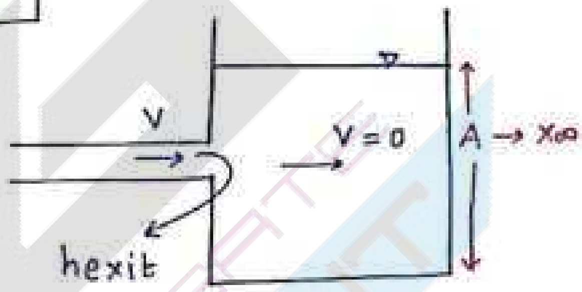
$$\boxed{h_{exp} = \frac{(V_1 - V_2)^2}{2g}}$$

② Exit loss



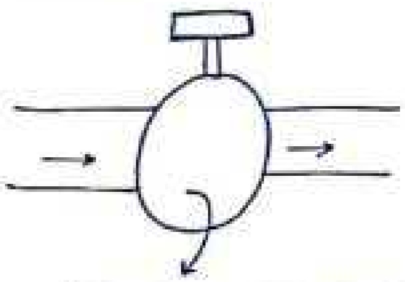
$$h_{exit} = \frac{(V-0)^2}{2g}$$

$$\boxed{h_{exit} = \frac{V^2}{2g}}$$



$$h_{exit} = \frac{V^2}{2g}$$

⑥ Valve loss



$$\boxed{h_v = k_v \frac{V^2}{2g}}$$

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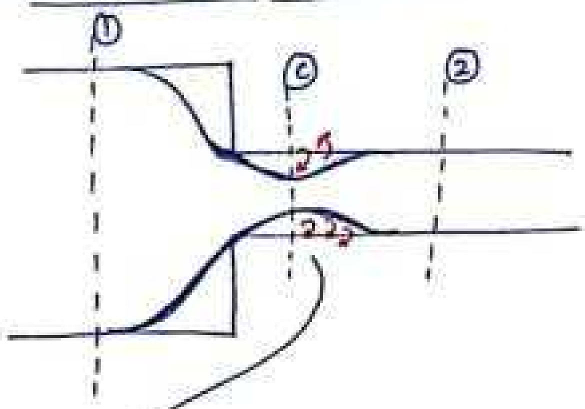
③ Sudden contraction loss



$$h_{cont} = \frac{(V_c - V_2)^2}{2g}$$

$$h_{cont} = \frac{V_2^2}{2g}\left[\frac{A_2}{A_c} - 1\right]^2$$

$$C_c = \frac{A_c}{A_2}$$

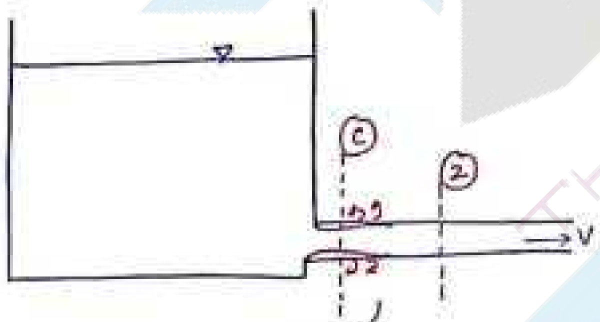
$$\boxed{h_{cont} = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2}$$

C_c is not given

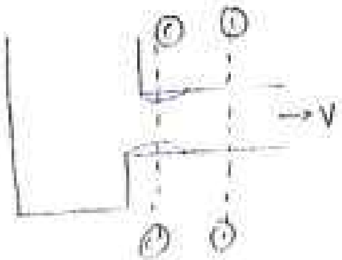
$$\left[\frac{1}{C_c} - 1\right]^2 \approx 0.5$$

$$\boxed{h_{cont} = \frac{0.5\,V_2^2}{2g}}$$

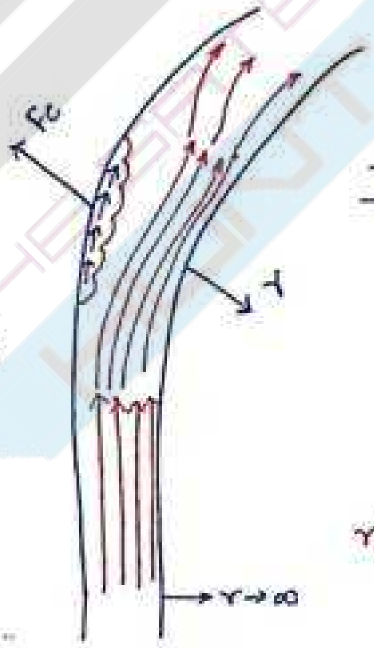
④ Entrance Loss

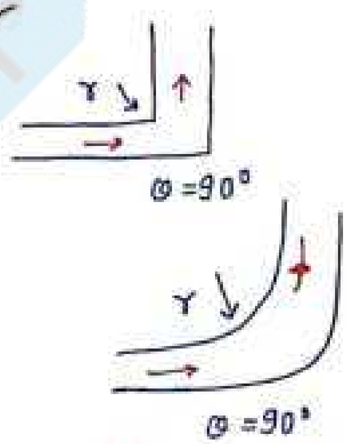


$$\boxed{h_{Ent} = \frac{0.5\,V^2}{2g}}$$



⑤ Bend loss :-





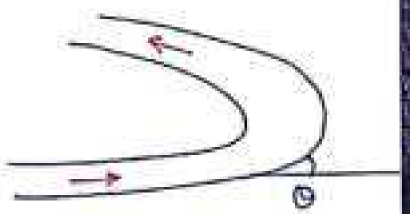
r↓ → K↑ → h_{bend}↑

$$\boxed{h_{bend} = \frac{KV^2}{2g}}$$

K → K bend

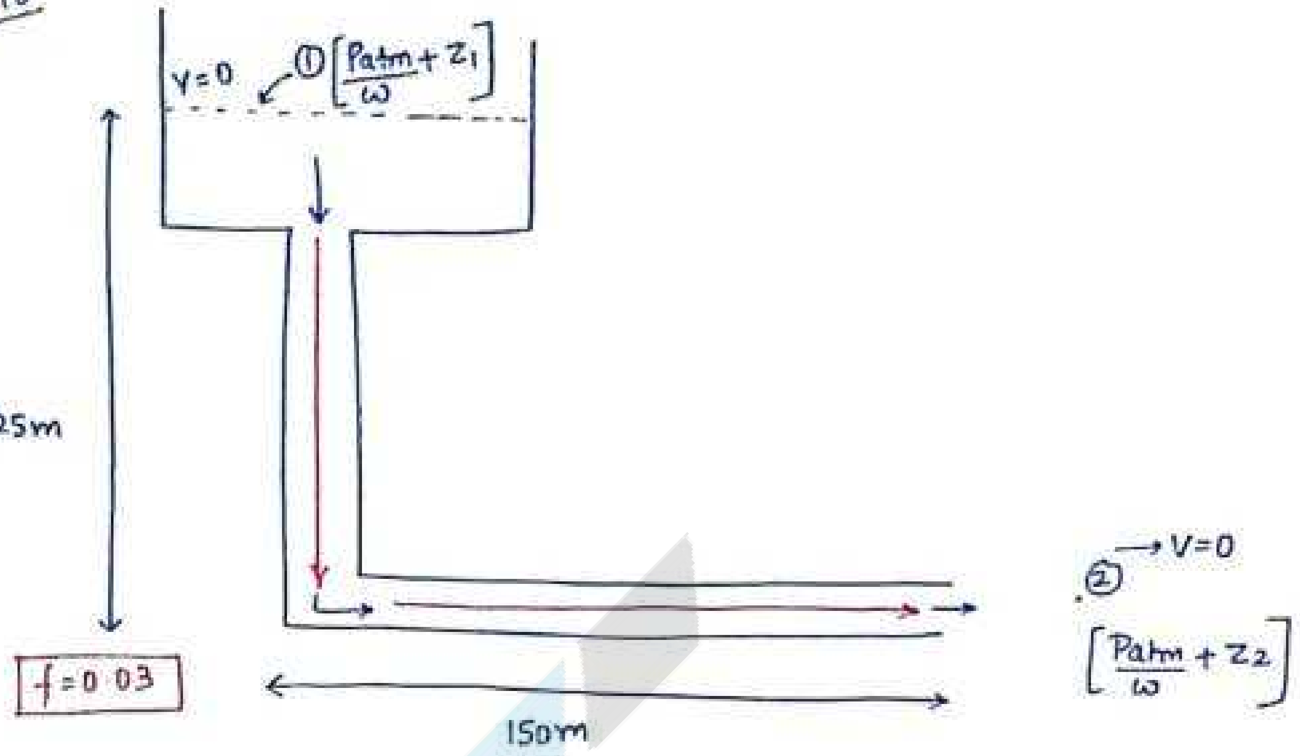
design parameter

[examiner द्वारा दिया given]



θ↑ → K↑

Pg 39
Q18



$$\left(\frac{P_1}{\omega} + z_1\right) - \left(\frac{P_2}{\omega} + z_2\right) = h_{loss}$$

$$(0+z_1) - (0+z_2) = \cancel{\frac{0.5V^2}{2g}} + \frac{fL_1Q^2}{12.1d^5} + \cancel{\frac{kV^2}{2g}} + \frac{fL_2Q^2}{12.1d^5} + \cancel{\frac{V^2}{2g}}$$

Sudden contraction losses

Major losses

Bend losses

Major losses

exit loss

$$25 = \frac{f(L_1+L_2)Q^2}{12.1d^5}$$

$$25 = \frac{0.03\,[175]\,Q^2}{12.1 \times (0.1)^5}$$

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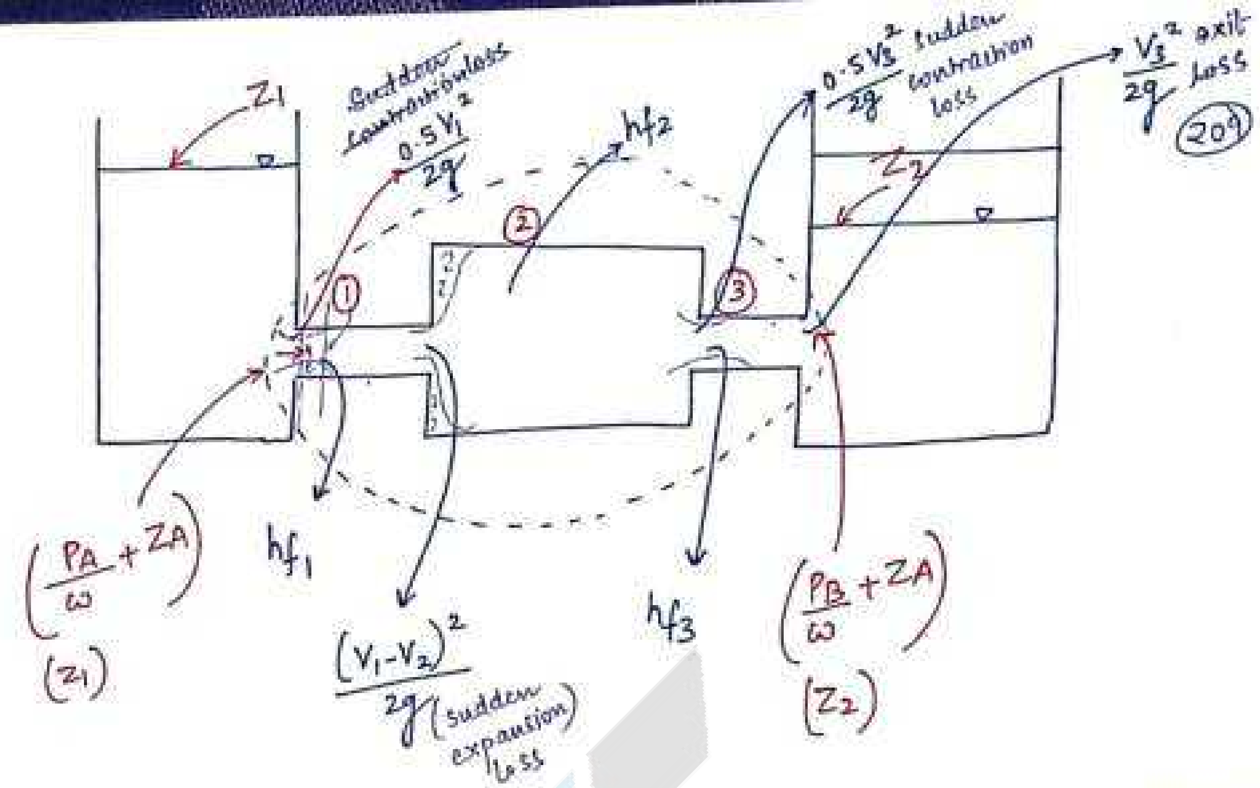
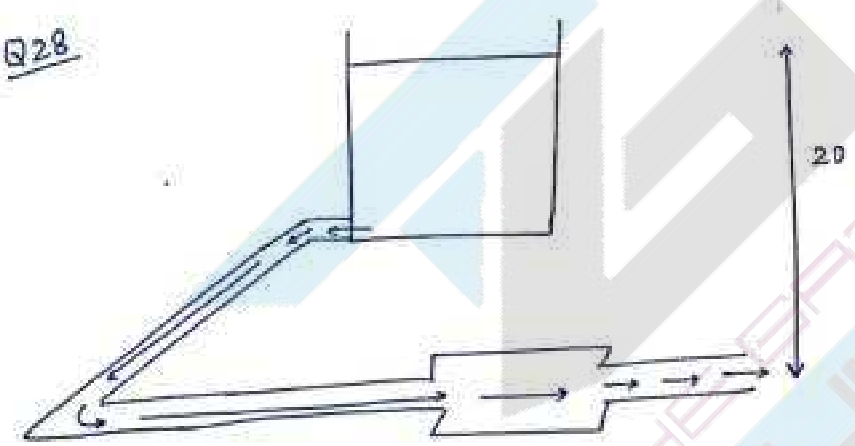
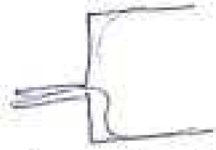


entrace loss + bend loss + major loss + bend loss + major loss + exp + contr + major loss + exit loss

Q28

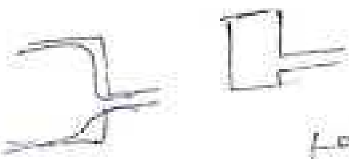




Sudden expansion loss $= \dfrac{(V_1-V_2)^2}{2g} = h_{exp}$

Exit loss $= \dfrac{V^2}{2g} = h_{exit}$

Valve loss $= K_v \dfrac{V^2}{2g} = h_{valve}$



Sudden Contraction loss $= 0.5\dfrac{V^2}{2g} = h_{contr.}$

Entrance loss $= 0.5\dfrac{V^2}{2g} = h_{entrance}$

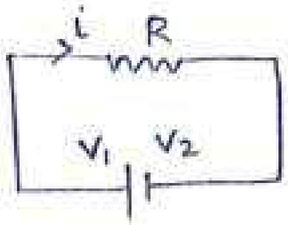
Bend loss $= \dfrac{KV^2}{2g} = h_{Bend}$

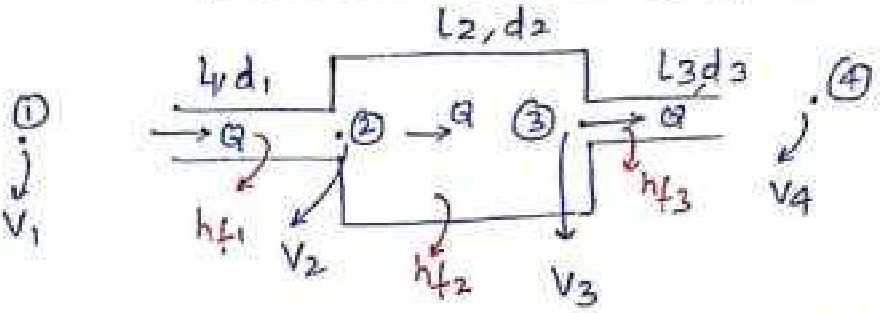
13/11/16

* Compound Pipes :-

① Series Connection → (discharge same, losses different)

$$\boxed{Q_1 = Q_2 = Q_3 = \cdots = Q} \quad ①$$





$$\boxed{V_1 - V_2 = iR}$$

$$\left[\frac{P_1}{\omega} + z_1\right] - \left[\frac{P_2}{\omega} + z_2\right] = h_f$$

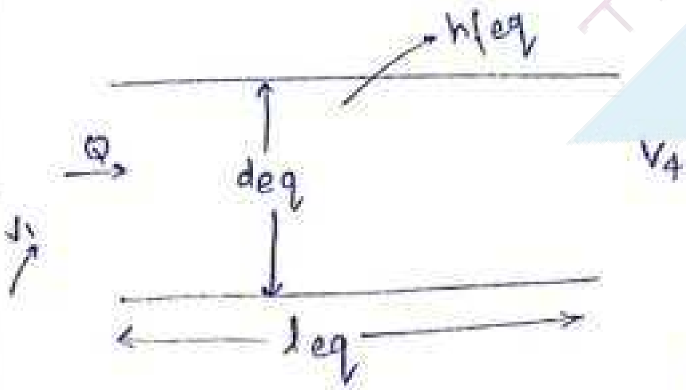
Assumptions :- ① Minor losses are neglected

② $\boxed{f_1 = f_2 = \cdots = f}$

$$V_1 - V_4 = (V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4)$$

$$\boxed{h_{feq} = h_{f_1} + h_{f_2} + h_{f_3}} \quad ③$$ → losses added

$$\frac{\cancel{f} L_{eq} \cancel{Q^2}}{\cancel{12.1} d_{eq}^5} = \frac{\cancel{f} l_1 \cancel{Q^2}}{\cancel{12.1} d_1^5} + \frac{\cancel{f} l_2 \cancel{Q^2}}{\cancel{12.1} d_2^5} + \cdots$$

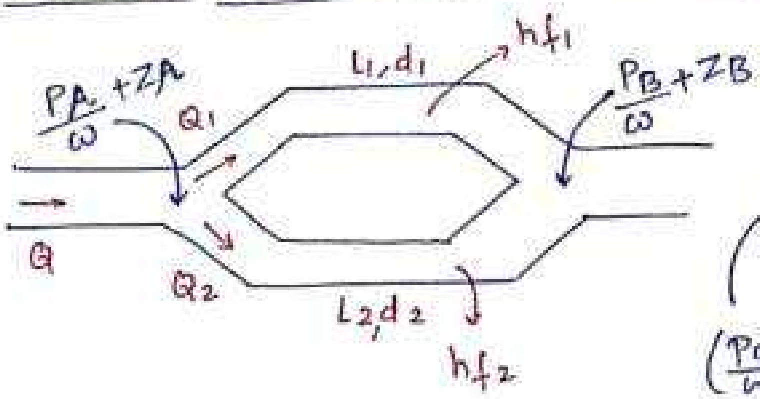


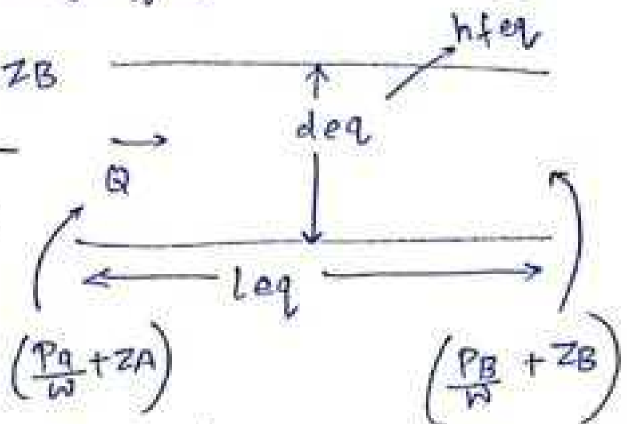
$$(V_1 - V_4 = h_{feq})$$

$$\boxed{\frac{L_{eq}}{d_{eq}^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \cdots}$$

Dupuit's eqn.

Parallel Connection → (losses same, discharge different)





Pipe I :- $\left[\frac{P_a}{w}+Z_A\right]-\left[\frac{P_B}{\omega}+Z_B\right]=h_{f_1}$

Pipe II :- $\left[\frac{P_a}{w}+Z_A\right]-\left[\frac{P_B}{w}+Z_B\right]=h_{f_2}$

$\left[\frac{P_A}{w}+Z_A\right]-\left[\frac{P_B}{w}+Z_B\right]=h_{feq}$

$\boxed{h_{f_1}=h_{f_2}=h_{feq}}$ — ①

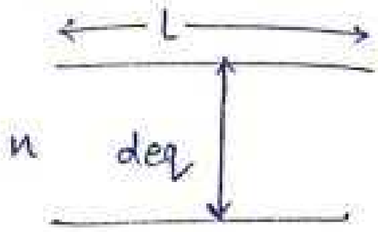
$\boxed{Q=Q_1+Q_2}$ — ②

After solving ① & ②

$$\boxed{\frac{d_{eq}^{5/2}}{L_{eq}^{1/2}}=\frac{d_1^{5/2}}{L_1^{1/2}}+\frac{d_2^{5/2}}{L_2^{1/2}}+\cdots}$$

$L_1=L_2=L_3=\cdots L_{eq}=L$ $\quad d_1=d_2=\cdots=d$

$$\frac{d_{eq}^{5/2}}{\cancel{L^{1/2}}}=\frac{d^{5/2}}{\cancel{L^{1/2}}}+\frac{d^{5/2}}{\cancel{L^{1/2}}}+\cdots$$



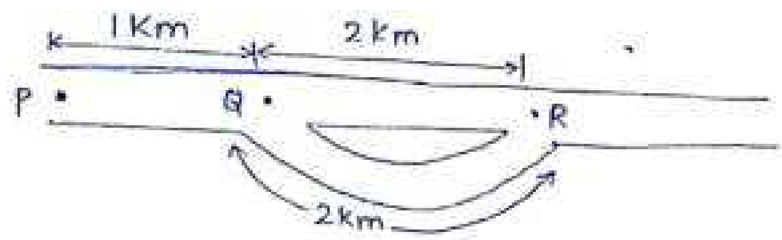
$\boxed{d_{eq}=n^{2/5}d}$

$n=3$

$\boxed{d_{eq}=9^{1/5}d}$ $\quad \boxed{d_{eq}=1.55d}$

Q WB Pg 38 (9) (10) (8) (11) ✓

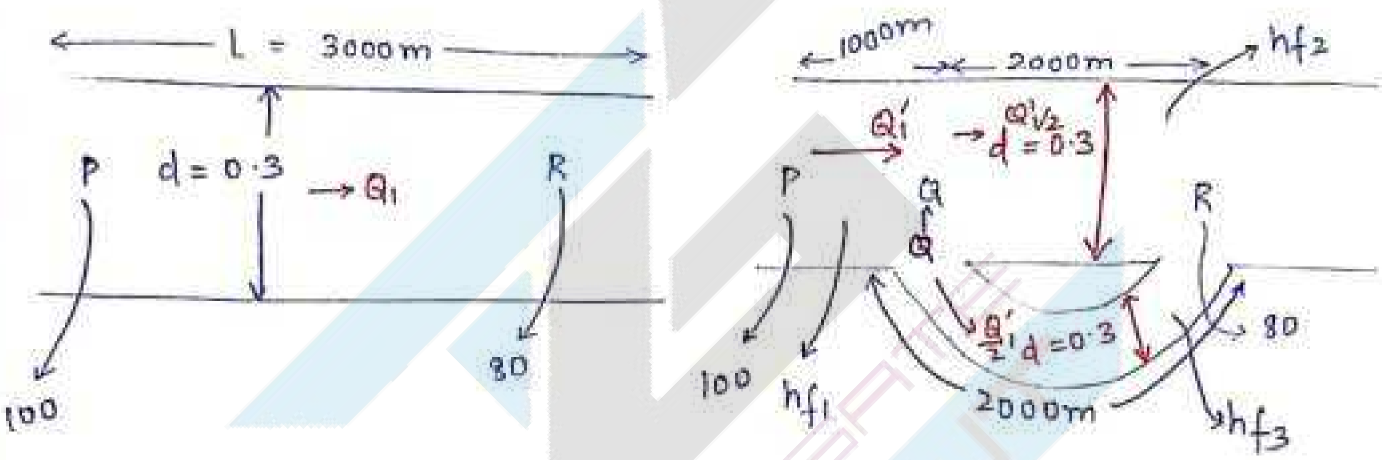
(19)



$D = 0.3m$ $P \rightarrow 100m$ $f = 0.04$

$L = 3KM$ $R \rightarrow 80m$

SIR



$f = 0.04$

$$100 - 80 = \frac{fLQ_1^2}{12.1d^5}$$

$$20 = \frac{0.04 \times 3000 \times Q_1^2}{12.1 \times (0.3)^5}$$

$$Q_1 = 0.07\ m^3/s$$

$$100 - 80 = h_{f_1} + h_{f_2} + \cancel{h_{f_3}}$$

In II connection,

h_f of one pipe लेते हैं

$$20 = \frac{f(1000)\,Q_1'^2}{12.1d^5} + \frac{f(2000)(Q_1'/2)^2}{12.1d^5}$$

$$Q_1' = 0.099\ m^3/s$$

$$\%\uparrow = \frac{Q_1' - Q_1}{Q_1} \times 100$$

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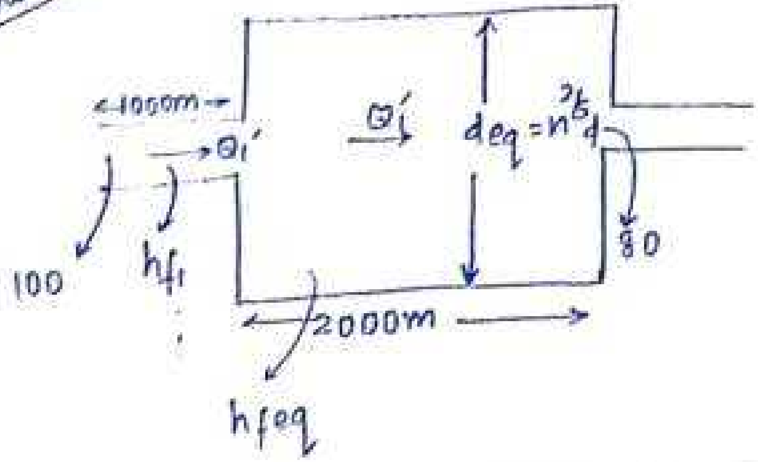
alternate



$$100 - 80 = h_{f_1} + h_{feq}$$

$$1 = \frac{f(1000) Q_1'^2}{12.1 d^5} + \frac{f(2000) Q_1'^2}{12.1 (n^{2/5} d)^5}$$

$n \to \infty$

$h_{feq} = 0$

$$20\% = \frac{f \times 1000 \times Q_1'^2}{12.1 d^5}$$

$Q_1' = ?$

(20) theoretically

(d)

Total energy line (TEL) joining the points, representing the values of total energy head $\left(P/\omega + z + \frac{V^2}{2g}\right)$ at different parts

Hydraulic gradient line (HGL) line joining the points representing the values of piezometric heads at different points along the flow

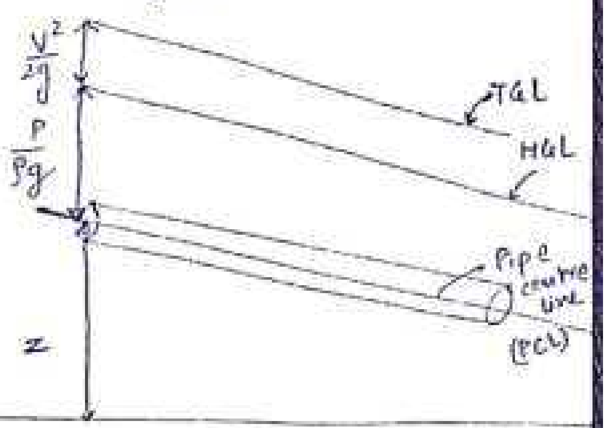
TOTAL ENERGY LINE and HYDRAULIC GRADIENT LINE :-

T.E.L. = Locus of Total Energy in flow

$$= \left(\frac{P}{\omega} + z\right) + \frac{V^2}{2g}$$

H G L. = locus of Piezometric heads along flow

$$= \left(P/\omega + z\right)$$

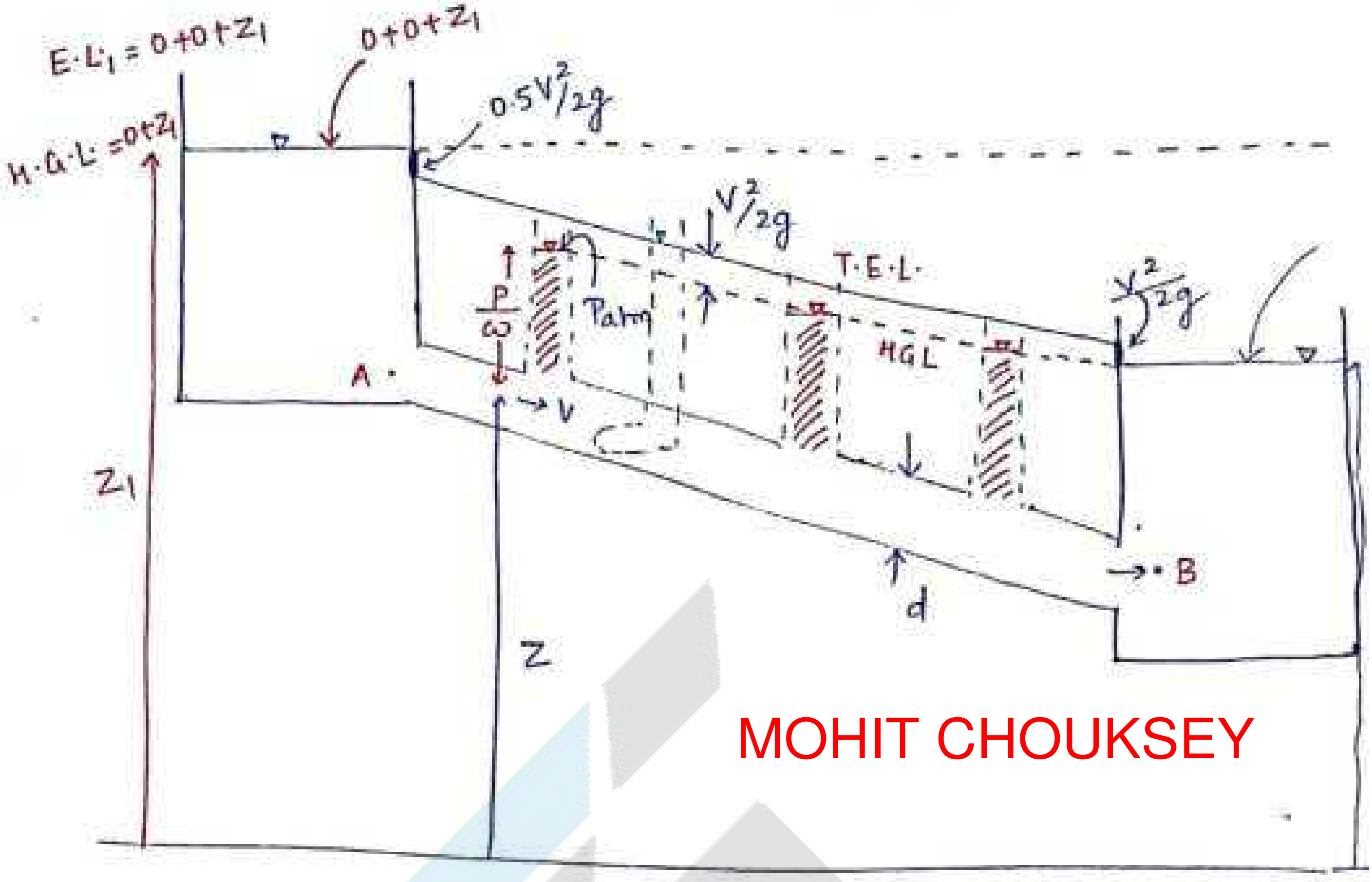


$$T.E.L. - H.G.L. = \frac{V^2}{2g}$$

$V = 0 \qquad V \geq 0$

$$T.E.L. - H.G.L. \geq 0$$

$$\boxed{T.E.L. \geq H.G.L.}$$



$EL_2 = Z_2$

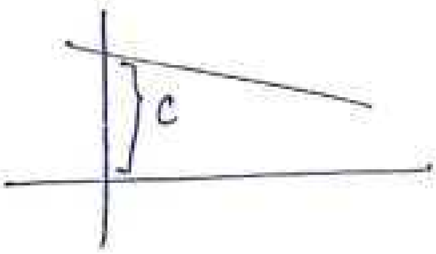
$H{\cdot}G{\cdot}L_2 = Z_2$

$$E{\cdot}L_1 = E{\cdot}L_2 + h_f$$

$$E{\cdot}L_2 = E{\cdot}L_1 - h_f$$

$$E{\cdot}L_2 = E{\cdot}L_1 - \frac{\cancel{f}LV^2}{\cancel{2}g\cancel{d}}$$

$$Y = c - mx$$



$\frac{0.5V^2}{2g}$ → entrace loss or sudden contraction loss

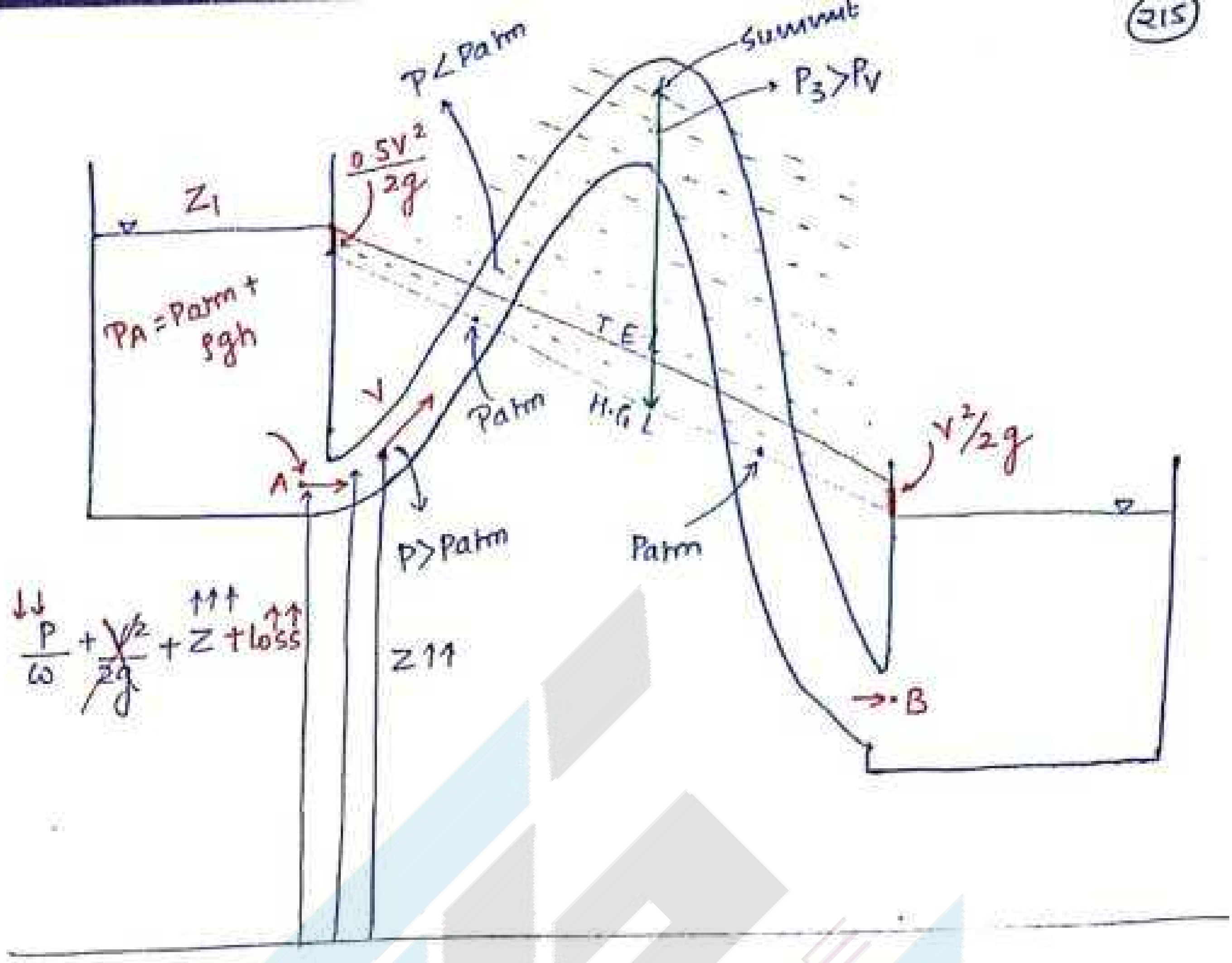
$\frac{V^2}{2g}$ → exit loss

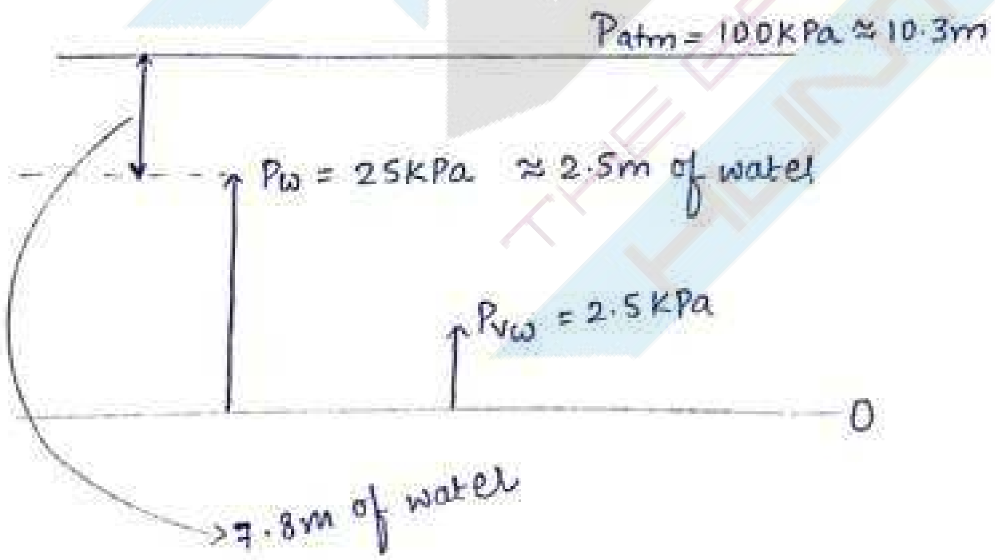
Note → ① If HGL is above the pipe centre line, flow is under +ve gauge pressure. If it coincides, the flow is under atm pressure. If it's below, the pressure is -ve gauge pressure.

② TEL is above HGL in flow but they coincides when there is no flow.

③ Pipe is converging. it is an accl^r flow, TEL & HGL diverges.

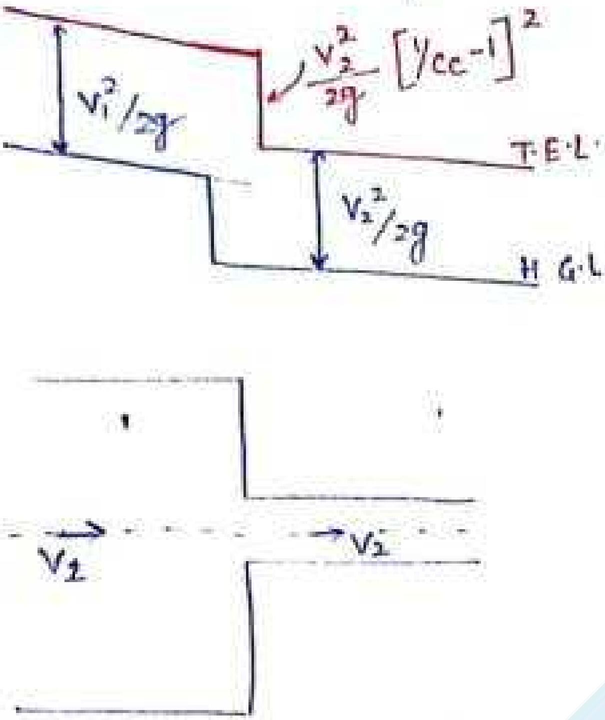
④ Pipe is diverging. it is decelerated flow, TEL & HGL converges.

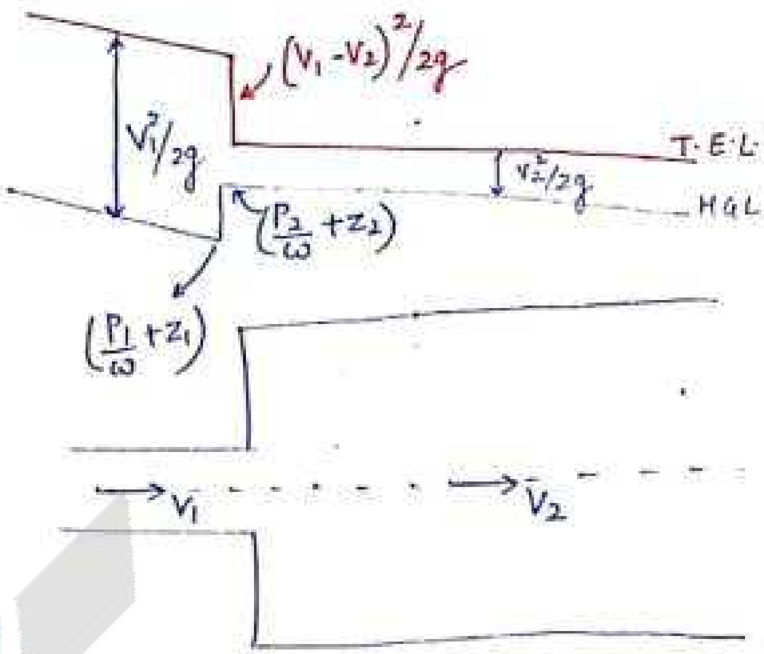




If HGL is above the pipeline, the pipe is in +ve gauge. if coincides, the pipe is at atmospheric pressure but if at below pipe is in -ve gauge.

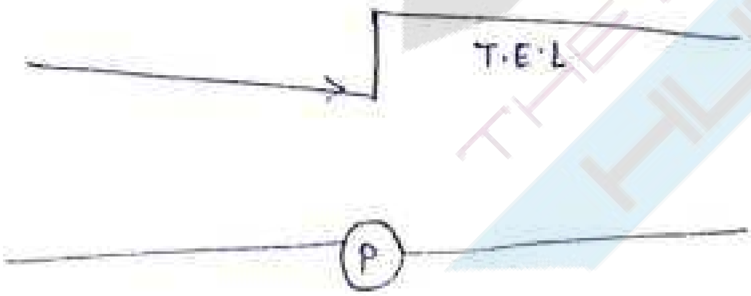
H.G.L. and T.E.L. for sudden expansion and contraction:-

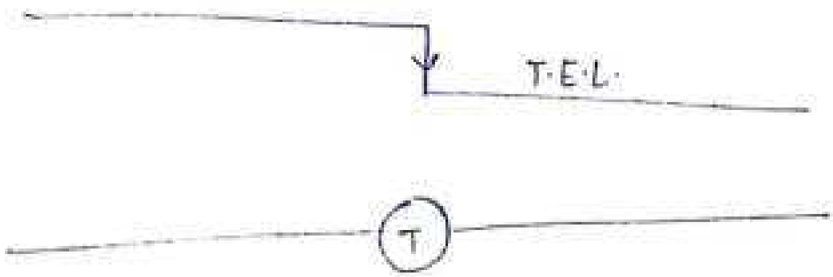




$$\text{T.E.L.} = \left(\frac{P}{\omega}+z\right) + \frac{V^2}{2g} + h_{exp}$$

$$\text{H.G.L} = \left(\frac{P}{\omega}+z\right)$$



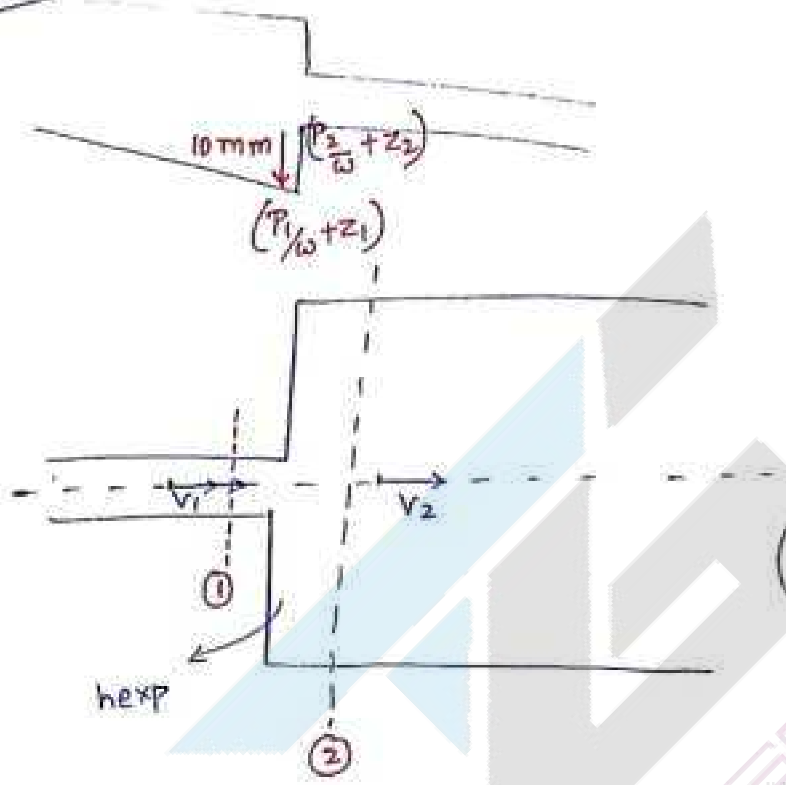


MOHIT CHOUKSEY

Note → HgL (HGL) can rise or fall along the flow through the pipe but TEL cannot rise unless there is some external energy input

Q At a sudden expansion of a water pipeline from a diameter of 0.24 m to 0.48 m. The H.G.L. rises by 10 mm, then find the discharge

Sol D = 0.24 m to 0.48



$$\left(\frac{P_2}{\omega}+Z_2\right)-\left(\frac{P_1}{\omega}+Z_1\right)=10\times10^{-3}$$

$$\left(\frac{P_1}{\omega}+Z_1\right)+\frac{V_1^2}{2g}=\left(\frac{P_2}{\omega}+Z_2\right)+\frac{V_2^2}{2g}+h_{exp}$$

$$\frac{V_1^2-V_2^2}{2g}-h_{exp}=\left(\frac{P_2}{\omega}+Z_2\right)-\left(\frac{P_1}{\omega}+Z_1\right)$$

$$A_1V_1=A_2V_2$$

$$\frac{V_2}{V_1}=\frac{A_1}{A_2}$$

$$\frac{V_2}{V_1}=\frac{d_1^2}{d_2^2}$$

$$\frac{V_2}{V_1}=1/4$$

$$\frac{V_1^2}{2g}\left[1-\frac{V_2^2}{V_1^2}\right]-\frac{V_1^2}{2g}\left[1-\frac{V_2}{V_1}\right]^2=10\times10^{-3}$$

$V_1=?$

$Q=A_1V_1=0.0327\ m^3/s$

32 l/s

very large Q

MOHIT CHOUKSEY

~~FRICTIONAL POWER~~ / ~~TOTAL~~ WATER POWER :-

WB
(14) Pg 43

Q ↑ ⟶ 20%

laminar regime

P ⟶ ?

SIR

$P_1 \propto Q^2$

$P_2 \propto (1.2Q)^2$

$P_2 - P_1 \propto 0.44Q^2$

$\frac{P_2 - P_1}{P_1} \times 100 \propto \frac{0.44 Q^2}{Q^2} \times 100 = 44\%$ } Avg. analysis

a 44
b 40% ✓ then True

But in Gate, use avg. analysis method then 44% ✓

if using $\log P_1 \propto 2 \log Q$ ⟶ 40% Ans

→ use linear assumption in Gate not log (assumption)

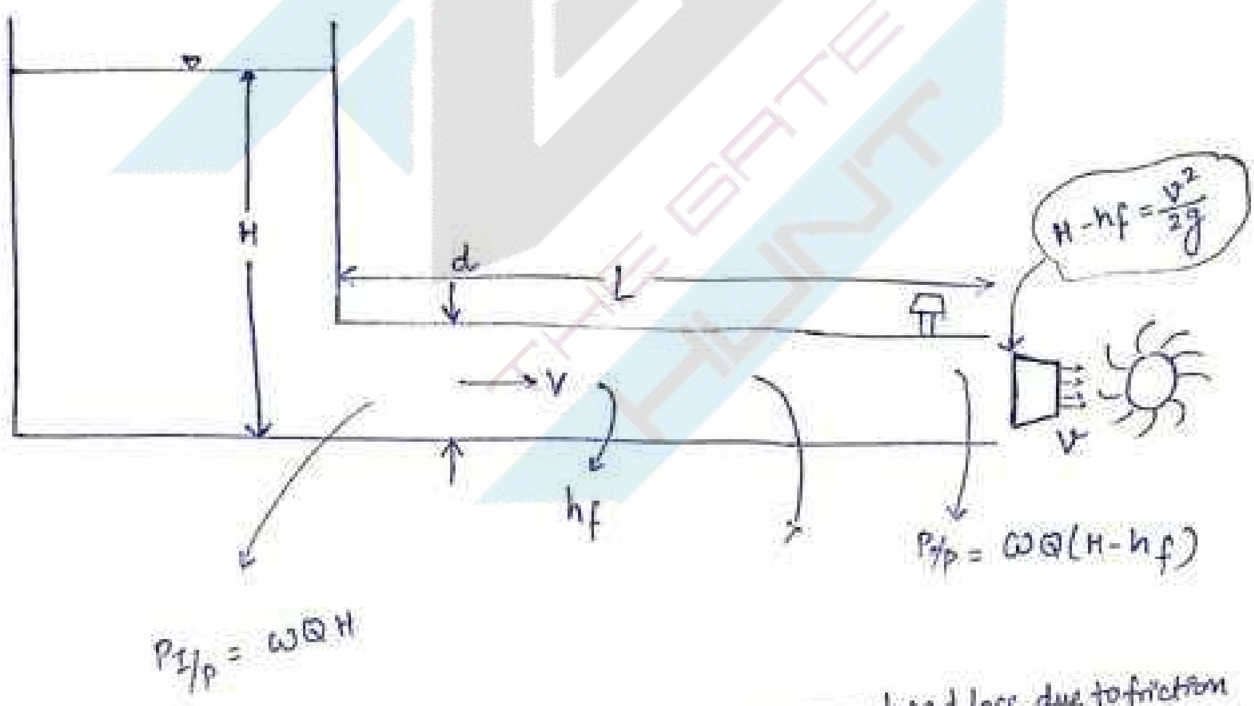
~~# POWER TRANSMISSION~~

Theory of frictional power or water power →

MOHIT CHOUKSEY

POWER TRANSMISSION THROUGH THE PIPE →

$P = F \times V = (P \times A) V = P \times Q = (wh) Q \quad (\because \frac{P}{w} = h)$



$\boxed{\text{Power} = wQ(H - h_f)}$ → head loss due to friction

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$$\boxed{\text{Power} = wQ(H - h_f)} = \omega Q\left[H - \frac{fLQ^2}{12.1D^5}\right]$$

maximum Power

$$P = \omega\left[QH - \frac{fLQ^3}{12.1d^5}\right]$$

$$H = \text{constant}$$

$$\frac{dP}{dQ} = \omega\left[H - \frac{3fLQ^2}{12.1d^5}\right]$$

maxima

$$\frac{dP}{dQ} = 0$$

$$\omega\left[H - 3h_f\right] = 0$$

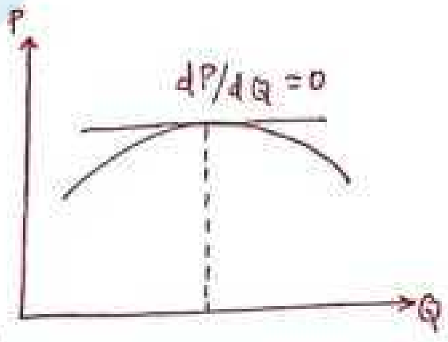
$$H - 3h_f = 0$$

$$\boxed{h_f = H/3}$$

$$P_{max} = \omega Q(H - h_f)$$

$$= \omega Q\left[H - H/3\right]$$

$$= 2/3\ wQH$$



$$h_f = H/3$$

$$\frac{\cancel{f}\cancel{L}Q^2}{121\,\cancel{d^5}} = H/3$$

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$$\eta = \frac{P_{o/p}}{P_{I/P}} = \frac{\cancel{\omega} Q (H - h_f)}{\cancel{\omega Q} H}$$

$$\boxed{\eta = \frac{H - h_f}{H}}$$

$$\eta_{max} = \frac{H - H/3}{H}$$

$$= 0.6667$$

$$\boxed{\eta_{max}\,\% = 66.67\%}$$

Kinetic head   Pressure head

*** WATER HAMMER :-** When a flowing fluid is brought to rest suddenly, dynamic energy of the fluid converted into the pressure energy i.e. pressure waves are formed. These pressure waves creates huge noise known as knocking known as water hammering. These pressure waves even may burst the pipe.

Surge Tanks are used to release the excess pressure rise as well as works as temporary reservoirs.

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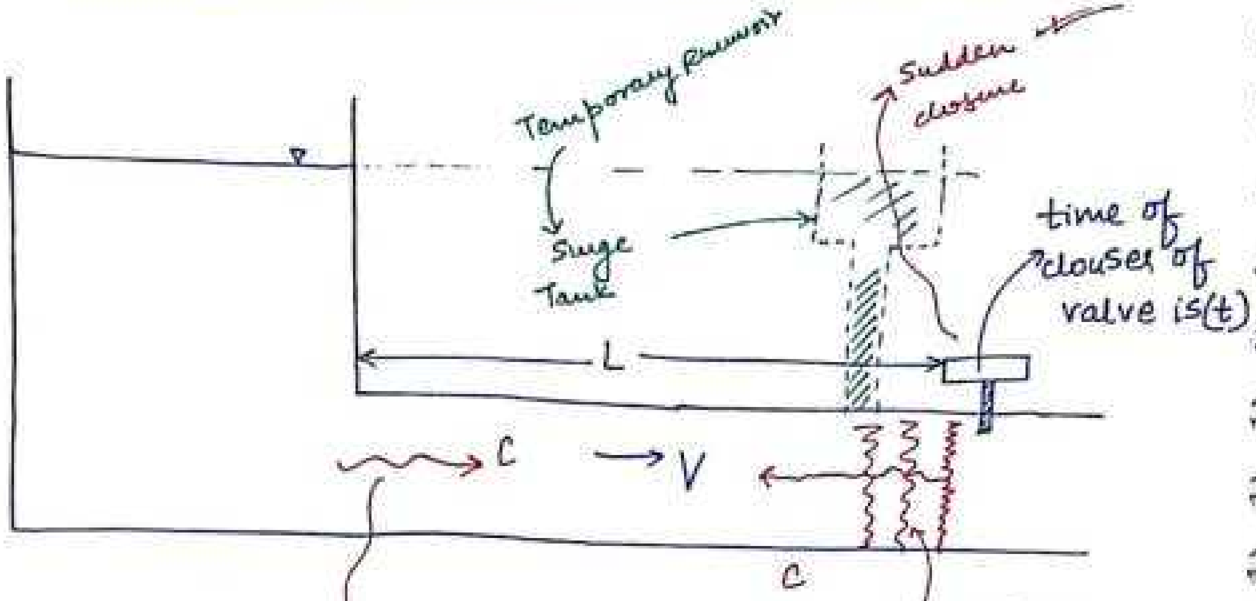


pressures generating & pressure waves moving ← but water को compress karna aasan nahin hai

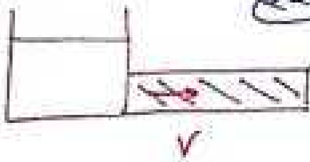
pressure waves खत्म jayengey due to capacity of the tank hence pressure rise not high.

pressure wave movement gradual it hoga
↓
gradual closure करोगे stoppage को

time travelled by wave

$$C = \sqrt{K/\rho}$$

$$\boxed{t' = \frac{2L}{C}}$$



Pressure rise

gradual closure

$t' < t$

(momentum analysis)

$$\boxed{P = \frac{\rho V L}{t}}$$

V → initial velocity of flow

t → time of closure

sudden closure

$t < t'$

(Energy analysis)

Rigid pipe

$(KE)_f = (SE)_f$

$\cancel{\frac{1}{2}}\rho(\cancel{vol})V^2 = \cancel{\frac{1}{2}}\frac{P^2}{K}\cancel{vol}$

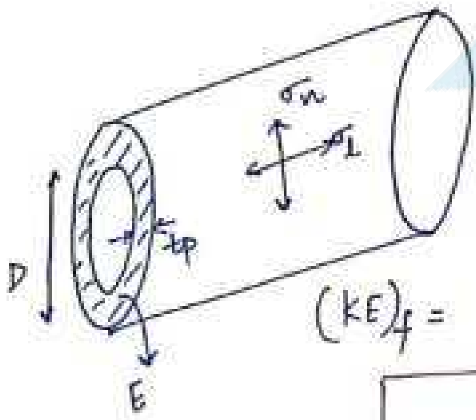
$P^2 = \rho.\rho.V^2.\frac{K}{\rho}$

$$\boxed{P = \rho V\sqrt{K/\rho}}$$

$\sqrt{K/\rho} = C$

$$\boxed{P = \rho V C}$$

Elastic pipe (steel)



$(KE)_f = (SE)_f + (SE)_p$ → C'

$$\boxed{P = \rho V\sqrt{\frac{K/\rho}{1 + \frac{DK}{E\,t_p}}}}$$

DK ← direct

t_p ← [illegible]

[illegible] → E

$E \longrightarrow \infty$ Rigid Pipe

$c' \Rightarrow$ water hammer wave velocity

WB
Pg 40
(22)

$1414 = \sqrt{K/\rho} \longrightarrow$ Refr. velocity

the velocity ------ pressure wave

$c' = ?$

D ✓
K ✓

data all given

1000 ← Ans.

ant. elastic pipe

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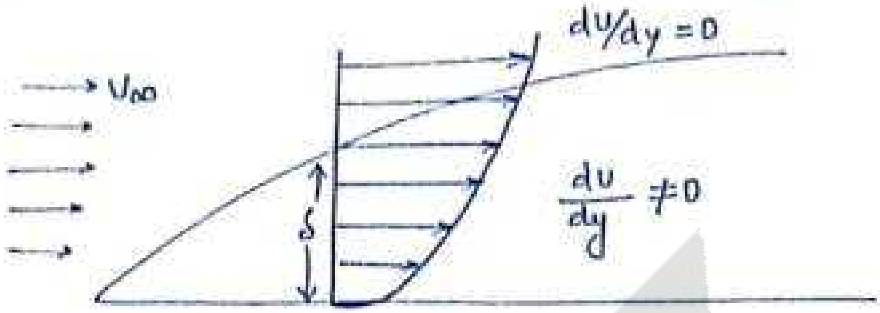
* BOUNDARY LAYER THEORY

when viscous fluid flows over the solid surface, near the surface, large velocity gradients occurs. The region where these gradients occurs is known as Boundary layer region.

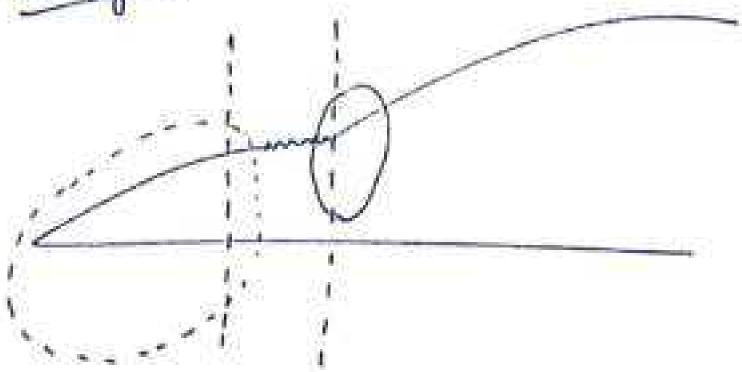


☆ Factors affecting transition from laminar to turbulent →

① Viscosity of the fluid.
② Velocity of the flow.
③ surface roughness.
④ Density of the fluid.
⑤ Pressure gradient over the surface.
⑥ External disturbance.
⑦ Tempr. of the plate or the fluid etc.
⑧ Geometry of the surface

☆ Boundary Conditions for laminar flow over the plate →

Q34(Pg 51) ✓



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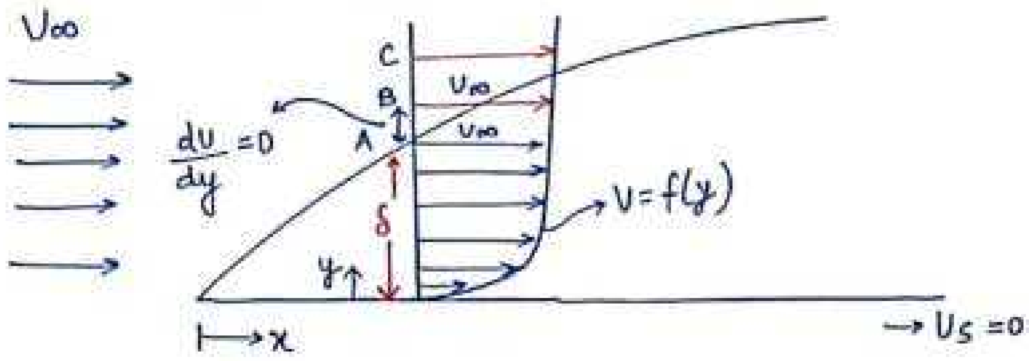

$$\boxed{\frac{U_A - U_s}{U_\infty - U_s} = 0.99}$$

$U_s = 0$

$$\boxed{\frac{U_A}{U_\infty} = 0.99}$$

calculation

$\boxed{y = \delta}$ $\boxed{V = V_\infty}$

at $\boxed{x = 0}$ $\boxed{\delta = 0}$

(i) $\boxed{y = 0}$ $\boxed{V = 0}$

(ii) $\boxed{y = \delta}$ $\boxed{V = V_\infty}$

(iii) $\boxed{y = \delta}$ $\boxed{\frac{du}{dy} = 0}$

(iv) $\boxed{y = 0}$ $\tau = \tau_{max}$

$$\tau = \mu \frac{du}{dy}$$

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maximum

$$\frac{d\tau}{dy} = 0 \quad y = 0$$

$$\mu \frac{d^2 u}{dy^2} = 0 \quad y = 0$$

$\boxed{y = 0}$ $\boxed{\frac{d^2u}{dy^2} = 0}$

Q $$\frac{u}{U_\infty} = a + b\left(y/\delta\right) + c\left(\frac{y^2}{\delta^2}\right) + d\left(\frac{y}{\delta}\right)^3$$

$y = 0, \quad u = 0, \; a = 0$

$b = 3/2$

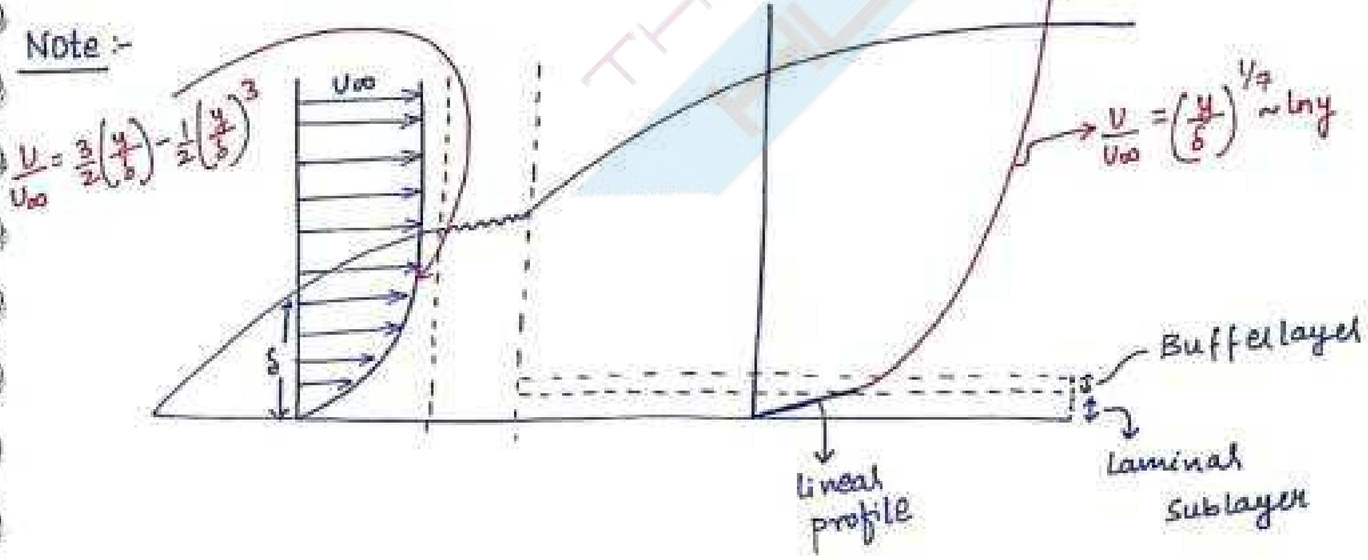
$c = 0$

$d = -1/2$

$$\boxed{\frac{u}{U_\infty} = 3/2\left(y/\delta\right) - 1/2\left(y/\delta\right)^3}$$

Laminar - Plate

Note :-

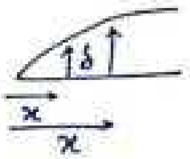


$$\tau_w = \mu \frac{du}{dy}\Big|_{y=0}$$

$$\boxed{\tau_w = \mu V_\infty \frac{3}{2} \times \frac{1}{\delta}}$$

$\tau_w \propto 1/\delta$

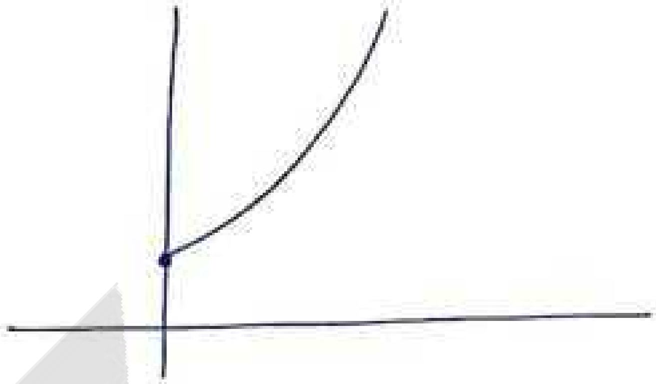
$x\uparrow \rightarrow \delta\uparrow \rightarrow \tau_w\downarrow$



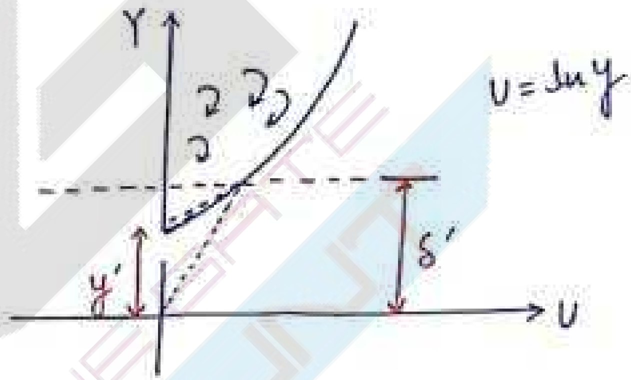
$$\tau_w = \mu \frac{du}{dy}\Big|_{y=0}$$

$$= \mu V_\infty \cdot \frac{1}{\delta^{1/7}} \cdot \frac{1}{7} \frac{1}{y^{6/7}}\Big|_{y=0}$$

$y \to 0 \quad \tau_w \to 0$



$\downarrow$



$\boxed{y = y'}$ $\boxed{u = 0}$

Pipe flow (Turbul.)

Smooth pipe

$\boxed{y = y'}$

$y' = \frac{\delta'}{107}$

$$\boxed{y' = \frac{11.6\nu}{107 V^*}}$$

Rough Pipe

$y' = \frac{k}{30}$

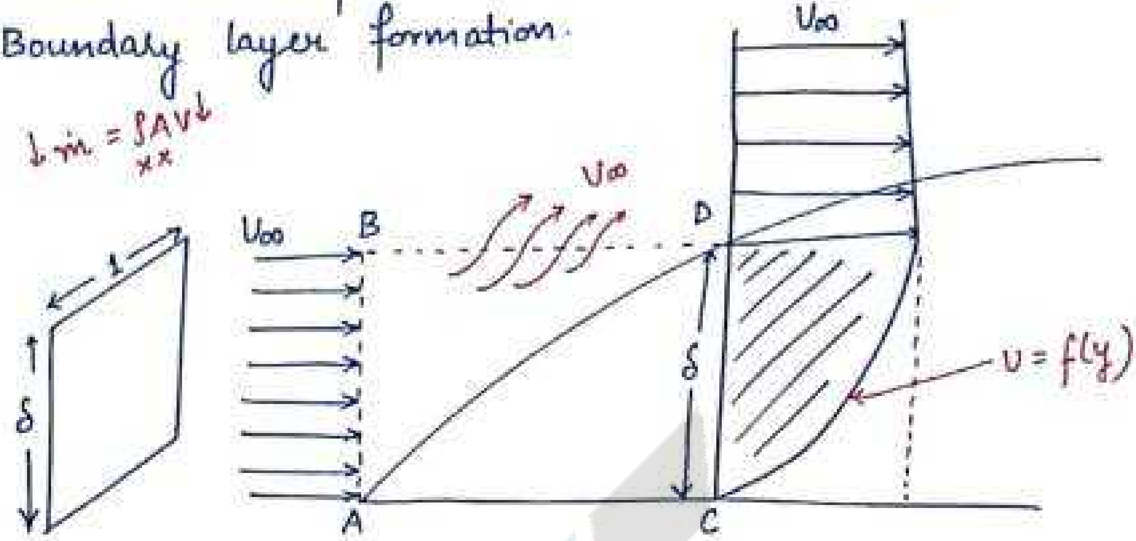
$$\boxed{y' = k/30}$$

MOHIT CHOUKSEY

* Displacement Thickness (δ^*) :-

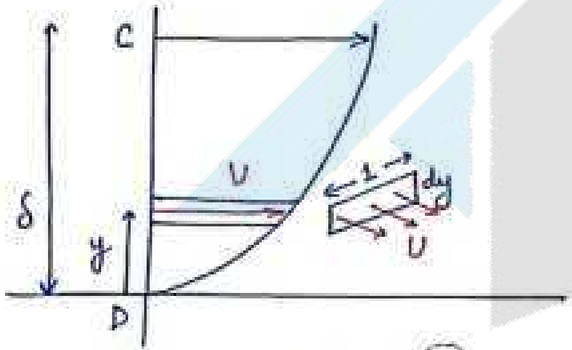
It is the distance by which the Boundary should be displaced in order to compensate the loss in mass flow rate due to Boundary layer formation.

$\downarrow \dot{m} = \rho A V \downarrow$



$\dot{m}_{AB} = \rho[\delta \times 1]U_\infty$

$\dot{m}_{BD} = \dot{m}_{AB} - \dot{m}_{DC}$



$d\dot{m} = \rho\, dA\, U$

$\int d\dot{m} = \int_0^\delta \rho U (dy \times 1) \Rightarrow \dot{m}_{DC} = \rho \int_0^\delta U\, dy$

Pg 50 (28)

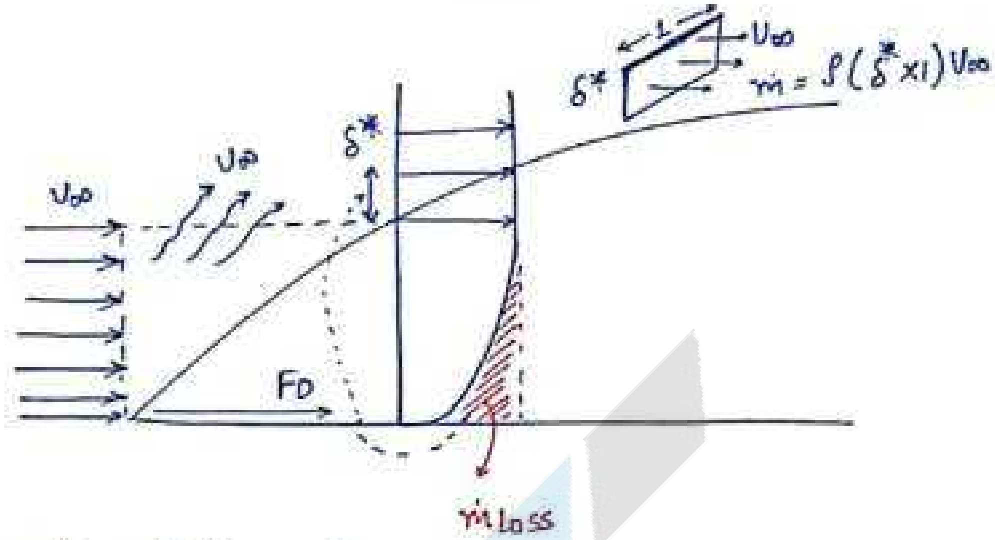
$$\dot{m}_{DC} = \rho \int_0^\delta U_0 \left[2\left(y/\delta\right) - \left(y/\delta\right)^2 dy \right]$$

$$\dot{m}_{DC} = \tfrac{2}{3} \rho U_0 \delta$$

$$\dot{m}_{BD} = \rho \delta U_0 - \tfrac{2}{3} \rho U_0 \delta$$

$$\dot m_{BD} = \tfrac{1}{3}\rho\,\delta\,U_0$$

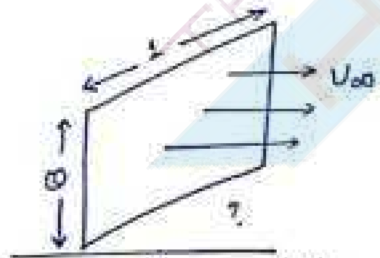
$$\frac{\dot m_{BD}}{\dot m_{AB}} = \frac{\tfrac{1}{3}\cancel{\rho\delta U_0}}{\cancel{\rho\delta U_0}} = \tfrac{1}{3}$$

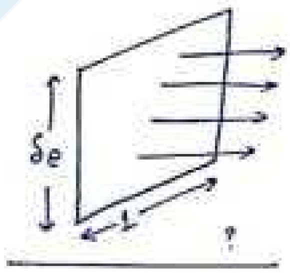


$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy \qquad \theta \rightarrow \text{momentum thickness}$$

$$\delta_e = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy \qquad \delta_e \rightarrow \text{energy thickness}$$





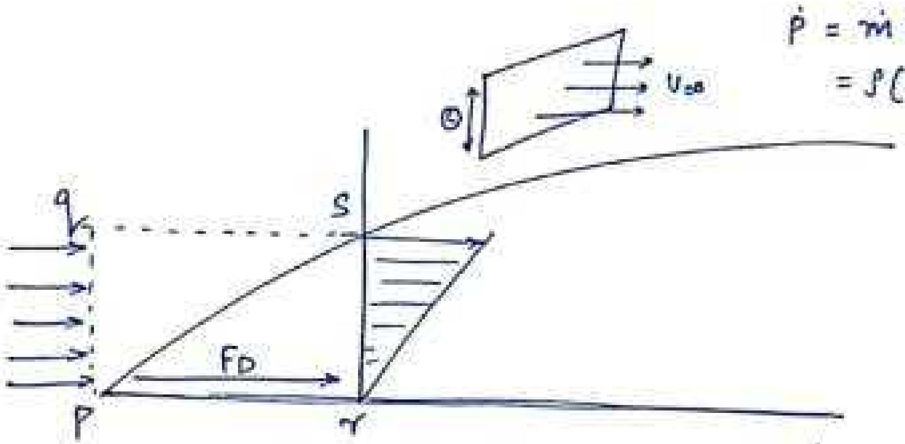
$$\dot P_{loss} = \dot m U_\infty = [\rho(\theta \times 1)U_\infty]U_\infty$$

$$F_D = \dot P_{loss} = \rho\,\theta\,U_\infty^2$$

$$KE = \tfrac{1}{2}\dot m U_\infty^2 = \tfrac{1}{2}[\rho(\delta_e \times 1)U_\infty]U_\infty^2$$

$$\dot{KE}_{loss} = \tfrac{1}{2}\rho\,\delta_e\,U_\infty^3$$

BB 264 Q5.22 (5.23) → profile → linear



$\dot{P} = \dot{m}\, U_\infty$

$= \rho(\theta \times 1)U_\infty \cdot U_\infty$

$\theta = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$

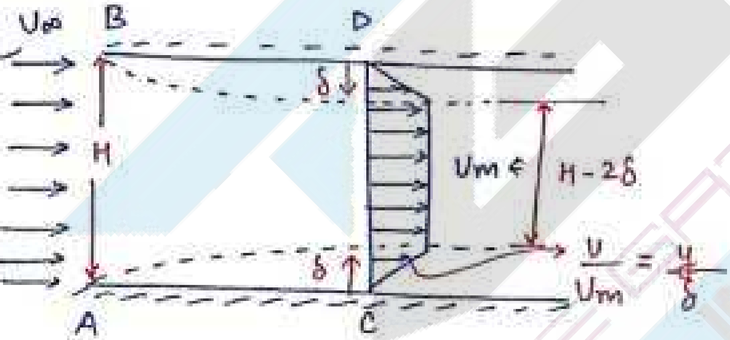
$\theta = \delta/6$

$\dot{P}_{loss} = \rho \frac{\delta}{6} \times U_\infty^2$

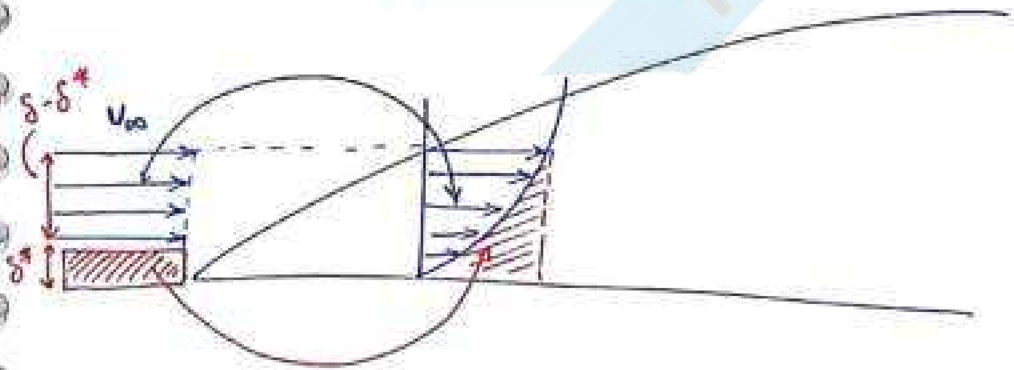
$F_D = \frac{1 \times 10 \times 10^{-3}}{6} \times 10^2$

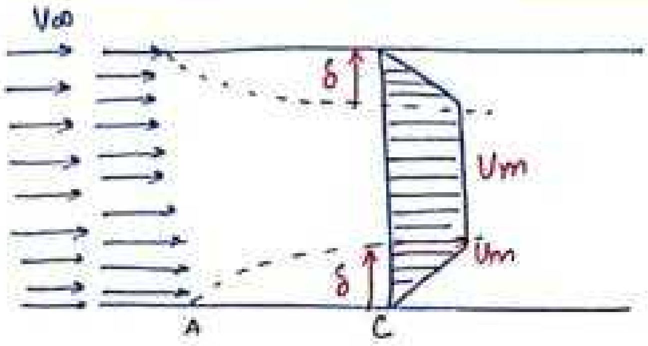
$F_D = 0.17\,N$

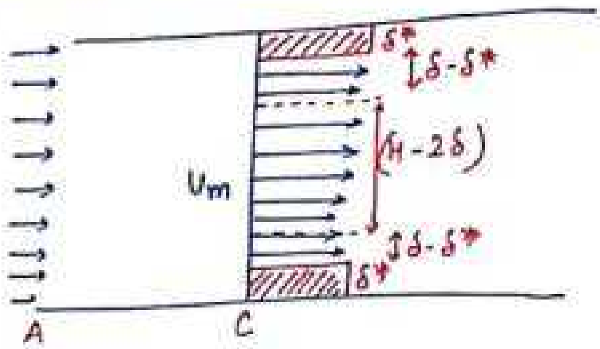
5.26



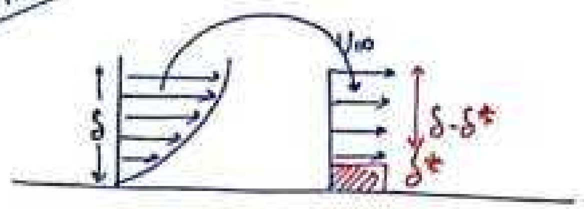
$\boxed{\dot{m}_{AB} = \dot{m}_{CD}}$





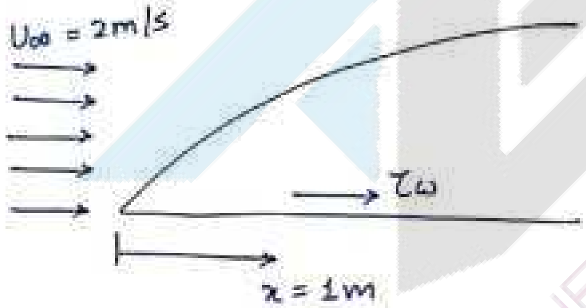


ow more



WB
(12) B.L.T.

$$\frac{U}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$



$\nu = 1.5 \times 10^{-5}$

$\rho = 1.23$

$$\delta = \frac{4.64\, x}{\sqrt{\frac{U_\infty x}{\nu}}}$$

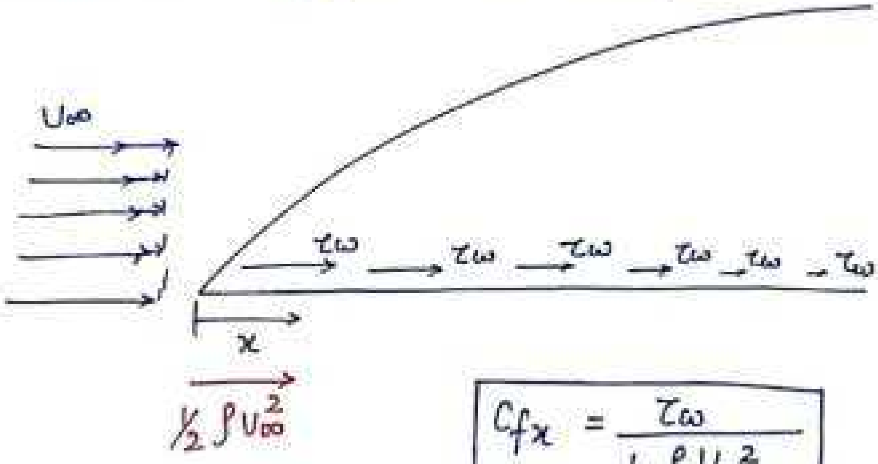
$$\delta = \frac{4.64 \times 1}{\sqrt{\frac{2\times 1}{1.5\times 10^{-5}}}}$$

$$\tau_\omega = \mu \frac{du}{dy}\bigg|_{y=0}$$

$$\tau_\omega = (\nu \cdot \rho)\, V_\infty \frac{3}{2} \times \frac{1}{\delta}$$

MOHIT CHOUKSEY

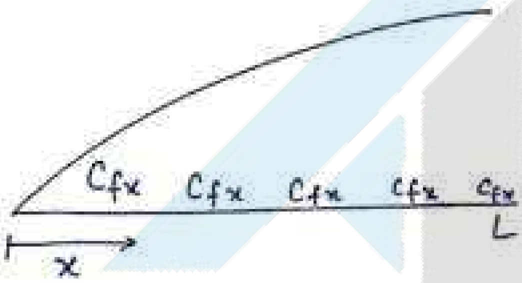
* Skin friction Coefficient and Average Drag coefficient :-



$$\boxed{C_{fx} = \frac{\tau_\omega}{\frac{1}{2}\rho U_\infty^2}}$$

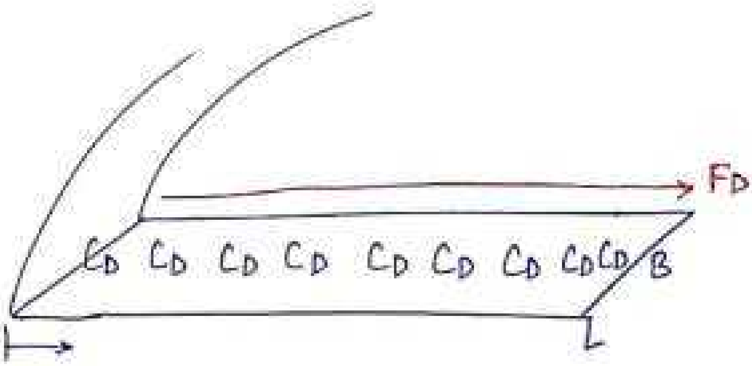
$$x\uparrow \rightarrow \tau_\omega \downarrow \rightarrow C_{fx}\downarrow$$

$$\boxed{\tau_\omega = C_{fx} \times \frac{1}{2}\rho U_\infty^2}$$



$$\boxed{C_D = \frac{1}{L}\int_0^L C_{fx}\,dx}$$

Average drag coefficient



$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\rho A U_\infty^2}}$$

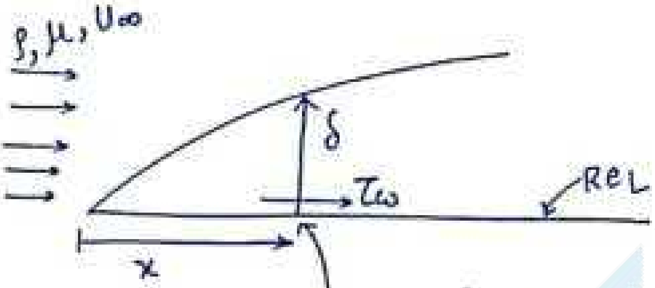
$$\boxed{F_D = C_D \frac{1}{2}\rho(B\times L)V_\infty^2}$$

BLT

Prandtl (1904)

Analytical solⁿ → Blasius (1908)

Flat plate, laminar flow



$Re_x = \frac{\rho U_\infty x}{\mu}$

$$\boxed{\delta = \frac{5x}{\sqrt{Re_x}}}$$

$$\boxed{C_D = \frac{1.328}{\sqrt{Re_L}}}$$

$$\boxed{C_{fx} = \frac{0.664}{\sqrt{Re_x}}}$$

$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$

(Approximate)

Vonkalman : Lam & turb

$$\boxed{\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_\infty^2}}$$

Flateplate laminar

$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

$$\boxed{\delta = \frac{4.64x}{\sqrt{Re_x}}}$$

$C_{fx} = \frac{0.646}{\sqrt{Re_x}}$

$C_D = \frac{1.292}{\sqrt{Re_L}}$

Turb

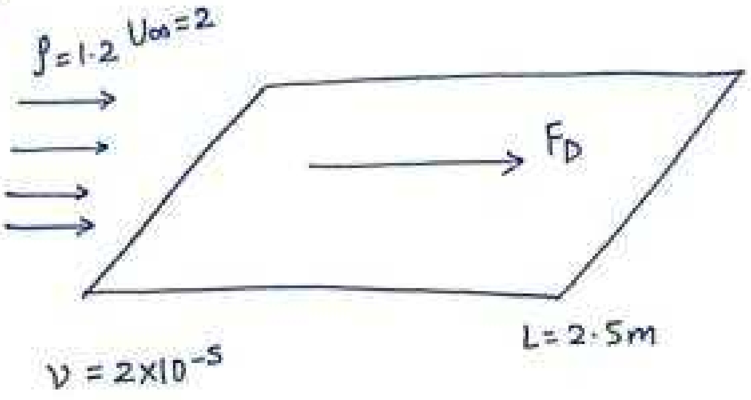
$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$

$\delta = \frac{0.375x}{Re_x^{1/5}}$

$C_{fx} = \frac{0.059}{Re_x^{1/5}}$

$C_D = \frac{0.074}{Re_L^{1/5}}$

Q27 WB



$$C_D = \frac{1.328}{\sqrt{Re_L}}$$

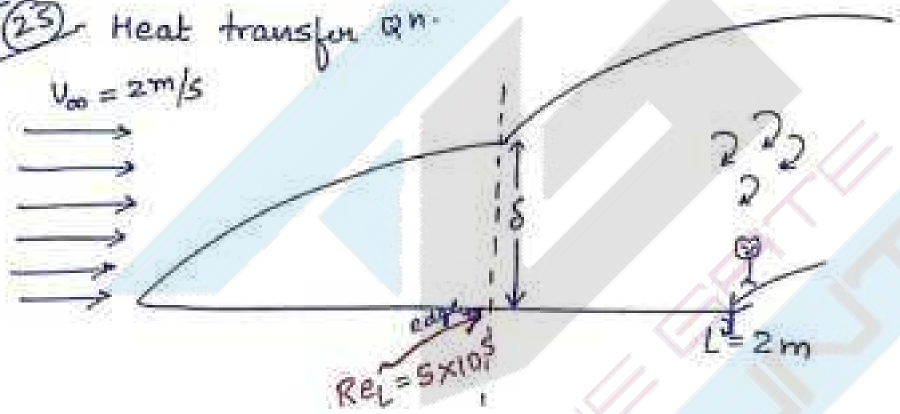
$$C_D = \frac{F_D}{\frac{1}{2}\rho A U_\infty^2}$$

$$F_D = \frac{1.328}{\sqrt{Re_L}} \times \frac{1}{2} \times \rho (B\times L) U_\infty^2$$

$$\sqrt{Re_L} \rightarrow \sqrt{\frac{U_\infty L}{\nu}}$$

0.159 Ans

Q22+3
=25 Heat transfer Qn.



at the edge of laminar

$$Re = \frac{U_\infty L}{\nu}$$

$$= \frac{2\times2}{10^{-6}} = 4\times10^6 > 5\times10^5$$

turb.

$$\delta = \frac{5x_c}{\sqrt{Re_{x_c}}}$$

$$\delta = \frac{5\ ?}{\sqrt{5\times10^5}}$$

$$\boxed{\delta = 1.76\ mm}$$

$$\frac{U_\infty x_c}{\nu} = 5\times10^5$$

$$2/10^{-6} = 5\times10^5$$

$$\boxed{x = 0.25}$$

Q30 Pg no. 51

$$\frac{U}{U_\infty} = \frac{y}{\delta}$$

$$\delta^* = \int_0^\delta \left(1 - y/\delta\right) dy$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta$$

$$= \delta/2$$

$$\theta = \int_0^\delta y/\delta \left[1 - y/\delta\right] dy$$

$$\theta = \left[\frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2}\right]$$

$$= \delta/6$$

$$\delta_e = \int_0^\delta y/\delta \left[1 - y^2/\delta^2\right] dy$$

$$\delta_e = \delta/4$$

$\boxed{\delta > \delta^* > \delta_e > \theta}$

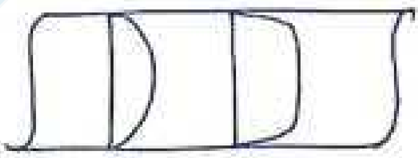
$\delta^* = \delta/2$

$\delta_e = \delta/4$

$\theta = \delta/6$

* Shape Factor :

$$\boxed{H = \left(\frac{\delta}{\theta}\right)^*}$$



Please don't see these qns → not in syllabus

Q18 (Stokes theorem)

Q21 (Pg 51)

Q15

Turbulent Pipe flow :-

$$\tau_{total} = \tau_v + \tau_e$$

$$= \mu \frac{du}{dy} + \eta \frac{d\bar{u}}{dy}$$

eddy viscosity

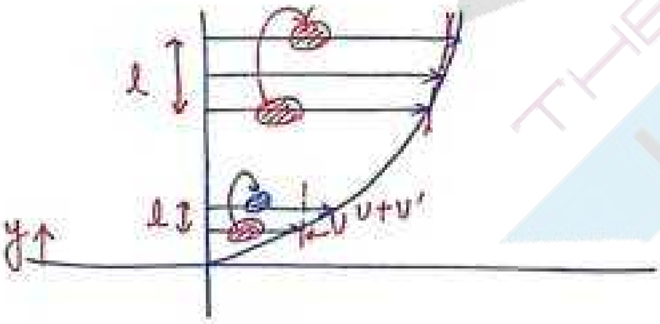
$$\boxed{\varepsilon = \frac{\eta}{\rho}}$$ Eddy kinematic viscosity

Reynold's :

$$\boxed{\tau_e = \rho u' v'}$$

Prandtl theory :

Define $u' = v'$ in terms of mixing length



$$\frac{d\bar{u}}{dy} = \frac{(\cancel{\bar{u}} + u') - \cancel{\bar{u}}}{l}$$

$$\boxed{u' = l \frac{d\bar{u}}{dy}}$$

$$\boxed{v' = l \frac{d\bar{u}}{dy}}$$

$$V_{rms} = \sqrt{\frac{\overline{u^2} + \overline{v^2} + \overline{w^2}}{3}}$$

$$\text{Degree of turbulence} = \sqrt{\frac{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}{3}} \quad m/s$$

<u>Vonkalman :-</u>

$l \propto y$

$\boxed{l = ky}$ → Universal Vonkalman constant

$l = 0.4y$

(pipe turb) $\boxed{k = 0.4}$

$$\tau_e = \rho\left(l\frac{du}{dy}\right)\left(l\frac{du}{dy}\right)$$

$$\sqrt{\frac{\tau_e}{\rho}} = 0.4y\frac{du}{dy}$$

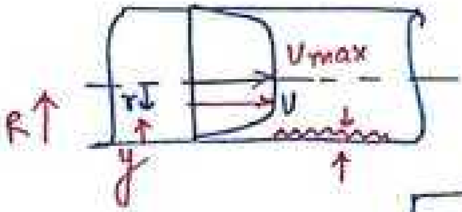
$\boxed{\tau_e \simeq \tau_w}$

$$\int dU = 2.5 V_* \int \frac{dy}{y}$$

$$U = 2.5 V_* \ln y + C$$

$$\boxed{U = 5.75 V_* \log_{10} y + C}$$

Turbulent flow



Smooth

Local velocity

$$\boxed{\frac{U}{V_*} = 5.5 + 5.75 \log_{10}\left(\frac{V_* y}{\nu}\right)}$$

$y = R \quad U = U_{max}$

$$U_m = \frac{2\int_0^R U r\,dr}{R^2}$$

$$\boxed{\frac{U_m}{V_*} = 1.75 + 5.75 \log_{10}\left(\frac{V_* R}{\nu}\right)}$$

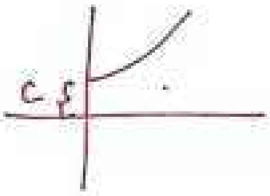
Roughness

+3

$$\boxed{\frac{U}{V_*} = 8.5 + 5.75 \log_{10}\left(\frac{y}{k}\right)}$$

+3

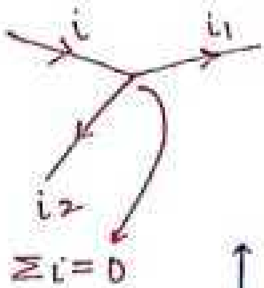
$$\boxed{\frac{U_m}{V_*} = 4.75 + 5.75 \log_{10}\frac{R}{k}}$$

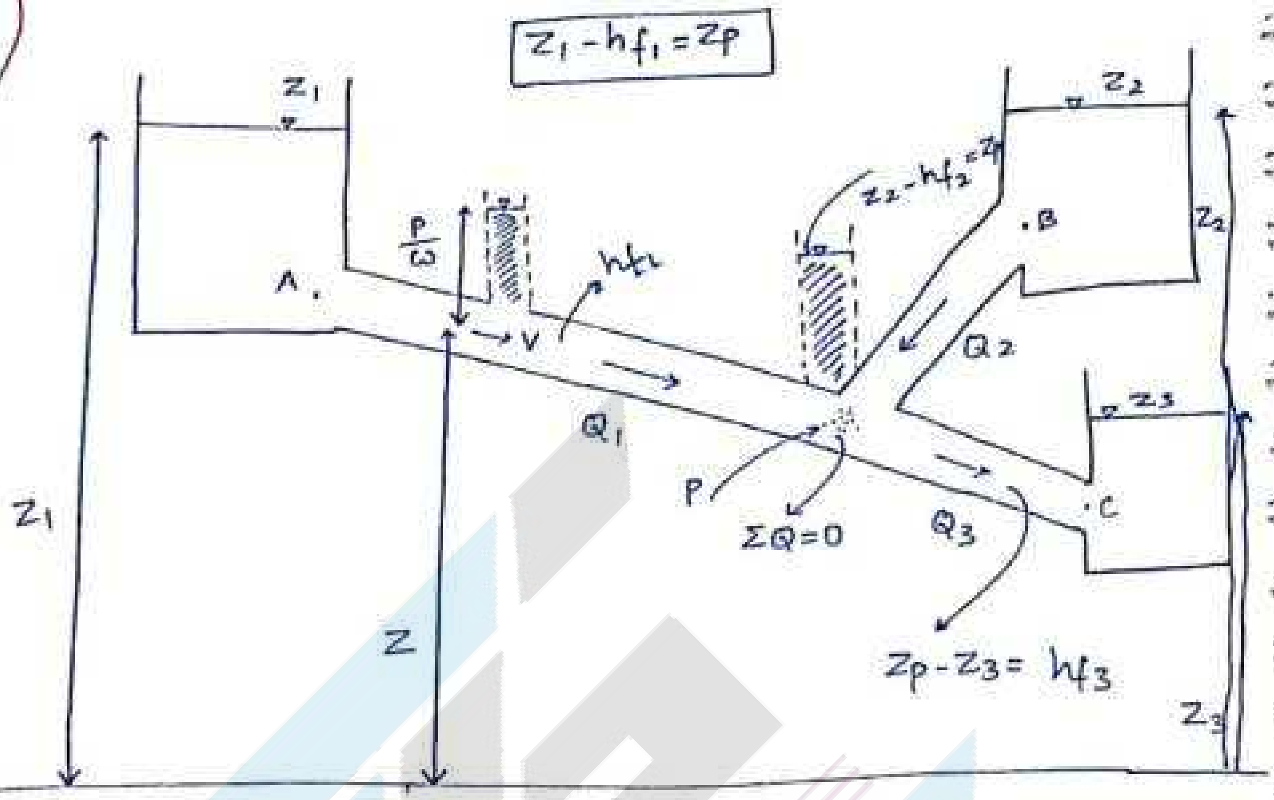


Mixing length :-

Average distance covered by bulk molecules to acquire the new environment velocity

Pipe Network





Pg 39 Q16

41

29 → $Q_2 = 0.0202\ m^3/s$

$Q_3 = 0.0802\ m^3/s$

$Z_3 = 32.288\ m$

Dimensional Analysis

$T \rightarrow f(l^a, g^b)$

[MLT]

$F_D = f[\rho^a, \mu^b, V_r^c, L^d]$

$\pi_1 = f(\pi \text{—})$.

$$\boxed{Re = \frac{\rho V_\infty L}{\mu}}$$

photocopy में - - - - - -

Pg 35

Q10

P	M
$\nu_p = 5\times10^{-4}$	$\nu_m = 10^{-6}\ m^2/s$

$\frac{L_p}{L_m} = \frac{1}{L_r}$

$L_r = \nu_r^{2/3}$

$L_r = \left[\left(\frac{\nu_m}{\nu_p}\right)\right]^{2/3} = \left[\frac{10^{-6}}{5\times10^{-4}}\right]^{2/3}$

$\frac{1}{L_r} = ?$

Shortcut

$Re_m = Re_p$ $\quad$ $Fr_m = Fr_p$

$\frac{Re_m}{Re_p} = 1$ $\quad$ $\frac{Fr_m}{Fr_p} = 1$

$Re_r = 1$ $\quad$ $Fr_r = 1$

$\frac{L_r}{\nu_r} = 1$ $\quad$ $\frac{V_r}{\sqrt{L_r}} = 1$

$\frac{\cancel{V_r} L_r}{\nu_r} = \frac{\cancel{V_r}}{L_r^{1/2}}$

$$\boxed{\nu_r = L_r^{3/2}}$$

* **Vortex Flow** :- vortex motion of fluid in which particles revolve around an axis which is fixed or curved.

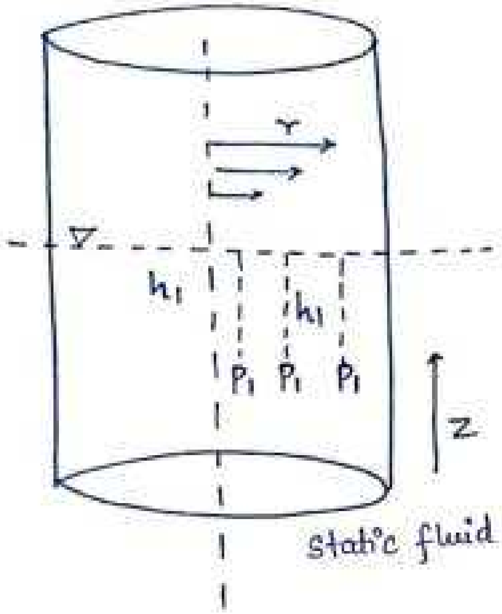
$P \neq f(r)$

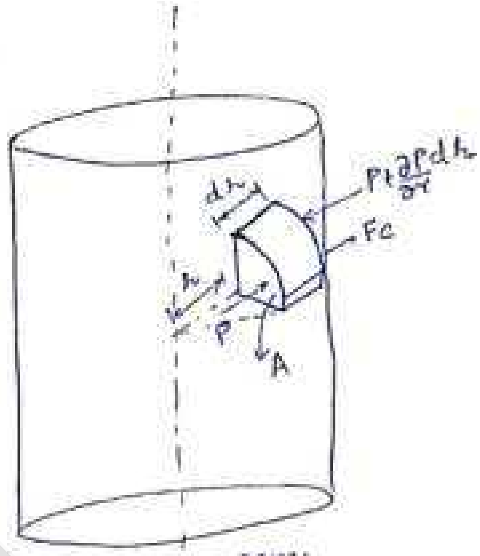
$P = f(z)$

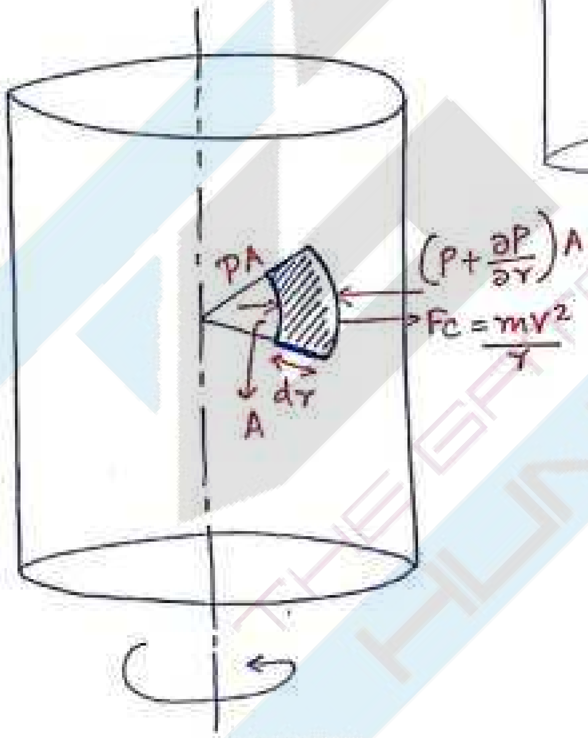
① Free vortex motion

② Forced vortex motion

$$\boxed{\frac{\partial P}{\partial z} = -\omega} \quad ①$$







eqm:

$\Sigma F_{net} = 0$

$(PA + F_c) - \left(P + \frac{\partial P}{\partial r} dr\right)A = 0$

$\frac{\partial P}{\partial r} dr\, A = \frac{\rho (A\, dr) V^2}{r}$

$$\boxed{\frac{\partial P}{\partial r} = \frac{\rho V^2}{r}} \quad ②$$

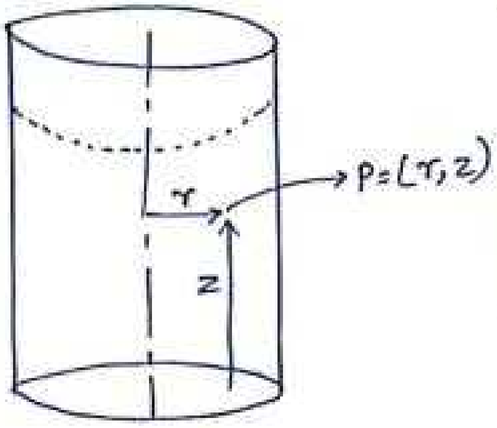
w.r.t radius, different variations of pressure

Eqm :-

$$\left(P + \frac{\partial P}{\partial r}\right)A = \cancel{PA} + \frac{mV^2}{r}$$

$$\frac{\partial P}{\partial r} \cdot \cancel{dr.A} = \frac{\rho(\cancel{A\,dr})V^2}{r}$$

$$\boxed{\partial P/\partial r = \frac{\rho V^2}{r}} \quad (ii)$$



vortex

Now, $P = f(r,z)$ $\left[\because \frac{\partial P}{\partial r} = \frac{\rho v^2}{r}\right]$ (243) here $P = f(r)$ also.

$$dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$$

$$\boxed{dP = \frac{\rho v^2}{r} dr - \omega dz}$$

→ Generalized Vortex motion Equation

General eqn. vortex

$$\boxed{dP = \frac{\rho v^2}{r} dr - \omega dz}$$

Free vortex motion ($v_\theta = c/r$)

$V_1 = \frac{c}{r_1}$; $V_2 = \frac{c}{r_2}$

$$\int_1^2 dP = \rho c^2 \int_1^2 \frac{dr}{r^3} - \rho g \int_1^2 dz$$

*** ***

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2}$$

Bernoullie's eqn (irrotational)

- ✓ irrotational
- ✓ $\Gamma = 0$, angular momentum is constant
- ✓ $Vr = C$

Forced vortex motion ($V_\theta = r\omega$)

$V_1 = r_1\omega$; $V_2 = r_2\omega$

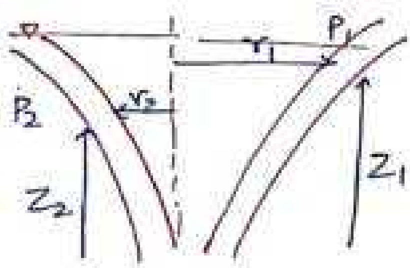
$$\int_1^2 dP = \rho\omega^2 \int_1^2 r\,dr - \rho g \int_1^2 dz$$

*** ***

$$\boxed{\frac{P_1}{\rho g} - \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} - \frac{V_2^2}{2g} + Z_2}$$

Forced Vortex motion eqn.

Vortex

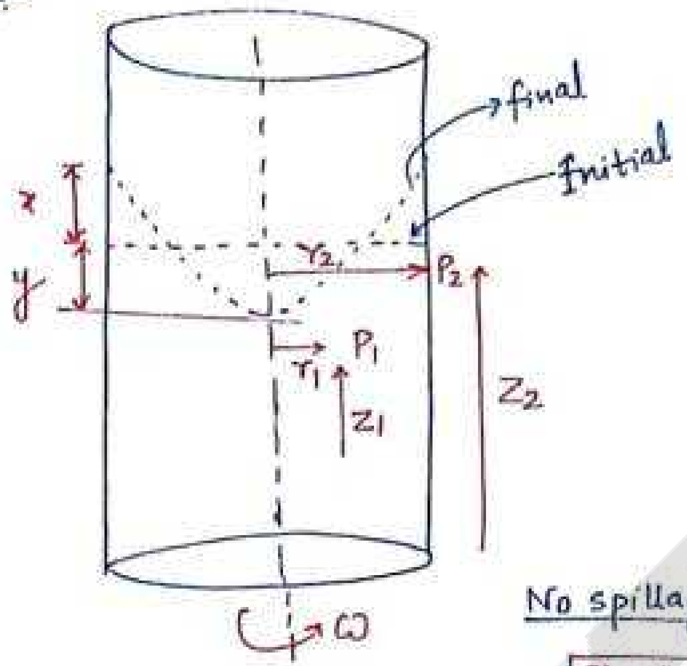


- ✓ Rotational
- ✓ particles also moves about their mass centers
- ✓ $\Gamma \neq 0$
- ✓ $V = r\omega$
- ✓ at the impeller of the pump, runner of turbine

Forced vortex के उदाहरण ↪ if Body moving with constant angular velocity.

- ✓ Flow in whirlpool
- ✓ Flow in washbasin

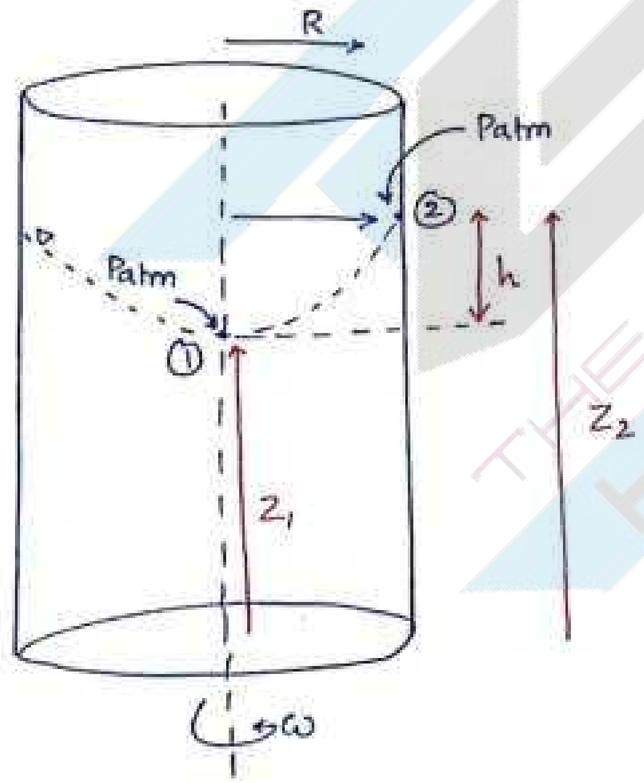
Q.



$$\frac{P_1}{\rho g} - \frac{r_1^2\omega^2}{2g} + z_1 = \frac{P_2}{\rho g} - \frac{r_2^2\omega^2}{2g} + z_2$$

No spillage :

$$\boxed{x = y}$$



$r_1 = 0$

$r_2 = R$

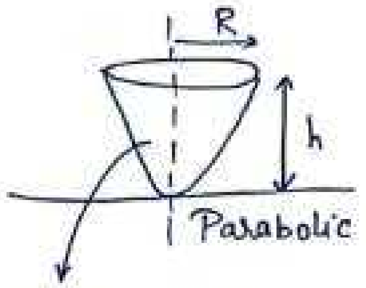
$P_1 = P_2 = P_{atm}$

$$0 - 0 + z_1 = 0 - \frac{R^2\omega^2}{2g} + z_2$$

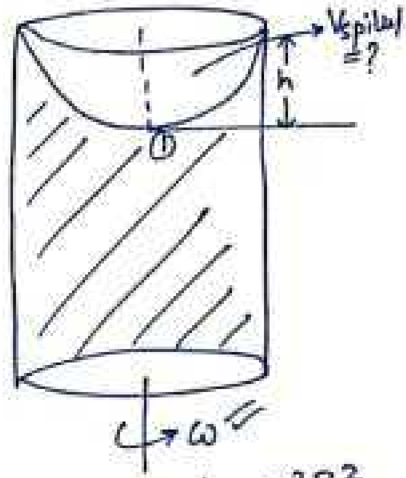
$$z_2 - z_1 = \frac{R^2\omega^2}{2g}$$

$$\boxed{h = \frac{\omega^2 R^2}{2g}}$$

$h \propto R^2$

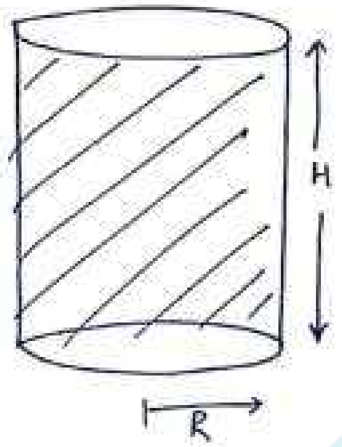


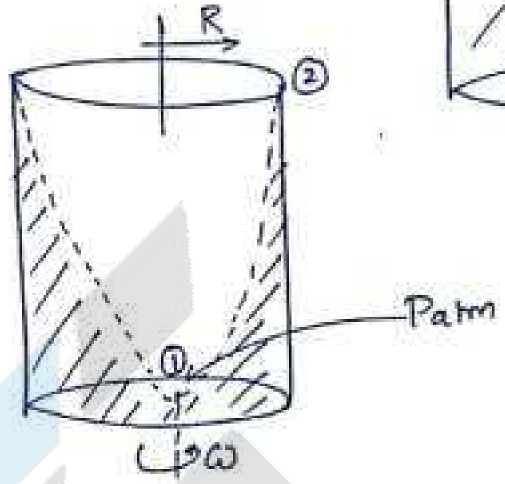
$V_P = \frac{1}{2}\pi R^2 h$



$h = \frac{\omega^2 R^2}{2g}$

$\boxed{V_P = \frac{1}{2}\pi R^2 h}$



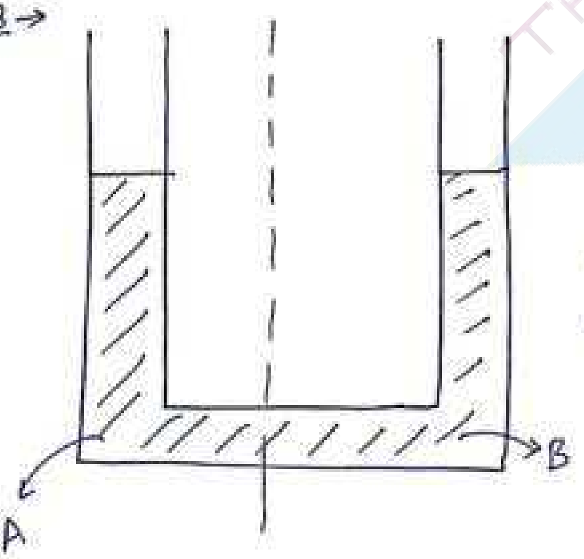


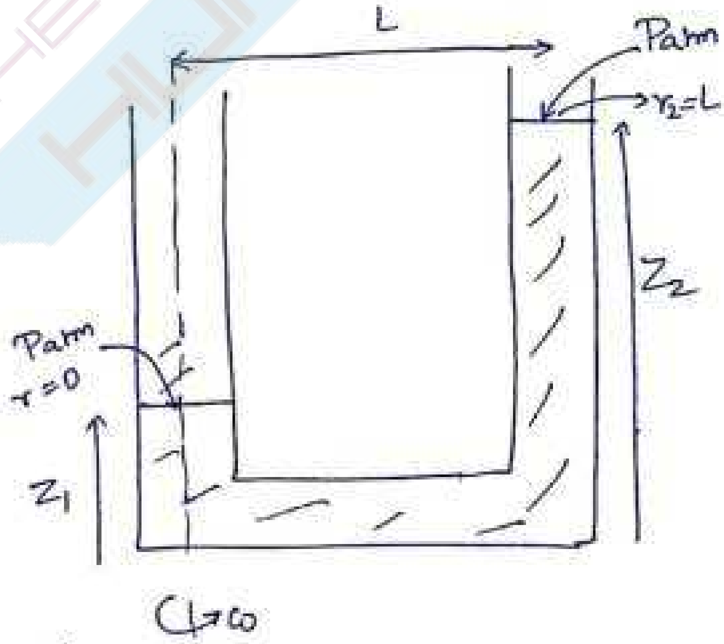
$H = \frac{\omega^2 R^2}{2g}$ $\quad \omega = \sqrt{\frac{2gH}{R^2}}$

26 → c

27 → a

28 →



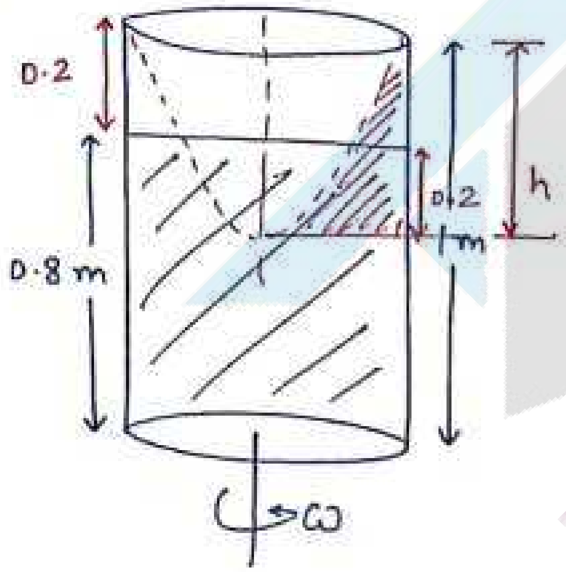


$\boxed{Z_2 - Z_1 = \frac{L^2 \omega^2}{2g}}$

$$\cancel{\frac{P_1}{\rho g}} - \cancel{\frac{r_1^2\omega^2}{2g}}^{\to 0} + Z_1 = \cancel{\frac{P_2}{\rho g}} - \frac{r_2^2 \omega^2}{2g}^{\to L} + Z_2$$

Remaining → 16 → c, 17 → b.

25



$$h = \frac{\omega^2 R^2}{2g}$$

$$0.4 = \frac{\omega^2 \times (0.15)^2}{2 \times 10}$$

$$\boxed{\omega = 18.65 \text{ rad/s}}$$